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CONTROL OF CHAOS IN DRIVEN FITZHUGH-NAGUMO CIRCUIT BY MEANS OF FILTERED FEEDBACK

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Abstract
We report on the feedback control of a periodically driven FitzHugh-Nagumo system displaying various dynamical regimes, regular and chaotic, including chaotic spiking. The feedback term is composed of a high-pass or an all-pass filter. We provide a characterization of the efficiency of the filters in controlling the dynamical state of the system, both experimentally (electronic circuit) and numerically (model equations). We demonstrate the better efficiency of the all-pass filter as we change the coupling strength of the feedback. Moreover, we show that both filters change the appearance of bifurcations, that is the main result of the controlling effect. However, all-pass filter revealed to have a stronger efficiency in the bifurcation control. Finally, we discuss the relation of the feedback method based on filters with the delayed feedback control scheme.

Key words
Control, electronic circuits, FitzHugh-Nagumo neurons, filters, bifurcation.

1 Introduction

The control of dynamical systems is a classical problem of engineering science. One method for stabilizing unstable periodic orbits is the Ott-Grebogi-Yorke (OGY) method [Ott, Grebogi and Yorke, 1990]. First the unstable low-period orbits that are embedded in the chaotic attractor are determined. After examination of these orbits, the one which guarantees the improved system performance is chosen. Then, small time-dependent parameter perturbations are applied to stabilize the already existing, but unstable, periodic orbit. This control method was for the first time successfully applied to control a parametrically driven magnetoelastic ribbon [Ditto, Rauseo and Spano, 1990]. Other successful experimental realizations of this method concerned many other different systems, e.g. in a chaotic Duffings oscillator [Chen, 1996] and a neural tissue [Schiff et al., 1994]. Another control method was introduced by Pyragas, the Delayed Feedback Control (DFC) method [Pyragas, 1992]. The method consists in applying to the system a term which is the difference between the current state of the system and its state one period in the past so that the control signal vanishes when the stabilization of the desired orbit is attained. For this reason the method is also called time-delay auto-synchronization, since the stabilization of the desired orbit occurs due to the synchronization between the current state of the system with its delayed one. The fact that the chaotic system is sensitive to small perturbations helps in controlling it. Thus by applying a perturbative, small control term one can obtain the desired effect. The advantage of this method over OGY is that it only requires a priori the knowledge of the period of the system, and does not require any previous computation. The DFC method has been applied to many systems, to mention the control of rhythmic activity in neural ensemble [Rosenblum, et al., 2006]. Recently, it has been shown that phase perturbations can be used to control different aspects of the dynamics of the driven FitzHugh-Nagumo (FHN) system, that is, to tame or enhance the spiking activity as well to control chaos [Zambrano et al., 2008; Zambrano et al., 2010]. This control technique, called Phase Control of Chaos (PCC), considers the phase difference ϕ between the periodic driving and a small harmonic perturbation

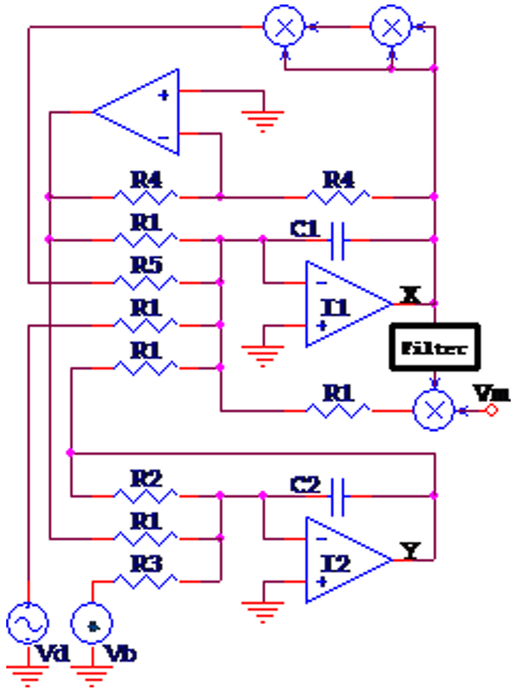

Figure 1. Electronic scheme of the FHN oscillator, with the feedback loop in the dashed green box. I1 and I2 are operational amplifiers, R resistors and C capacitors. V_d is the sinusoidal driving signal, V_b is the bias. V_x and V_y are the voltage signals representing the x and y variables respectively. The feedback loop consists in a passive filter (HPF or APF) whose output $F_{1,2}$ modulated by the signal V_m and attenuated by R_1. $R_1 = 100$K,$R_2 = 125$K,$R_3 = 143$K, $R_4 = 1$K, $R_5 = 48$K, $C_1 = 3$n, $C_2 = 37.5$n.

added to it. The phase control scheme relies on an appropriate selection of ϕ once the perturbation frequency and its amplitude are selected, in order to lead the system to the desired dynamical regime. This limitation, which is typical of the non-feedback methods to control chaos, can be overcome by a suitable filter inserted in a feedback loop. This feedback control offers several advantages because once the frequency response of the system is known, a filter can be implemented and inserted in a feedback loop without a preliminary scanning of the phase difference. A simple approach to this selective control is the wash-out filter [Hassouneh, Lee and Abed, 2004] easily implemented by a High Pass Filter (HPF) in a feedback loop with a particularly selected cutoff frequency. When such a filter is inserted in a feedback loop the high frequencies are rejected. The control methods based on filters have been used to control the appearance of bifurcations, what include delaying the onset of an inherent bifurcation, stabilizing a bifurcated solution or branch, and other objectives, described in details in Ref. [Chen, Moiola and Wang, 2002]. In particular, this strategy has been used to stabilize the fixed points of the systems.

The well-known FHN model [FitzHugh, 1961] used to demonstrate the phase control, has been the subject of intensive studies in engineering science [Lemasson, Lemasson and Moulins, 1995] due to its relative simplicity for analytical study, numerical simulations, and electronic realization. In spite of its simplicity, many mechanisms responsible for generating complex patterns in the neural information processes [Scott, 2002] are contained in it. Chaotic spiking has been theoretically investigated in FHN model by introducing a third slow refractory variable [Doi and Kumagai, 2005]. The chaotic behaviour can be also observed in the periodically driven FHN system [Pankratova, Polovinkin and Spagnolo, 2005]. Such chaotic mixed-mode oscillations (chaotic small amplitude oscillations interrupted by large spikes) were first discovered in the Belousov-Zhabotinsky reaction [Petrov et al, 1992] and, since then, have been frequently observed in experiments and models of chemical and biological rhythms [Iglesias et al, 2011] as well as in optical systems [Marino et al., 2011]. Moreover, the FHN model has been widely used to investigate the effects of noise. Stochastic resonance effects, that optimize information transmission have been investigated in Ref. [Stocks and Mannella, 2001] and noise enhanced stability phenomena due to the correlation time of the noise have been investigated in Ref. [Valenti, Augello and Spagnolo, 2008].

In this paper we study the control method based on filters in FHN model equations as well as in the experimental implementation of these equations in the electronic circuit. We consider HPF in the feedback with the cut-off frequency slightly above the spiking frequency components. We show that the control may be improved by using an All Pass Filter (APF). To assess this problem we consider the coupling constant and the cut-off frequency of the feedback term as the control parameters and look at the bifurcation diagrams for different values of the driving signal amplitude. We found that it is possible to delay the onset of bifurcation of the system, and we demonstrate the better performances of the APF with respect to the HPF. Finally, we show that the feedback method based on filters may be expressed in terms of the delayed variable what reveals the relation with the delayed feedback control scheme.

2 Model and its implementation

The driven FHN is ruled by the following equations:

$$\begin{aligned} \dot{x} &= x - y - x^3/3 + V_d \\ \dot{y} &= 0.08(x - 0.8y + 0.7) \end{aligned} \tag{1}$$

where $x(t)$ is the voltage variable, $y(t)$ is the recovery variable and $V_d = A\sin(2\pi\nu t)$ is an external driving term with amplitude A and frequency $\nu = 1/T$. The circuit implementing the model in Eq. 2 is shown in Fig. 1 (excluding filters). It consists of an electronic analog simulator implemented by commercial semiconductor devices. The spiking regime, consisting

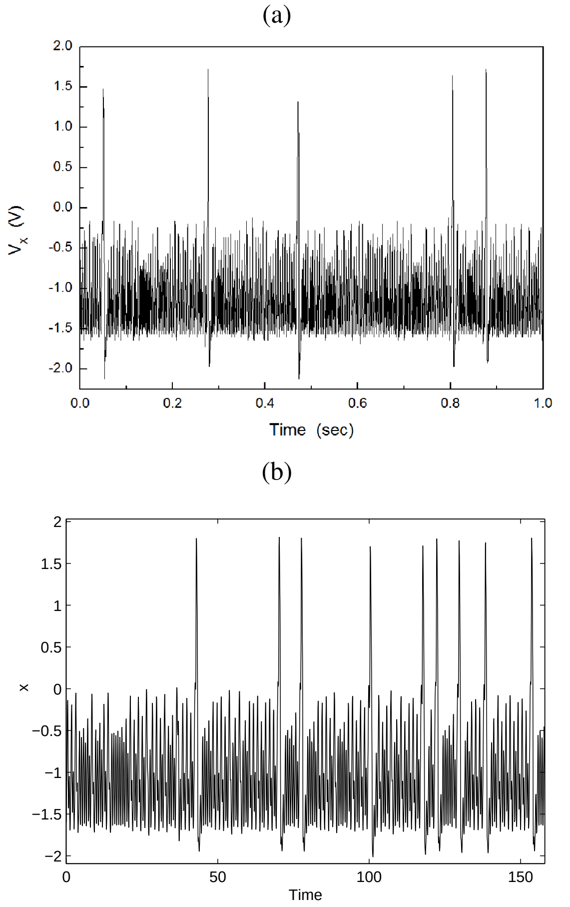

Figure 2. Time series of the x variable in open loop conditions obtained (a) experimentally for $A = 570$mV and $T = 2.5$ms, and (b) numerically for $A = 0.4267$ and $T = 0.667$.

of chaotic small amplitude oscillations interrupted by large spikes measured experimentally and numerically, is shown in Fig. 2 (a) and (b), respectively. A small amplitude chaotic regime is reached approximately at $A = 0.57$ V (for the experiment) and $A = 0.4$ (for the model). We consider two filters through which the output is fed back to the system, that is, a HPF and an APF. Eq. 2 are modified as follows:

$$\dot{x} = x - y - x^3/3 + V_d - \alpha F_{1,2}$$
$$\dot{y} = 0.08(x - 0.8y + 0.7) \qquad (2)$$

where $F_{1,2}$ are the control signals for HPF and APF, respectively, and α is the coupling strength. The position of filters in the set up is shown by a box in Fig. 1. The dynamical evolution $F_1(t)$ of the HPF is ruled by the following first order differential equation:

$$\dot{F}_1 = \dot{x} - \frac{F_1}{\tau} \qquad (3)$$

where τ is the time constant. In the case of the APF the dynamical evolution of the control signal $F_2(t)$ is

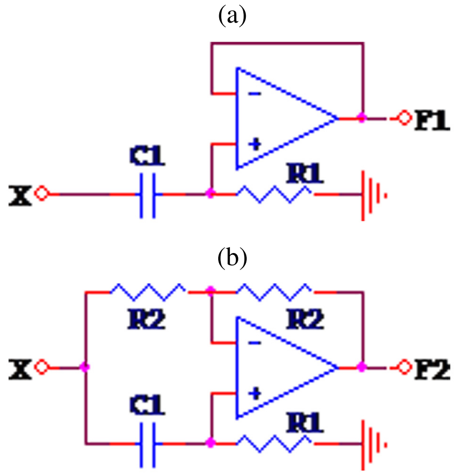

Figure 3. Electronic schemes of (a) HPF and (b) APF.

given by:

$$\dot{F}_2 = \dot{x} - \frac{x + F_2}{\tau} \qquad (4)$$

The electronic schemes for HPF and APF are shown in Fig. 3 (a) and (b), respectively. In both filters, the time constant $\tau = R_1 C_1$ is adjusted in order to choose a suitable cut-off frequency value.

3 Results

Modulation of parameter A, starting from the zero value, induces the period-doubling route to chaos with the sharp windows where chaotic spiking emerges, as shown in the bifurcation diagrams of Figs. 4 (a) and (c) for experimental circuit and model equations, respectively. When the HPF is introduced in the feedback loop, we observe a change of parameter value A at which the bifurcation onset from period-1 to period-2 occurs. In Figs. 4 (b) and (d) we marked by an arrow the shift in the bifurcation onset with respect to the bifurcation diagrams shown in Figs. 4 (a) and (c). We observed the same phenomena also in the case of APF used instead of HPF. In order to quantify the effects of filters on bifurcation onsets, we explore the parameters A, α and τ and estimate the corresponding dynamical regimes observed in the system. In Fig. 5 we show the parameter space $\alpha - A$ for a fixed value of τ, where different dynamical regimes: period-1, period-2, period-4, chaos and chaotic spiking, are marked with the scaled colors starting from the brightest (period-1) ending on the black one (chaotic spiking). The horizontal lowest part of the figures show the dynamical regimes for $\alpha = 0$. In the case of HPF (Fig. 5 (a)), as α is increased, at a critical value $\alpha = \alpha_c$ the chaotic and period-N dynamics disappears completely and is replaced by a period-1 regime. Also, a slight delay in the bifurcation onset is observed (notice a slight skew to the right of the region surrounded by period-1 area).

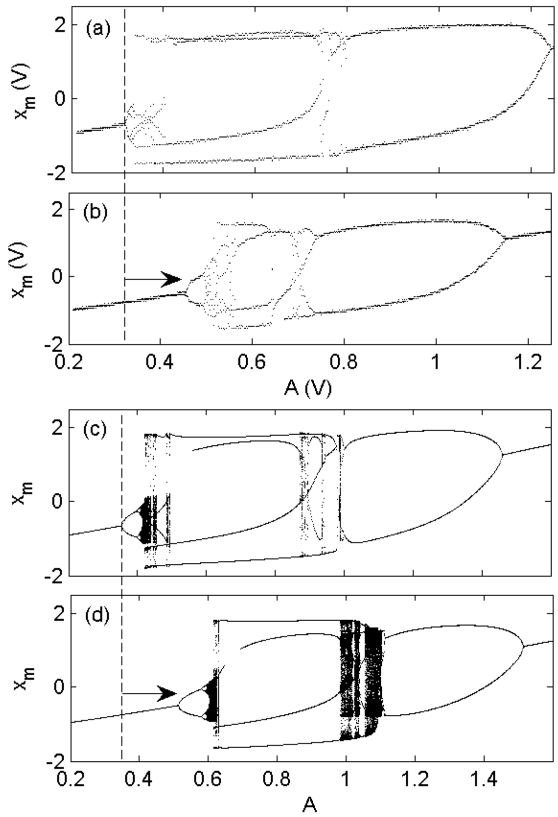

Figure 4. Bifurcation diagrams for the varying amplitude A of the driving force in the open loop conditions ((a) the circuit and (c) the model) and with a non-zero HPF feedback ((b) the circuit and (d) the model). The parameters are: (a) $\alpha = 0V, T = 2.5$ms; (b) $\alpha = 0.4V, T = 2.5$ms; (c) $\alpha = 0, T = 0.667$; (d) $\alpha = 0.203, T = 0.667$. x_m denotes maxima of oscillations.

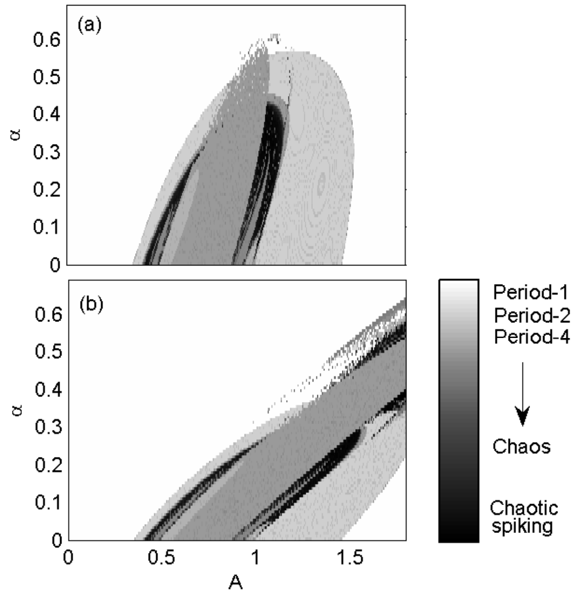

Figure 5. Dynamical regimes of the FHN system in the parameter space α - A in the case of: (a) HPF and (b) APF, calculated numerically with the use of the model equations for $T = 0.667$ and $\tau = 0.1$.

On the other hand, in the case of APF (Fig. 5(b)), the effect of a control has different characteristics. The delay in the bifurcation onset is larger. This delay allows to achieve control of chaos at smaller values of the coupling α, and thus showing the better performance of APF over HPF. The bifurcation onsets for the transition from period-1 to period-2 dynamics for HPF and APF has been estimated also experimentally as shown in Fig. 6 (a), and has been compared with the numerical results shown in Fig. 6 (b). The results obtained experimentally are in a good accordance with numerical data.

Modulation of the time constant of filters versus A at fixed value of α (see Fig. 7), reveals the highest variations at smaller values of τ, while for higher τ's the dynamical regimes remain relatively unchanged. The situation is the same, both, for HPF (Fig. 7 (a)) and APF (Fig. 7 (b)). The similar situation is observed in Fig. 8, where we calculated numerically the dynamical regimes of the system being initially in the chaotic spiking state (for fixed parameters of driving force), under the effect of HPF and APF in the feedback loop. Also in this case the highest variation in the dependence between the parameters α and τ occurs at smaller values of τ. This dependence is inverse and in approximation has a form $\tau = a/(\alpha - \alpha_0)$, where α_0 and a are constant parameters. The transition to period-1 regime in the case of the HPF (see Fig. 8 (a)) occurs at the highest value of α_0 than in the case of APF (see Fig. 8 (b)). In the view of the above analysis we can conclude that APF is more efficient than HPF in controlling the onset of bifurcations in the FHN system.

The inverse dependence of the constant time τ on coupling α resembles the stability condition for the simple delayed differential equation of the form $\dot{x} = -\alpha x(t-\tau)$ [Murray, 1993]. In this case the critical values of α and τ, at which the stability of solutions changes, are defined by an equation $\tau = 1/\alpha$. Let us expand in a Taylor series the variable $x(t-\tau)$ delayed in time:

$$x(t-\tau) \approx x(t) - \tau\dot{x}(t) + \mathcal{O}(\tau^2) \qquad (5)$$

where we consider only zero and first order terms. Now, we rewrite Eq. 3 taking into account the above approximation:

$$\begin{aligned} \dot{F_1} &= \dot{x} - \frac{F_1}{\tau} \\ &= -\tau^{-1}(F_1 - \tau\dot{x}) \\ &= -\tau^{-1}(F_1 - x + x - \tau\dot{x}) \\ &\approx -\tau^{-1}(F_1 + x(t-\tau) - x) \qquad (6) \end{aligned}$$

where we introduced $-x + x$ in order to retrieve the Taylor expansion sum. From the above equation we

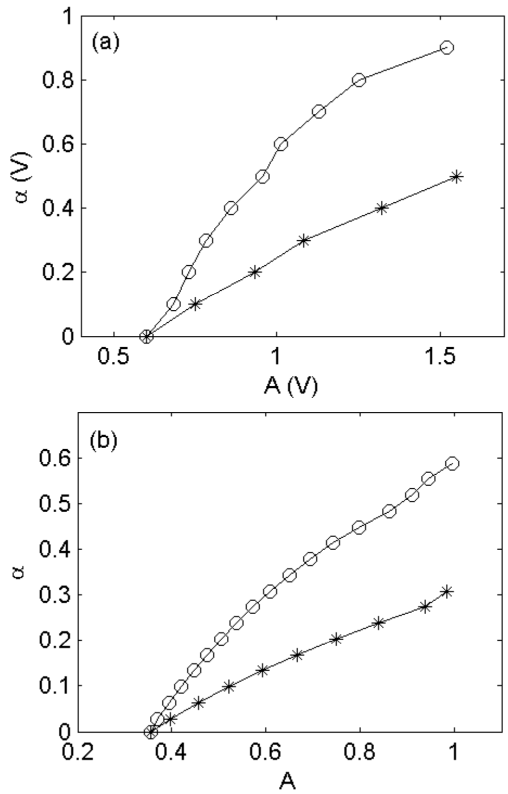

Figure 6. Bifurcation points for the transition from period-1 to period-2 for the FHN system in the parameter space α - A in the case of (a) experiment for $T = 2.5$ms and $\tau = 0.0152$s, and (b) model for $T = 0.667$ and $\tau = 0.1$. The two curves in each panel are for HPF (o) and APF (*).

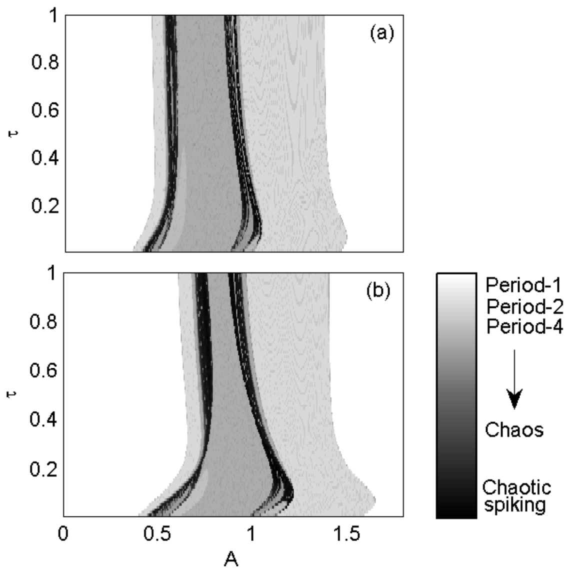

Figure 7. Dynamical regimes of the FHN system in the parameter space τ - A in the case of (a) HPF, and (b) APF, calculated numerically with the use of the model equations for $T = 0.667$ and $\alpha = 0.1$.

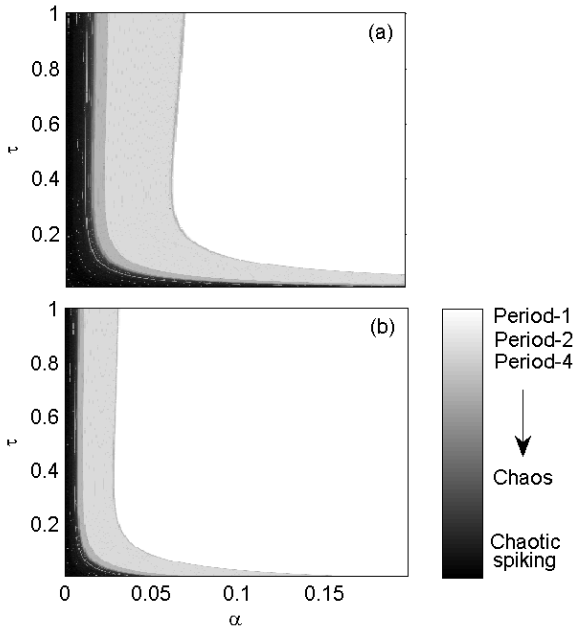

Figure 8. Dynamical regimes of the FHN system in the parameter space α- τ in the case of (a) HPF, and (b) APF, calculated numerically with the use of the model equations with parameters for the driving force $A = 0.4267$ and $T = 0.667$.

obtain the feedback term composed of the low-pass filtered difference between the delayed $x(t-\tau)$ and non-delayed $x(t)$ variables, that corresponds to a modified DFC method based on auto-synchronization between the present and the past state of the system. We apply the same analysis to Eq. 4:

$$\begin{aligned}\dot{F}_2 &= \dot{x} - \frac{x + F_2}{\tau} \\ &= -\tau^{-1}(F_2 + x - \tau\dot{x}) \\ &\approx -\tau^{-1}(F_2 + x(t-\tau)) \qquad (7)\end{aligned}$$

where we obtain the feedback term composed of the low-pass filtered delayed variable $x(t-\tau)$. Thus, the feedback method based on filters may be expressed in terms of the delayed variable that reveals its relation to DFC method. In our case however, the delayed variable is additionally low-pass filtered and does not enter the FHN system directly, at variance with the original DFC method.

4 Conclusion

In this paper we have examined a feedback method based on filters to control the dynamics of a periodically driven FHN displaying complex behaviour. We have shown that APF is more efficient than HPF in achieving suppression of chaos and spiking behavior. The control leading to a period-1 oscillations is achieved for a certain range of control parameters α and τ, which we have explored in details. The advantage of this feedback method with respect to the open loop PCC method is related to the fact, that it is not necessary to scan the relative phase up to when the desired behavior is obtained. From this point of view the method is similar to DFC method, where a priori knowledge of the period of the system and any previous computation are not required. In fact, we have shown, that for small values of time τ, our control scheme may be expressed in terms of a delayed variable, which does not enter directly the system but is low-pass filtered beforehand.

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