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AN ALGORITHM OF ROUTE PLANNING THROUGH THE SET OF NONCONVEX OBSTACLES

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Abstract

Navigation and routes planning for a group of vessels jointly moving in complex environment, including circumvention of seashore and islands, are important applications of computer-based decision-making support systems. A group of objects as an open complex system includes several levels of a hierarchy. The processes of control and decision-making involve an information-analytical soft and methodical maintenance. A significant stage of control is planning, that includes an a priori phase – determination routes for all objects in the group, coordinated in both time and directions of approach to the target set restricted by a set of obstacles. A discrete system of priorities reflects the relative preferences of the decision-maker and allows the choice of routes optimal in different senses. Problem statement may be given in terms of different mathematical models including control problems of formation motion, the theory of extreme networks and interval analysis. In the paper on the guaranteed approach to control of dynamic objects under uncertainty is used. Unified mathematical descriptions of seashores, routes of isolated objects and the whole group may be given in terms of hierarchic (i)-systems. That allows reconciling data on geography, environment, object characteristics, peculiarities of control systems and data transmission, including sources and causes of uncertainty. A movement of a group is described as an extremal problem of control and estimation. The results of computer simulation are considered. Similar models possess to explore economic aspects of application a heterogenic complex of surface and underwater vessels.

Key words

Hierarchical systems, optimal control and estimation, uncertainty, duality, control algorithms.

1 Introduction

Problems of formation control and of team behaviour simulation are significant items in modern agenda of optimal control and game theories [Kurzhanski, 2006]. These problems also include a wide range of statements modelling real-time interaction of teams with finite membership.

Research is motivated by applications in navigation of heterogenic complex of surface and underwater vessels [Murray-Smith, 2010; Kruglikov, 2005], where practical problems may be stated as geometrical ones. The crucial point for trajectory planning algorithms design is choice of adequate information structure. The requirement is to describe regularly complex circumstances with multiple obstacles and routes of team motion in case of objects limited in perception and conflicting interests. The application analysis shows that various interactions one may considered as individual or common movement provided with restricted resources, in particular, information; supplying, distributing and transforming under hierarchically organized control. Further, it is convenient to discuss the obstacles as islands.

The motion of objects one may treat in terms of system trajectories reflecting state, spatial, conceptual and organizational structures, results of observation and management. Objects may change their positions in accordance with consequent control decisions, stepwise formed on a symmetrical and discrete positional grid.

Hence, the common interaction one may split into multiple layers of respectively independent processes for couples of symmetrical systems. Note that logics of economic aspects of application a heterogenic complex of surface and underwater vessels is the same and decision-making tree techniques are evident example.

The problem under consideration in the paper is that of trajectory planning for the team of objects constrained in dynamics and overcoming obstacles under coordinated control.

Different mathematical approaches are known to be examined for trajectory design. The reference presented below does not even sketch the variety of ideas but outlines some restrictions to realize them.

Interval analysis techniques based on the description of the obstacles as the union of rectangles proves to be efficient. However, corresponding constructions depending on the level of investigation are not regularly hereditable that complicates the hierarchy analysis. Variation techniques developed in [Berdyshev, 2007] allow full scale route modelling based on contingent constructions. The significant manoeuvrability is required [Zenor and others, 2009].

A wide range of situations allows imbedding restrictions of object dynamics and observation in uncertainty of state position. Then the analysis of tubes of admissible trajectories is possible via guaranteed approach. In [Kruglikov, 2011] was shown that a priori choice of optimal tube and parameter approximation of obstacle are symmetrical problems.

The structural symmetry and duality of problem statements and solutions for conjugate systems are essential properties in the optimal control theory under uncertainty [Kurzhanski, 2009]. The duality properties of control problems with set-membership description of disturbances were investigated on the base of operator presentation. Cases of integral or extremal performance index were examined.

The possible applications of results on duality of guaranteed estimation and control problems for the route design and choice are considered below.

Presented models, based on a notion of hierarchical *(i)*-system, may provide the unified description of organizational structure, routes and geography. Then the problem of shoreline description is dual with the problem of admissible route design. Main constructions are given as finite combinations of chains with cylindrical branches. Complex shape of shoreline one may describe in advance in the form of sea graph and further the admissible routes may chose on the graph. The admissible route corresponds to the tube of trajectories, sections of which involve accumulated errors.

2. Problem Statement

Control algorithms, providing coordinated motions, are based on a separation property for problems of guaranteed control and estimation [Kruglikov, 2011].

2.1. Main assumptions

Suppose that object motions one may describe as trajectories reflecting state, conceptual and organizational structures, results of observation and control. Objects may change their positions in accordance with consequent decisions stepwise formed in accordance with positions on discrete grid. Hence, a common interaction one may split into multiple layers of relatively independent processes for pairs of symmetrical systems. To describe adequately features of such problems mathematical notions have to satisfy a number of key assumptions. Among the most important cases is one connected with the internal information model, describing data available, and constructed via inverse scheme. That reflects shifts of perception with the centre on the image of system.

The base for our discussion forms the approach of the theory of guaranteed control and estimation for systems with uncertainty in cases where uncertainties in dynamics and location of object are possible to imbed in description of space state.

Main idea is to separate from the very beginning the problem of description of the obstacles and the route choice one. Rather evident supposition for practitioners of navigation is not so natural for the statement of optimal control problems.

2.2 Separation of ensured problems of control and estimation

Duality of extremal problems means the symmetry in the problem statement and solutions for conjugate systems. Throughout the paper the symmetrical operator representation of extremal a priori problems stated for linear systems with unknown in advance parameters is used. The following notations are used below. $\mathbf{B}(X,Y)$ is the set of linear bounded operators mapping X in Y; $R\# = R^1 \cup \{-\infty,+\infty\}$. The symbols o,* mean a superposition and conjunction respectively. A scalar product in a Hilbert space X is denoted by $<\cdot,\cdot>_X$. E_Y is the unity operator in Y, $E_Y : Y \to Y$.

Suppose that ϕ is a closed functional. Elements κ and ν of corresponding Hilbert spaces $\Psi, Z\#$ satisfy the linear system

$$\kappa=F\eta+G_0\nu;\ \nu=U\zeta,\ \kappa\in Y,\ \nu\in Z\#;\ \zeta=A(U)\eta=A_0\eta+B\ \nu;$$

and may be interpreted as realizations of an observed signal and control.

The control procedure U, $U\in\mathbf{U}$, is fixed before a performance of the system (1) with an uncertain parameter η, $\eta\in X$, is started. Then the problem of a priori design of control acting on the base of incomplete or imperfect observations one may state as the following.

Problem 2.1. Find an operator U^*, $U^*\in\mathbf{U}$, satisfying the condition

$$-\infty < SUP_X \{ \Phi(U\_*)\} = MIN_U\ SUP_X \{\Phi(U)\} < +\infty,$$

where $\Phi(U)=\phi(F\eta + G_0 \text{ o } U \text{ o } A(U)\xi) - \theta(\xi)$.

A functional θ describes the quality restrictions on uncertain parameter η. If θ is defined by $\theta(\eta) = \delta(\eta|W)$, where $\delta(\cdot|W)$ is an indicator function for a convex weakly compact set W, $W\subseteq X$, then problem 2.1 may be interpreted as an a priori problem of ensured control and/or estimation.

Similar problems are investigated in the H_∞ -theory, where $\phi(\zeta) =<\zeta,\zeta>_Z$ and $\theta(\eta) = \gamma^2 < \eta,\eta>_X$, $\gamma > 0$.

The assumptions on operators B and U, $U \in \mathbf{U}$, mean that mappings $\Psi_Y(U)$, $\Psi_{Z\#}(U)$ are homeomorphisms;

$\Psi_Y(U)=E_Y - B \circ U$, $\Psi_Y(U): Y \to Y$;

$\Psi_{Z\#}(U)=E_{Z\#}-U_o \circ B$, $\Psi_{Z\#}(U): Z \to Z$.

Hence, for every $U \in \mathbf{U}$ the equalities

$U \circ \Psi_Y^{-1}(U)= \Psi_{Z\#}^{-1}(U) \circ U$, $G_0 \circ U \circ A(U)=G(U) \circ U \circ A_0$, holds. Here $G(U) = G_0 \circ \Psi_{Z\#}^{-1}(U)$.

Moreover, a singleton correspondence exists between sets $\mathbf{U}$ and $\mathbf{U}_0$. It is defined by equivalent expressions

$\mathbf{U}_0= \{U_0 = U \circ \Psi_Y^{-1}(U) = \Psi_{Z\#}^{-1}(U) \circ U \mid U \in \mathbf{U}\}$;

$\mathbf{U}=\{U=U_0 \circ (E_Y+B \circ U_0)^{-1}=(E_{Z\#}+U_0 \circ B)^{-1} \circ U_0 \mid U_0 \in \mathbf{U}_0\}$.

Suppose that the following properties hold.

i) Sets $\boldsymbol{G0},\boldsymbol{S0} \subseteq \boldsymbol{B}(V)$ present the initial information for description of islands and sea. Here $V \subseteq R^2$ and $\boldsymbol{B}(V)$ is a power set. Subsets $G0=\cup\{G \mid G \in \boldsymbol{G0}\}$, $S0=\cup\{S \mid S \in \boldsymbol{S0}\}$ are nonintersecting; $G0 \cap S0=\varnothing$, $V=G0 \cup S0$.

ii) Suppose that a chain with linear branches L_k approximates an admissible trajectory. The inequality (1) restricts manoeuvrability of the object

$$|L_i| \geq 2RR[\mathrm{Tan}(\alpha_I/2)]+ lmin. \qquad (1)$$

Here α_i, RR are angle and radius of return, $lmin$ is a length of the minimal straight branch. Accumulated errors of dynamics RR and position observation R^* are estimated by parameter $Rmax$; $Rmax=Max\{R^*, RR\}$. Then a tube of possible trajectories may be constructed as a chain of $(L_k,Rmax)$-cylindrical branches.

iii) Quality indices ψ_K, ψ_L, ψ_E evaluate to multiplicity of turns over trajectory, length and efficiency correspondently.

3 The basic algorithm design

The paper deals with axiomatic description of hierarchical systems, the choice of adequate statements of optimization problems under uncertainty for such systems, the study of analogues of the basic structural properties (duality, separation) that are obtained earlier for the control and estimation problems in the operator form. The mathematical formalism is motivated by applied research, including the simulation of control of objects team motion and processes of decision making in modernization of high-tech engineering enterprises.

Estimation problem in the case under consideration consists of a priori preparation of initial data. A solution forms a dual description situation.

3.1 The notion of hierarchical (*i*)-system

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Definition 1. The triple $(i)CG= (i)\{\boldsymbol{X},\boldsymbol{P},\boldsymbol{Q}\}$ is called a hierarchical (i)-system if the following components are included.

1.1) *Topological region* $\boldsymbol{X} = \{X|Sc\}$. Here $X \subseteq V$ and a polar coordinate system $Lc= \{cc,rc| \boldsymbol{ec}\}$ corresponds to a fixed point cc and a given vector $\boldsymbol{ec}$.

1.2) *Graph of organizational structure* $\boldsymbol{P} =\{P, \mathrm{P}\}$. Here P lists $(i\text{-}1)$-systems presenting vertices $P=\{CG.m=\{\boldsymbol{X}|\boldsymbol{P},\boldsymbol{Q}\}.m\}$ and a binary relation $\mathrm{P}=\{(C,C^*)\} \subseteq P \times P$ reflects a structure.

1.3) *Positional approximation*. $\boldsymbol{Q}=\{Q,\mathrm{Q}\}$; $Q=\{q=L_k\}$ is a list of links q, Lc-ordered by index θ : $\mathrm{Q} \to \mathrm{N}$, $\theta(q)=k$; $\theta(Q)= K=\{1,...,k\} \subseteq \mathbf{N}$. The border of the level (i) is an ordered subset of $(Rmax, L_k)$-branches.

Following parameter values $[i,kc,K,ok]$ evaluate basic properties of hierarchical (i)-system $C_{kc,K}= \{\boldsymbol{X}|\boldsymbol{P},\boldsymbol{Q}\}$.

(1) Level (i) corresponds to the scale $is=[rc/Rmax]$; $is=2^i$, corresponding to the scaling of nautical charts.

$$[0 \leq i \leq I]: 0 \sim \varepsilon^* \leq \varepsilon^* 2^i \leq 2Rmax,$$

where $I=[\mathrm{Log}_2\, 2Rmax/\varepsilon^*]$.

$[0 \leq i^* \leq I^*]: LMI=\mathrm{MIN}\{lmin,2Rmax\}$,

$LMA=\mathrm{MAX}\{Lmax,2rc0\}$

$[LMI\ 2^{i^*} \leq LMA]$, where $I^*=[\mathrm{Log}_2\, LMA/LMI]$.

(2) Power $kc=m(Q)$ is the quantity of actual relations.

(3) Multiplicity of vertices $K=m(P)$.

The typology of (i)-systems includes components, subsystems such as chain $CS_{2,K}$ with cylindrical branches, arc, orbit, admissible route, star, net.

3.2 The circumstances description

The Definition above allows to formulate operations over (i)-system.

(+) Combination of standard routes $CS_{2,K}$ via smooth closure of (i)- links, $qL(j)$.

$$CS_{2,K}=(+|k \in K)\ CS_{2,1}.$$

(-) Decomposition, extraction subsystems $C_j=\{X|L,S\}_j$; $X=\cup X_j$, $X_j \cap X_i=\varnothing$.

A hierarchical (i)-system $CG=\{XG|\boldsymbol{PG},\boldsymbol{QG}\}$ provides the construction and obstacles as a combination of islands.

1.G) $(i)\boldsymbol{XG} =(i)\{XG|Sc\}$, where $XG \cap G0 \neq \varnothing$ and $Lc=\{cc,rc|\boldsymbol{ec}\}$ is the internal polar coordinate system.

2.G) $(i)\ \boldsymbol{PG}=(i)\{PG, \mathrm{PG}\}$.

The list of $(i\text{-}1)$-systems presents islands

$(i)PG=\{(i\text{-}1)CG.m=\{\boldsymbol{XG}|\boldsymbol{PG},\boldsymbol{QG}\}.m\}$, and a binary relation $\mathrm{PG} \subseteq PG \times PG$ reflects a structure.

$$\mathrm{PG}=\{(CG,CG^*)|\Pi G(\pi G(CG), \pi G(CG^*) < \delta\},$$

where $\pi G(CG) \leq \pi G(CG^*)$. An injective mapping $\pi G: PG \to \mathbf{N}$, $\pi G(CG)=m$; gives an ordering of vertices with respect to Sc. $\pi G(PG)=MG=\{1,..., mG\}$, $mG \in \mathbf{N}$.

Let $(i)\Pi G=\{\delta_+(ki,kj)\}$, $(i)\Pi G{:}\mathbf{N}\times\mathbf{N}\to R^1$; be a matrix scaling object couples $(i)CG.ki,(i)CG.kj$, $(i)\Pi G(ki,kj) = \delta_+(ki,kj)$; $\lambda s(ki,kj) = |c.ki\text{-}c.kj|\geq 0$.

*H*ere $\delta_-(ki,kj)$ evaluates X-separability of $(i)CG.ki$ and $(i)CG.kj$. $\varepsilon s(ki,kj) = \lambda s(ki,kj) - (rc.ki+rc.kj) > 0$.

3.G) $\boldsymbol{Q}G=(i)\{QG, \text{QG}\}$; $(i)QG=\{qG\}$ is a Sc-ordered list $L_k=\{\, l,r|\ \boldsymbol{l}\}_k$ of links, chains forming a border of the main system and $(i)qG'$ are borders of nearby systems of the same level (i). A border of level (i) is subset of $(Rmax, L_k)$-cylindrical branches, ordered by $\theta G{:}QG\to\mathbf{N}$, $\theta G(qG)=k$; $\theta G(QG)=KG=\{1,\ldots, kG\}\subseteq\mathbf{N}$.

3.3 The admissible route tubes description

A hierarchical (i)-system $CS=\{\boldsymbol{XS}|\boldsymbol{PS},\boldsymbol{QS}\}$ gives construction for sea and admissible route tubes.

1.S) *Region* $\boldsymbol{XS} = \{\, XS|SS\}$, where $XS\cap S0\neq\varnothing$. An external polar coordinate system $SS=\{cS,rS|\boldsymbol{e}0\}$ corresponding to the point cS and zero direction $\boldsymbol{e}0$ is fixed.

2.S) Graph $\boldsymbol{P}\text{S}=\{PS,\text{PS}\}$ provides the *network description*. Here $PS=\{CSj=\{\, \boldsymbol{XS}|\boldsymbol{PS},\boldsymbol{QS}\}j\}$ is a list of $(i\text{-}1)$- systems, a structure PS is a binary relation.

3.S) *position*: $\boldsymbol{Q}S$ presents a SS- ordered list $L_j=\{\, l,r|\ \boldsymbol{l}\}(j)$ of links, describing bays, straits, fjords.

(QS/P) $\forall\ QS_2=\langle L1,L2\rangle\in QS\ \ \exists\, CS|QS_2=$

$\{\, P_K=\{S(k)|1\leq k\leq K\}\}=\Phi(QS_2)\subseteq P$:

$(S(k),S(k+1))\in P\ \&\ LO\in QS(1)\ \&\ LF\in QS(K)\ \forall 1\leq k<K$.

(P.3) $QS_2=\{L1,L2|(L1,L2)\in QS\}\Rightarrow[(l1,lD)*(l2,lD)<0 \vee L2=l1l2+L1\_\text{-}]$,

where $lD=\mathbf{A}(\phi/2)l12$, $l12=l1l2/\,d(l1,l2)$.

$(i)XS= S(cs, rs\sim rg+2Rmax){:}\ X'\sim XG\ XS= XG(+)SS\sim X$.

3.4 Problems under consideration

The admissible route tube description as a chain of branches.

Problem 3.1. [Control.A|W|I]. Under assumptions given above, find an admissible route optimal in accordance with the corresponding criterion:

(Con.A) minimal multiplicity K;

(Con.W) minimal route length $L(K)$,

(Con.I) efficient route MIN$\{L(K)/K\}$.

Definition 3. $(i)CG\in\ \boldsymbol{B}G$ and $(is)CS^*\in\boldsymbol{BS}$ are conjugated hierarchical (i)-systems, if $\forall\ (is)CS\in\boldsymbol{BS}$: $XG\subseteq XS$.

$(i)CS^*$ is the convex hull, depending on (i)-G, QG.

Symmetry of hierarchical conjugated (i)-systems based on the directed links couples may be given. The notion of dual (i)-systems $CG=\{\boldsymbol{XG}|\boldsymbol{QG},\boldsymbol{PG}\}$, and $CS(G;K, Rmax) = \{\, \boldsymbol{XS}|\ \boldsymbol{QS}, \boldsymbol{PS}\}$.

Input information. (1) V $=(0)XS=\{c0,rc0|c0\}$: $V=G\cup S$; $G\in B(U)$, $\text{Int}(G)\neq\varnothing$; connected set $S\in\tau U$.

(2) List of closed sets $PG=\{CG0.ki|\ ki\leq kk\}$ defining finite cover.

Values $Rmax,rr,\alpha^*,lmin$ and object qualities (range constraints $Lmax$, positioning accuracy and so on).

Output information. Set **BS** of hierarchical (i)-systems $(i)CS=(i)\{\boldsymbol{XS},\boldsymbol{PS},\boldsymbol{QS}\}$ presenting dual network of passages. Control problem solution gives a series of routes admissible for objects and forms tube of trajectories. Sets of possible routes.

3.5 Solution visualisation

Basic T-procedure allows determine set of trajectory tubes different in main properties but coordinated in starting and target positions (Fig.1). Obstacles are presented by unconnected set. Operations over hierarchical (i)-systems form a wide range of nonconvex obstacles. Below are presented solutions for obstacles with star structure.

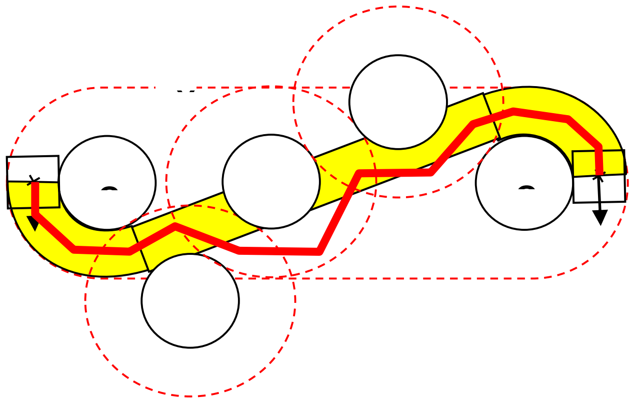

Figure. 1. A tube going trough the nonconvex set

An important method of solving particular problems is the procedure of routing around obstacles having a non-convex topology of a star. Software implementation of route planning around a star with an arbitrary number of beams is based on the application of the external contour of the outer shell of the star and the T-procedure (developed previously).

The procedure provides an iterative selection of routes around obstacles $[IS]CU$, represented by a combination of sets with the topology of the star. Analysis is based on the structure of a hierarchical system $(0)CG.m=\{\boldsymbol{XG},\boldsymbol{PG},\ \boldsymbol{QG}\}.m$, that generates a family $\boldsymbol{P}\otimes=\{P\otimes,\Pi\otimes\}$: $P\otimes=\{(0)CG.m|m\in I\otimes\}$, $(0)CG.m=\{XG,PG,QG\}$. Here $(0)\Pi\otimes$ is a matrix; $\boldsymbol{X}G.m=\{B.m|\boldsymbol{e}=\boldsymbol{e}U\}$, $\boldsymbol{P}G.m=\{\{B.m\},diag_1\{1\}\}$.

Planning of motion for a specified period of time, taking into account the discrete update. Simulation of avoiding a collision, provided that the information is limited to opponents hypotheses about organizational affiliation, objectives and areas of possible location of targets.

Table 1. Statements of particular problems for design of a route

Problem 1.0. The region description *[IS] CU* formalizing restrictions and obstacles
Input. ***V***, an absolute coordinate system *L0=(l0,r0|**l**0)*. Set *P0*, matrix of structure *Π0*. **Output.** *[IS]XU* $\subseteq \mathbf{R}^2$*,P⊗,Π⊗,Π⊕.* **Content.** Region *[IS]CU=[IS]{**XU**,∅,**BS**}. Here [IS]**BS** is a finite set of objects (i≤0)CG.m.*
Problem 0.1. *(auxiliary)* The region description *[IS#]CU* in case of fixed particular set *[IS#]XU*.
Input. *[IS]CU;* $XU \in \tau_{XU}$ *a convex set.* $XU \subseteq$ ***V*** **Output.*BG*,*BS*.** **Content.** *[Is]CU=[Is]{**XU**,**BG**,**BS**}.*
Problem 0.2. *(auxiliary)* The problem statement for the whole group
Input. *[IS]CU*. Couple Q_2*={LO,LF}* **Output.** *(A Pr)*: *{A Po}* **Content.** T-procedure.
Problem 2.0 Coordination of data on the problem and the associated constraints CQ_2*={ (A Pr)// <Si> [is]CU=[is]{**XU**,**BG**, **BS**}// {A Po}}*
Input. The problem statement $(Q_2, \Phi_2$*: λs#≤ Lmax*)*. **Output.** *[Is]XUE=E*(Q_2, Φ_2) ellipse. **Content.** *[⇒**3.0.1**] Corresponding region [Is]CUE=[Is]{**XU**,**BG**,**BS**}.* Zone description *σ={(θ,κ)}* for obstacles in accordance with region *[Is]CUE* and *[is#]CU0.m*. Obstacles are symmetrical in values of potential *κ*; (±).
Problem 2.1. Design of tubes of admissible routes
Input. CQ_2*={(A Pr)//<Si>//{A Po}}* **Output.** Set of tubes *OT(0;±;±∝)*. **Content.** Obstacle encircling algorithm based on T-procedure.
Problem 2.1. Opposite *[is]* refinement of the objects routes *PM*.
Input. *SU:{(A Pr)/<Si>/{A Po}} /*/{{A Po}*/<Si>*/(A Pr)*}* **Output.** A set ***M***(Q_2) of admissible routes. **Content.** Coordination of numbering of the outer shells and ways on base of directions ***l***, ***l****OF*. Calculation of the joints smooth conjugation of routes and containers.

4 Conclusion

Models presented above provide unified descriptions of organizational structure, trajectories and geography. Symmetry of hierarchical *(i)*-systems allows to describe complex shape of shoreline in advance as a solution of ensured estimation problem. Quality is evaluated by extremal induces. A priori procedures of control and estimation are descripted by non-anticipating operators. Then choice of trajectories one may interpret in terms of control problem. Separation property of ensured control/ estimation problems allows to split algorithmically procedures of coordinated control. Similar models may be used to explore economic aspects of application a heterogenic complex of surface and underwater vessels.

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