MINIMAX APPROACH PROBLEM WITH INCOMPLETE INFORMATION FOR TWO-LEVEL HIERARCHICAL NONLINEAR DISCRETE-TIME DYNAMICAL SYSTEM

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Abstract

We consider a discrete–time dynamical system consisting of three controllable objects. The motions of all objects are given by the corresponding vector nonlinear discrete–time recurrent vector relations, and control system for its has two levels: basic (first or I level) that is dominating and subordinate level (second or II level) and both have different criterions of functioning and united a priori by determined informational and control connections defined in advance. For the dynamical system in question, we propose a mathematical formalization in the form of solving a multistep problem of two-level hierarchical minimax program control over the terminal approach process with incomplete information and give a general scheme for its solving.

Key words

Hierarchical discrete-time dynamical system, minimax control.

1 Introduction

In this paper, we consider a discrete–time dynamical system consisting of three controllable objects. The motions of all objects are given by the corresponding vector nonlinear discrete–time recurrent vector relations, and control system for its has two levels: basic (first or I level) that is dominating and subordinate level (second or II level) and both have different criterions of functioning and united a priori by determined informational and control connections defined in advance. We assume that on a given integer-valued time interval the dynamics of each object I, I_1, and II in this system, controlled by the dominating player-pursuer P, subordinate player S, and player-evader E respectively, are given by the corresponding vector nonliner discrete-time recurrent relations. Phase states of all objects, all their controlling parameters, and all parameters in the considered system that have not been defined a priori are restricted to given constraints that have the form of a finite sets or convex and bounded polyhedrons in the corresponding finite-dimensional vector spaces. For the dynamical system in question, we propose a mathematical formalization in the form of solving a multistep problem of two-level hierarchical minimax program control over the terminal approach process with incomplete information and give a general scheme for its solving. Results obtained in this paper are based on the works [Krasovskii, 1968]–[Bazaraa and Shetty, 1979] and can be used for computer simulation and for designing of optimal digital controlling systems for actual technical, robotics, economic, and other multilevel control processes. Mathematical models of such systems are presented, for example, in [Chernousko, 1994]–[Tarbouriech and Garcia, 1997].

2 Object's dynamics in the two-level hierarchical control system

Suppose that on a given integer-valued time interval (simply interval) $\overline{0,\mathrm{T}} = \{0, 1, \cdots, \mathrm{T}\}$ $(\mathrm{T} > 0,\ \mathrm{T} \in \mathbf{N},$ where $\mathbf{N}$ is the set of all natural numbers) we consider a controlled multistep dynamical system that consist of three objects. Dynamics of object I, i.e., the basic object controlled by the dominating player P, the player-pursuer, is described by a vector nonlinear discrete-time recurrent relation of the form

$$y(t+1) = f(t, y(t), u(t), u^{(1)}(t)),\ y(0) = y_0; \quad (1)$$

dynamics of object I_1, auxiliary object controlled by the subordinate player S, is described with the following analogy relation:

$$y^{(1)}(t+1) = f^{(1)}(t, y^{(1)}(t), u(t), u^{(1)}(t)),$$

$$y^{(1)}(0) = y_0^{(1)}; \qquad (2)$$

dynamics of object II controlled by the player-evader E is described by a vector nonlinear discrete-time recurrent relation of the form

$$z(t+1) = F(t, z(t), v(t)),\ z(0) = z_0. \qquad (3)$$

Here $t \in \overline{0, \mathrm{T}-1}$; $y \in \mathbf{R}^r$, $y^{(1)} \in \mathbf{R}^{r_1}$, and $z \in \mathbf{R}^s$ are phase vectors of the objects I, I_1, and II respectively ($r, r_1, s \in \mathbf{N}$; for $n \in \mathbf{N}$, $\mathbf{R}^n$ is an n-dimensional Euclidean vector space of column vectors); $u(t) \in \mathbf{R}^p$, $u^{(1)}(t) \in \mathbf{R}^{p_1}$ and $v(t) \in \mathbf{R}^q$ are vectors of controlling influences (controls) of the players P, S, and E respectively, restricted to given constraints:

$$u(t) \in \mathrm{U}_1,\ u^{(1)}(t) \in \mathrm{U}_1^{(1)},\ v(t) \in \mathrm{V}_1; \qquad (4)$$

where the set U_1 is a finite set in the space $\mathbf{R}^p$, and $\mathrm{U}_1^{(1)}$, and V_1 are a convex sets in the spaces $\mathbf{R}^{p_1}$, and $\mathbf{R}^q$ respectively; for all fixed $t \in \overline{0, \mathrm{T}-1}$ the vector-function $f : \overline{0, \mathrm{T}-1} \times \mathbf{R}^r \times \mathbf{R}^p \times \mathbf{R}^{p_1} \longrightarrow \mathbf{R}^r$ is continuous by collection of the variables $(y, u, u^{(1)})$, and for all fixed convex set $Y \subset \mathbf{R}^r$ and vector $u \in \mathrm{U}_1$ the set $f(t, Y, u, \mathrm{U}_1^{(1)}) = \{f(t, y, u, u^{(1)}),\ y \in Y,\ u^{(1)} \in \mathrm{U}_1^{(1)}\}$ is convex; the vector-function $f^{(1)} : \overline{0, \mathrm{T}-1} \times \mathbf{R}^{r_1} \times \mathbf{R}^p \times \mathbf{R}^{p_1} \longrightarrow \mathbf{R}^{r_1}$ is continuous by collection of the variables $(y^{(1)}, u, u^{(1)})$, and for all fixed convex set $Y^{(1)} \subset \mathbf{R}^{r_1}$ and vector $u \in \mathrm{U}_1$ the set $f^{(1)}(t, Y^{(1)}, u, \mathrm{U}_1^{(1)}) = \{f^{(1)}(t, y^{(1)}, u, u^{(1)}),\ y^{(1)} \in Y^{(1)},\ u^{(1)} \in \mathrm{U}_1^{(1)}\}$ is convex; the vector-function $F : \overline{0, \mathrm{T}-1} \times \mathbf{R}^s \times \mathbf{R}^q \longrightarrow \mathbf{R}^s$ is continuous by collection of the variables (z, v), and for all fixed convex set $Z \subset \mathbf{R}^s$ the set $F(t, Z, \mathrm{V}_1) = \{F(t, z, v),\ z \in Z,\ v \in \mathrm{V}_1\}$ is convex.

We also assume that for all instant $t \in \overline{0, \mathrm{T}}$, phase vectors $y(t)$, $y^{(1)}(t)$, and $z(t)$ of the objects I, I_1, and II respectively, combined with initial conditions in relations (1)–(3), are restricted to given constraints

$$y(t) \in \mathrm{Y}_1,\ y^{(1)}(t) \in \mathrm{Y}_1^{(1)},\ z(t) \in \mathrm{Z}_1, \qquad (5)$$

where Y_1, $\mathrm{Y}_1^{(1)}$, and Z_1 are convex sets in the spaces $\mathbf{R}^r$, $\mathbf{R}^{r_1}$, and $\mathbf{R}^s$ respectively.

Players P, S, and E together form the basic (first or I) level that is dominating control level of the considered control process, and they are interested in the values of final (terminal) phase states of objects I, I_1, and II. Player S alone forms subordinate (second or II) level (that is subordinating to the control level I) in the considered control process, and he is interested in the final phase states of object I_1 only, states that depend on the behavior of player P.

3 Information conditions for players in the control systems

The control process in the discrete-time dynamical system (1)–(5) operates under the following informational conditions.

At every instant $\tau \in \overline{1, \mathrm{T}}$, player P measures and stores the values of the following parameters: $y(0) = y_0$, $y^{(1)}(0) = y_0^{(1)}$; $u(\cdot) = \{u(t)\}_{t \in \overline{0, \tau-1}}$; $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{0, \tau-1}}$ $\overline{0, \tau}$; realizations of the informational signal $\omega(\cdot) = \{\omega(t)\}_{t \in \overline{0, \tau}}$ ($\omega(t) \in \mathbf{R}^m$; $m \in \mathbf{N}, m \leq s$), whose values $\omega(t)$ ($\omega(0) = \omega_0$ is fixed) are formed, at every instant $t \in \overline{0, \tau}$, according to the following discrete-time relation (signal measurement equation):

$$\omega(t) = G(y(t))z(t) + S(t)\xi(t), \qquad (6)$$

where $\xi(t)$ is a measurement error satisfying to given constraint

$$\xi(t) \in \Xi_1. \qquad (7)$$

Here for all instant $t \in \overline{0, \mathrm{T}}$, and vectors $y(t) \in \mathbf{R}^r$ we assume that $G(y(t))$ and $S(t)$ are real matrices of sizes $(m \times s)$, and $(m \times l)$ respectively, and for all vectors $y(t) \in \mathbf{R}^r$ the rank of each matrix $G(y(t))$ equals m, that is, the dimension of vector ω; the set Ξ_1 is a convex polyhedron in the space $\mathbf{R}^l$ (here and below, by a convex polyhedron we mean a convex hull of a finite set of vectors in the corresponding Euclidean vector space).

During the control process, player P also knows the set $\mathrm{Z}(0) = \mathrm{Z}_0 \subseteq \mathrm{Z}_1$ of all possible states of the initial phase vector $z(0) = z_0$ of object II that is consistent (see [Krasovskii, 1968]) with the initial informational signal ω_0 and is a nonempty convex set in the space $\mathbf{R}^s$.

Suppose that player P also knows a formation principle of the controls $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{\tau, \mathrm{T}-1}}$ ($\forall\ t \in \overline{\tau, \mathrm{T}-1}$: $u^{(1)}(t) \in \mathrm{U}_1^{(1)}$) of player S on the interval $\overline{\tau, \mathrm{T}}$ which will be described below.

We also assume that player P knows a choice of realization of the control $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{\tau, \mathrm{T}-1}}$ ($\forall\ t \in \overline{\tau, \mathrm{T}-1}$: $u^{(1)}(t) \in \mathrm{U}_1^{(1)}$) of player S on any interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$, and he can use it for construct of his control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \mathrm{T}-1}}$ on this interval ($\forall\ t \in \overline{\tau, \mathrm{T}-1} : u(t) \in \mathrm{U}_1$).

At every instant $\tau \in \overline{1, \mathrm{T}}$, player S measures and stores the values of the following parameters: $y^{(1)}(0) = y_0^{(1)}$; $u^{(1)}(\cdot) = \{u^{(1)}(t)\}_{t \in \overline{0, \tau-1}}$.

Here we assume that at every instant $t \in \overline{0, \mathrm{T}-1}$ of the basic interval $\overline{0, \mathrm{T}}$ the choice of the control $u^{(1)}(t)$ of player S depend not only to constraint (4) but also depends on the choice of control $u(t) \in \mathrm{U}_1$ of player P based on the mapping Ψ_1 specified a priori. Namely, suppose that mapping Ψ_1 is such that

$$\Psi_1 :\ \mathrm{U}_1 \rightarrow\ \mathrm{comp}(\mathrm{U}_1^{(1)});\ \forall\ t \in \overline{0, \mathrm{T}-1},$$

$$\forall\, u(t) \in \mathrm{U}_1,\ u^{(1)}(t) \in \Psi_1(u(t)) \in \mathrm{comp}(\mathrm{U}_1^{(1)}),\quad (8)$$

where $\Psi_1(u(t))$ is a convex set in the space $\mathbf{R}^{p_1}$ for all $u(t) \in \mathrm{U}_1$. Therefore, the constraint (8) that we have introduced establishes that the behavior of player S explicitly depends on the behavior of player P.

We also assumed that in the considered control process for every instant $t \in \overline{0, \mathrm{T}}$ player P knows all relations and constraints (1)–(8), and player S knows relations and constraints (2), (4), (5), and (8).

In the considered process we assume that player E can have full information about all parameters of the discrete-time dynamical system (1)–(8), and about realizations of phase vectors for objects I, I_1, and II on the interval $\overline{0, \mathrm{T}}$.

4 Definitions and auxiliary properties for parameters of the dynamical system

For a strict mathematical formulation of a multistep problem of two-level hierarchical minimax program control over the terminal approach process with incomplete information for the discrete-time dynamical system (1)–(8) we introduce some definitions.

For a fixed number $k \in \mathbf{N}$ and an integer-valued time interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau \leq \vartheta)$, we denote by $\mathbf{S}_k(\overline{\tau, \vartheta})$ the metric space of functions $\varphi:\ \overline{\tau, \vartheta} \longrightarrow \mathbf{R}^k$ of an integer argument t where the metric ρ_k is defined as

$$\rho_k(\varphi_1(\cdot), \varphi_2(\cdot)) = \max_{t \in \overline{\tau, \vartheta}} \parallel \varphi_1(t) - \varphi_2(t) \parallel_k$$

$$((\varphi_1(\cdot), \varphi_2(\cdot)) \in \mathbf{S}_k(\overline{\tau, \vartheta}) \times \mathbf{S}_k(\overline{\tau, \vartheta}));$$

by $\mathrm{comp}(\mathbf{S}_k(\overline{\tau, \vartheta}))$ we denote the set of all nonempty and compact (in the sense of this metric) subsets of the space $\mathbf{S}_k(\overline{\tau, \vartheta})$. Here for $x \in \mathbf{R}^k$ in what follows $\parallel x \parallel_k$ denotes the Euclidean norm of vector x in the space $\mathbf{R}^k$.

Based on constraint (4) we define the set $\mathbf{U}(\overline{\tau, \vartheta}) \in \mathrm{comp}(\mathbf{S}_p(\overline{\tau, \vartheta - 1}))$ of all admissible program controls $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \vartheta - 1}}$ of player P on the interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \vartheta)$ with relation

$$\mathbf{U}(\overline{\tau, \vartheta}) = \{u(\cdot):\ u(\cdot) \in \mathbf{S}_p(\overline{\tau, \vartheta - 1}),$$

$$\forall\, t \in \overline{\tau, \vartheta - 1},\ u(t) \in \mathrm{U}_1\}.$$

Similarly, according to constraints (4)–(7) we define the following sets: $\mathbf{U}^{(1)}(\overline{\tau, \vartheta})$ is the set of all admissible program controls of player S; $\mathbf{V}(\overline{\tau, \vartheta})$ is the set of all admissible program controls of player E; $\Xi(\overline{\tau, \vartheta})$ is the set of all admissible program errors in the measurements of the informational signal modeled by relations (6), and (7); all sets together correspond to the interval $\overline{\tau, \vartheta}$.

Based on constraints (4), and (8), for a fixed admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$ of player P we define the set $\Psi_{\overline{\tau, \vartheta}}(u(\cdot)) \in \mathrm{comp}(\mathbf{S}_{p_1}(\overline{\tau, \vartheta - 1}))$ of all admissible program controls $u^{(1)}(\cdot) \in \mathbf{U}^{(1)}(\overline{\tau, \vartheta})$ of player S on the interval $\overline{\tau, \vartheta}$ corresponding to an admissible program control $u(\cdot)$ of player P, by relation

$$\Psi_{\overline{\tau, \vartheta}}(u(\cdot)) = \{u^{(1)}(\cdot):\ u^{(1)}(\cdot) \in \mathbf{U}^{(1)}(\overline{\tau, \vartheta}),$$

$$\forall\, t \in \overline{\tau, \vartheta - 1},\ u^{(1)}(t) \in \Psi_1(u(t))\}.$$

Now, by virtue of (1)–(7), we denote by $\hat{\Omega}(\overline{\tau, \vartheta}) \subset \mathbf{S}_m(\overline{\tau + 1, \vartheta})$ the set of all admissible program realizations of the informational signal $\omega(\cdot) = \{\omega(t)\}_{t \in \overline{\tau + 1, \vartheta}}$ on the interval $\overline{\tau, \vartheta}$.

Then for any instant $\tau \in \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$ let $\hat{\mathbf{W}}(\tau) = \{\tau\} \times \mathbf{R}^r \times \mathbf{R}^{r_1} \times \mathrm{comp}(\mathbf{R}^s)$ $(\hat{\mathbf{W}}(0) = \hat{\mathbf{W}}_0 = \{w(0) = w_0 :\ w_0 = \{0, y_0, y_0^{(1)}, Z_0\} \in \{0\} \times \mathbf{R}^r \times \mathbf{R}^{r_1} \times \mathrm{comp}(\mathbf{R}^s)\})$ is the set of all admissible τ-positions $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \overline{0, \mathrm{T}} \times \mathbf{R}^r \times \mathbf{R}^{r_1} \times \mathrm{comp}(\mathbf{R}^s)$ of player P in the discrete-time dynamical system (1)–(8) (where $Z(\tau)$ is the set of all admissible phase vectors $z(\tau) \in \mathbf{R}^s$ of object II at instant τ; $w(0) = w_0 = \{0, y_0, y_0^{(1)}, Z_0\}$; $w^*(0) = w_0^* = \{0, y_0, y_0^{(1)}, Z_0^*\}$), where, by virtue (6), (7), the nonempty set Z_0^* is defined by relation

$$Z_0^* = \{z_0:\ z_0 \in Z_0,\ \exists\, \xi_0 \in \Xi_1,$$

$$\omega_0 = G(y_0) z_0 + S(0)\xi_0\}.$$

And let $\hat{\mathbf{W}}^{(1)}(\tau) = \{\tau\} \times \mathbf{R}^{r_1}$ $(\hat{\mathbf{W}}^{(1)}(0) = \hat{\mathbf{W}}_0^{(1)} = \{w^{(1)}(0) = w_0^{(1)}:\ w_0^{(1)} = \{0, y_0^{(1)}\} \in \{0\} \times \mathbf{R}^{r_1}\})$ is the set of all admissible τ-positions $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \overline{0, \mathrm{T}} \times \mathbf{R}^{r_1}$ $(w^{(1)}(0) = w_0^{(1)} = \{0, y_0^{(1)}\})$ of player S in the discrete-time dynamical system (1)–(8).

Let for fixed time interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \vartheta)$, admissible according (1)–(7) realizations of the τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ of player P, controls $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$, and $u^{(1)}(\cdot) \in \Psi_{\overline{\tau, \vartheta}}(u(\cdot))$ of players P, and S respectively, and informational signal $\omega(\cdot) \in \hat{\Omega}(\overline{\tau, \vartheta})$ for player P (available for him on this interval), we denote by $\mathbf{R}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot))$ the set of all collections $(\tilde{z}(\tau), \tilde{v}(\cdot)) \in Z(\tau) \times \mathbf{V}(\overline{\tau, \vartheta})$ that are consistent (see [Krasovskii, 1968], [Kurzhanskii, 1977], [Shorikov, 1997]) with this information on the interval $\overline{\tau, \vartheta}$, which is defined by relation

$$\mathbf{R}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) = \{(\tilde{z}(\tau), \tilde{v}(\cdot)):$$

$$(\tilde{z}(\tau), \tilde{v}(\cdot)) \in Z(\tau) \times \mathbf{V}(\overline{\tau, \vartheta}),\ \forall\, t \in \overline{\tau+1, \vartheta},$$

$$\exists\, \xi_*(t) \in \Xi_1,\ \omega(t) = G(y(t))\tilde{z}(t) + S(t)\xi_*(t)$$

$$(y(t) = y_t(\overline{\tau, \vartheta}, y(\tau), u(\cdot), u^{(1)}(\cdot)),$$

$$\tilde{z}(t) = z_t(\overline{\tau, \vartheta}, \tilde{z}(\tau), \tilde{v}(\cdot)))\}, \qquad (9)$$

where by $y(t) = y_t(\overline{\tau, \vartheta}, y(\tau), u(\cdot), u^{(1)}(\cdot))$, and $\tilde{z}(t) = z_t(\overline{\tau, \vartheta}, \tilde{z}(\tau), \tilde{v}(\cdot))$ we have denoted the solutions of systems (1), and (3) respectively, at time moment $t \in \overline{\tau+1, \vartheta}$, generated by collections $(y(\tau), u(\cdot), u^{(1)}(\cdot))$, and $(\tilde{z}(\tau), \tilde{v}(\cdot))$ respectively.

We call the set

$$\mathbf{Z}^{(e)}_{\vartheta}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) =$$

$$\{z^{(e)}(\vartheta) :\ z^{(e)}(\vartheta) \in \mathbf{R}^s,$$

$$z^{(e)}(\vartheta) = z_{\vartheta}(\overline{\tau, \vartheta}, z(\tau), v(\cdot)), (z(\tau), v(\cdot)) \in$$

$$\in \mathbf{R}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot))\} \qquad (10)$$

the informational set of player P for the posterior minimax filtering process by relatively of player E and of object II (see [Krasovskii, 1968], [Kurzhanskii, 1977], [Shorikov, 1997]) in the discrete-time dynamical system (1)–(7) on the interval $\overline{\tau, \vartheta}$ corresponding to the instant ϑ and admissible collection $(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) \in \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau, \vartheta}) \times \Psi_{\overline{\tau,\vartheta}}(u(\cdot)) \times \hat{\Omega}(\overline{\tau, \vartheta})$. We must note that by definition this set is the set of all admissible realizations of the phase vector of object II at instant ϑ that are consistent with all information about the system in question that player P possesses on the interval $\overline{\tau, \vartheta}$ about a behavior of player E, and motion of object II.

For any fixed interval $\overline{\tau, \vartheta} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \vartheta)$, τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ of player P, and program controls $u(\cdot) \in \mathbf{U}(\overline{\tau, \vartheta})$, and $u^{(1)}(\cdot) \in \Psi_{\overline{\tau,\vartheta}}(u(\cdot))$ of players P and S respectively, we introduce the following sets:

$$\Omega(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot)) = \{\omega(\cdot) :\ \omega(\cdot) \in \hat{\Omega}(\overline{\tau, \vartheta}),$$

$$\forall\, t \in \overline{\tau+1, \vartheta},\ \omega(t) = G(y(t))z(t) + S(t)\xi(t),$$

$$y(t) = y_t(\overline{\tau, \vartheta}, y(\tau), u(\cdot), u^{(1)}(\cdot)),$$

$$z(t) = z_t(\overline{\tau, \vartheta}, z(\tau), v(\cdot)),\ \xi(t) \in \Xi_1,$$

$$(z(\tau), v(\cdot)) \in Z(\tau) \times \mathbf{V}(\overline{\tau, \vartheta})\}; \qquad (11)$$

$$\mathbf{W}(\tau, w(\tau), \vartheta, u(\cdot), u^{(1)}(\cdot)) = \{w(\vartheta) :$$

$$w(\vartheta) \in \hat{\mathbf{W}}(\vartheta),\ w(\vartheta) = \{\vartheta, y(\vartheta), y^{(1)}(\vartheta), Z(\vartheta)\},$$

$$y(\vartheta) = y_{\vartheta}(\overline{\tau, \vartheta}, y(\tau), u(\cdot), u^{(1)}(\cdot)),$$

$$y^{(1)}(\vartheta) = y_{\vartheta}(\overline{\tau, \vartheta}, y^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot)),$$

$$Z(\vartheta) = \mathbf{Z}^{(e)}_{\vartheta}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)),$$

$$\omega(\cdot) \in \Omega(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot))\}, \qquad (12)$$

and call them, respectively, the set of all admissible informational signals on the interval $\overline{\tau, \vartheta}$, and the set of all admissible ϑ-positions of player P corresponding to τ-position $w(\tau)$ of player P, and controls $u(\cdot)$ and $u^{(1)}(\cdot)$ of players P and S respectively.

It is known (see [Shorikov, 1997]) that the informational set $\mathbf{Z}^{(e)}_{\vartheta}(\overline{\tau, \vartheta}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot))$ is the basic element in solving the a posteriori minimax filtering problem for the discrete-time dynamical system (1)–(8), and it is a convex, closed, and bounded, and may be approximate by a convex polyhedron in the space $\mathbf{R}^s$, and construct by way to realization of a finite sequence of one-step operations only (here and below, by a convex polyhedron we mean the convex hull of a finite set of vectors in the corresponding Euclidean vector space). Note that this informational set will be needed to formalization and solve the main multistep problem of two-level hierarchical minimax program control over the terminal approach process with incomplete information that we consider in this paper.

5 Quality criterions for control over the approach process

For estimating a quality of the considered dynamical approach process on control level I, we introduce the terminal functional

$$\alpha: \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{U}^{(1)}(\overline{\tau, \mathrm{T}}) \times \hat{\Omega}(\overline{\tau, \mathrm{T}}) =$$

$$= \Gamma(\overline{\tau, \mathrm{T}}, \alpha) \longrightarrow \mathbf{E} =]-\infty, +\infty[, \qquad (13)$$

which, for a collection of realizations $(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) \in \Gamma(\overline{\tau, \mathrm{T}}, \alpha)$ admissible on the interval $\overline{\tau, \mathrm{T}}$ has the following form:

$$\alpha(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) =$$

$$= \mu_1 \cdot \beta(w^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot)) +$$

$$+\mu \cdot \gamma(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)). \qquad (14)$$

Here the functional

$$\beta: \hat{\mathbf{W}}^{(1)}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{U}^{(1)}(\overline{\tau, \mathrm{T}}) =$$

$$= \Gamma(\overline{\tau, \mathrm{T}}, \beta) \longrightarrow \mathbf{E} \qquad (15)$$

and its values for realizations of collections $(w^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot)) \in \Gamma(\overline{\tau, \mathrm{T}}, \beta)$ admissible on the interval $\overline{\tau, \mathrm{T}}$ are defined by the following concrete relation:

$$\beta(w^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot)) =$$

$$= \parallel \{y^{(1)}(\mathrm{T})\}_{k_1} - \{y_*^{(1)}\}_{k_1} \parallel_{k_1}, \qquad (16)$$

where $\{y^{(1)}(\mathrm{T})\}_{k_1} = \{y_{\mathrm{T}}^{(1)}(\overline{\tau, \mathrm{T}}, y^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot))\}_{k_1}$ is the k_1-projection $(k_1 \leq r_1)$ of the solution of system (2) on the interval $\overline{\tau, \mathrm{T}}$ at final (terminal) instant T that corresponds to the collection $(y^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot))$.

Note that functional β defines a measure of how much an admissible realization of the k_1-projection $(k_1 \leq r_1)$ of the final state (at the final instant T) of the phase vector $y^{(1)}(\mathrm{T}) \in \mathbf{R}^{r_1}$ of object I_1 can deviate from the corresponding projection of a given fixed vector $y_*^{(1)} \in \mathbf{R}^{r_1}$, and lets players P and S estimate the quality of program terminal control for the approach process on the control levels I, and II of this two-level hierarchical control system for discrete-time dynamical system (1)–(8) on the considered interval $\overline{\tau, \mathrm{T}}$.

In formula (14) functional

$$\gamma: \hat{\mathbf{W}}(\tau) \times \mathbf{U}(\overline{\tau, \mathrm{T}}) \times \mathbf{U}^{(1)}(\overline{\tau, \mathrm{T}}) \times \hat{\Omega}(\overline{\tau, \mathrm{T}}) =$$

$$= \Gamma(\overline{\tau, \mathrm{T}}, \gamma) \longrightarrow \mathbf{E}, \qquad (17)$$

defines a measure of how much an admissible realization of the k_1-projection $(k_1 \leq r;\ k_1 \leq s)$ of the final state phase vector $y(\mathrm{T}) \in \mathbf{R}^r$ of object I can deviate from the corresponding projection of an admissible realization of the final state phase vector $z(\mathrm{T}) \in \mathbf{R}^s$ of object II, and lets player P estimate the quality of program terminal control for the approach process on control level I of this two-level hierarchical control system on the considered interval $\overline{\tau, \mathrm{T}}$. Values of this functional are defined by the following formula:

$$\gamma(w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) =$$

$$= \max_{\{z(\mathrm{T})\}_k \in \{Z^{(e)}(\mathrm{T})\}_k} \parallel \{y(\mathrm{T})\}_k - \{z(\mathrm{T})\}_k \parallel_k, \qquad (18)$$

where the set $Z^{(e)}(\mathrm{T}) = \mathbf{Z}_{\mathrm{T}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), u^{(1)}(\cdot), \omega(\cdot)) \neq \emptyset$, and $\{Z^{(e)}(\mathrm{T})\}_k$ is its k-projection.

In formula (14), $\mu^{(1)} \in \mathbf{R}^1$ and $\mu \in \mathbf{R}^1$ are any fixed numerical parameters that satisfy the following condition: $\mu + \mu_1 = 1$.

Also note that on control level I of this two-level hierarchical control system we not exclude situation when the parameter $\omega(\cdot) \in \Omega(\overline{\tau, \mathrm{T}})$ may be realized on the interval $\overline{\tau, \mathrm{T}}$ by worst form for the player P, namely, when it determine a maximal admissible value of the functional γ.

The quality of program terminal control for player S in the approach process for the final phase state of object I_1 with a given fixed vector $y_*^{(1)}$ on control level II is estimated by the functional β that we have already introduced by the relations (15), and (16).

We should note that if we introduce a vector-functional $\delta = (\beta, \gamma)$ such that

$$\delta: \Gamma(\overline{\tau, \mathrm{T}}, \beta) \times \Gamma(\overline{\tau, \mathrm{T}}, \gamma) \longrightarrow \mathbf{E} \times \mathbf{E}, \qquad (19)$$

whose every parameter's values are defined for admissible realizations of their arguments on interval $\overline{\tau, \mathrm{T}}$ according to formulas (17), and (18), we can conclude that functional α defined by relations (13)–(18) is a convolution of the vector-functional δ obtained with the method of scalarization (see, e.g., [Bazaraa and Shetty, 1979]) of vector functionals.

Thus, functional α lets us estimate in general, from player P's point of view, the operation in the approach process on the interval $\overline{\tau, \mathrm{T}}$ in two-level hierarchical control system for discrete-time dynamical system (1)–(8) considered as a collection of objects I, I_1, and II, players P, S, and E that define control level I. Functional α is a convolution of the vector terminal functional δ; for its scalarization, we use numerical parameters μ, and μ_1 that estimate how important each of the functionals β, and γ for player P. Note that these parameters can be determined, e.g., with an expert estimate based on experimental data about the considered dynamical system (1)–(8).

6 Optimization problems for control over the approach process

Based on the considerations above, the objective of the player S, who defines control level II together with the object I_1 in the considered two level hierarchical control system for discrete-time dynamical system (1)–(8) for program terminal control of an approach process on any fixed interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, can be formulated as follows. We will assume that player S, using his informational and control possibilities on control level II, is interested in such result of the program terminal control in the approach process defined by dynamical system (1)–(8) on a given interval $\overline{\tau, \mathrm{T}}$ for which functional β defined by relations (15), and (16) for all admissible realizations of his τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$, and program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P on this interval takes the least possible value with a possible choice of his admissible program control $u^{(1)}(\cdot) \in \Psi_{\overline{\tau, \mathrm{T}}}(u(\cdot))$.

To carry out this idea, below we formulate for player S, i.e., for control level II of the two-level hierarchical control system for discrete-time dynamical system defined by (1)–(8), the following optimization problem for the program terminal control over the approach process of object I_1 with a given fixed vector $y_*^{(1)}$.

Problem 1. For a given interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, a realization of the τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S which is admissible on control level II of the two-level hierarchical control system for discrete-time dynamical system defined by (1)–(8), and any admissible realization of the program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P on control level I, find the set $\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot)) \subseteq \Psi_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ of optimal program controls $u^{(1,e)}(\cdot) \in \Psi_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ of player S corresponding to control $u(\cdot)$ of player P which is defined by relation

$$\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot)) = \{u^{(1,e)}(\cdot) :$$

$$u^{(1,e)}(\cdot) \in \Psi_{\overline{\tau, \mathrm{T}}}(u(\cdot)),\ c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot)) =$$

$$= \beta(w^{(1)}(\tau), u(\cdot), u^{(1,e)}(\cdot)) =$$

$$= \min_{u^{(1)}(\cdot) \in \Psi_{\overline{\tau, \mathrm{T}}}(u(\cdot))} \beta(w^{(1)}(\tau), u(\cdot), u^{(1)}(\cdot))\}, \quad (20)$$

where functional β is defined by the relations (15), and (16).

We call the set $\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot)) = \{u^{(1,e)}(\cdot)\} \subseteq \Psi_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ $(w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau),\ w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ which is formed by the solution of Problem 1, the set of optimal program controls of player S on control level II of the two-level hierarchical control system for discrete-time dynamical system defined by (1)–(8), and the corresponding numerical value $c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))$ we call the optimal value of the program control result in the approach process for player S on control level II in this control system. Note that both this elements correspond to fixed and admissible interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, τ-position $w^{(1)}(\tau) \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S on control level II, and control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P on control level I.

We also note that the solution of Problem 1 on the interval $\overline{\tau, \mathrm{T}}$ defines the principle of constructing optimal program controls $u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot)) \subseteq \Psi_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ for player S on control level II that correspond to a realization of his τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ and are subordinate to the choice of the admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P on control level I.

According to the definitions and assumptions made above about all parameters and informational connections in the two-level hierarchical control system for discrete-time dynamical systems defined by (1)–(8), the objective of player P on control level I of this control system in the realization of the considered program terminal control over the approach process of object I with object II, and object I_1 with a given fixed vector $y_*^{(1)}$ on a given interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, i.e., control of objects I, and I_1, can be summarized as follows. We assume that player P, using his informational and controls possibilities, is interested in such result of program terminal control in the approach process defined by the dynamical system (1)–(8) on interval $\overline{\tau, \mathrm{T}}$ for which the functional α defined by relations (13)–(18) for every admissible realizations of his τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ takes minimal admissible value due to the possible choice of his admissible program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$, and optimal program control $u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))$ of player S which is subordinate to P (where τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, y_0^{(1)}\} = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S that defines the state of object I_1 on control level II at instant τ, is constructed from the τ-position $w(\tau)$).

Note that in the analysis of system (1)–(8), we cannot rule out the situation when parameters $v(\cdot) \in \mathbf{V}(\overline{\tau, \mathrm{T}})$, and $\xi(\cdot) \in \Xi(\overline{\tau, \mathrm{T}})$ can be realized in the worst possible way for player P, i.e., they define the maximal possible value of functional α for any fixed and admissible realizations of elements $w(\tau), u(\cdot)$, and $u^{(1)}(\cdot)$. Also note that the influence of parameters $v(\cdot)$, and $\xi(\cdot)$ on the considered approach process is reflected in the values of admissible realizations of the informational signal $\omega(\cdot) \in \Omega(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), u^{(1)}(\cdot))$.

To achieve this objective for player P, we formulate the minimax program terminal control problem with incomplete information for the approach process for objects I, I_1, and II on control level I of the two-level hierarchical control system for discrete-time dynamical system defined by (1)–(8).

Problem 2. For a given interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, and a realization of the τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ of player P which is admissible on control level I of the two-level hierarchical control system for discrete-time dynamical system defined by (1)–(8), find the set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$ of minimax program controls of player P defined as follows:

$$\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \{u^{(e)}(\cdot) :\ u^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}}),$$

$$c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \min_{u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u^{(e)}(\cdot))} \{$$

$$\max_{\omega(\cdot) \in \Omega_1(u^{(e)}(\cdot))} \alpha(w(\tau), u^{(e)}(\cdot), u^{(1,e)}(\cdot), \omega(\cdot))\} =$$

$$= \min_{u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})} \min_{u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))} \{$$

$$\max_{\omega(\cdot) \in \Omega_1(u(\cdot))} \alpha(w(\tau), u(\cdot), u^{(1,e)}(\cdot), \omega(\cdot))\}\}. \quad (21)$$

Here the functional α is defined by relations (13)–(18); τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w(0) = \{0, y_0^{(1)}\} = w_0 \in \hat{\mathbf{W}}_0^{(1)})$ of player S is constructed from the τ-position $w(\tau)$ of player P and defines, at instant τ, the realization of the phase vector of object I_1 on control level II of the two-level hierarchical control system, while the set $\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot)) = \{u^{(1,e)}(\cdot)\} \subseteq \Psi_{\overline{\tau, \mathrm{T}}}(u(\cdot))$ of optimal program controls of player S on control level II of the considered control system for any admissible realizations of τ-position $w^{(1)}(\tau) \in \hat{\mathbf{W}}^{(1)}(\tau)$ of player S and program control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P can be found from the solution of Problem 1; the sets $\Omega_1(u(\cdot)) = \Omega(\overline{\tau, \mathrm{T}}, w(\tau), u(\cdot), u^{(1,e)}(\cdot))$, and $\Omega_1(u^{(e)}(\cdot)) = \Omega(\overline{\tau, \mathrm{T}}, w(\tau), u^{(e)}(\cdot), u^{(1,e)}(\cdot))$.

We call the set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$, which is constructed from the solution of Problem 2, the set of minimax program controls of player P for the approach process on control level I of this two-level hierarchical control system for the discrete-time dynamical system defined by (1)–(8), while the corresponding number $c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ we call the optimal guaranteed value of the result of a minimax program control of player P for the approach process on control level I of this control system. Here the set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ and number $c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ correspond to fixed and admissible interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, and τ-position $w(\tau) \in \hat{\mathbf{W}}(\tau)$ $(w(0) = w_0 \in \hat{\mathbf{W}}_0)$ of player P on control level I.

Note that the solution of problem 2 defines, on the interval $\overline{\tau, \mathrm{T}}$, the principle of constructing minimax program controls $u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) \subseteq \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P on control level I of two-level hierarchical control system for the discrete-time dynamical system (1)–(8), corresponding to realization of his τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$.

Based on solutions of Problems 1 and 2 formulated above, we also consider the following problem.

Problem 3. For any fixed interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, and a realization of the τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{W}(\tau)$ $(w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ of player P admissible on control level I of the two-level hierarchical control system for the discrete-time dynamical system (1)–(8), find the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of minimax program controls of player P, which is defined as follows:

$$\hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) = \{\hat{u}^{(e)}(\cdot) :$$

$$\hat{u}^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)),$$

$$\min_{u^{(1)}(\cdot) \in\ \Psi_{\overline{\tau, \mathrm{T}}}(\hat{u}^{(e)}(\cdot))} \beta(w^{(1)}(\tau), \hat{u}^{(e)}(\cdot), u^{(1)}(\cdot)) =$$

$$= \min_{u^{(e)}(\cdot) \in \mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))} \min_{u^{(1)}(\cdot) \in \Psi_{\overline{\tau, \mathrm{T}}}(u^{(e)}(\cdot))} \{$$

$$\beta(w^{(1)}(\tau), u^{(e)}(\cdot), u^{(1)}(\cdot))\}\}; \quad (22)$$

for realization the τ-position $w^{(1)}(\tau) \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of the player S which is admissible on control level II of this two-level hierarchical control system and constructed from the τ-position $w(\tau)$, and an admissible realization of the minimax program control $\hat{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of player P on control level I which can be constructed from the solution of Problem 2 and the problem defined by (22), find the set $\hat{\mathbf{U}}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) \subseteq \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) \subseteq \Psi_{\overline{\tau, \mathrm{T}}}(\hat{u}^{(e)}(\cdot))$ of minimax program controls $\hat{u}^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))$ of player S on control level II, and number $c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))$ that is the optimal result value for player S for the approach process on control level II of this two-level hierarchical control system corresponding to control $\hat{u}^{(e)}(\cdot)$ of player P, which are defined by the following relations:

$$\hat{\mathbf{U}}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) = \{\hat{u}^{(1,e)}(\cdot) :$$

$$\hat{u}^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)),$$

$$c_\alpha^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau)) =$$

$$= \max_{\omega(\cdot) \in \Omega_2(\hat{u}^{(1,e)}(\cdot))} \alpha(w(\tau), \hat{u}^{(e)}(\cdot), \hat{u}^{(1,e)}(\cdot), \omega(\cdot)) =$$

$$= \min_{u^{(1,e)}(\cdot) \in \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))} \max_{\omega(\cdot) \in \Omega_2(u^{(1,e)}(\cdot))} \{$$

$$\alpha(w(\tau), \hat{u}^{(e)}(\cdot), u^{(1,e)}(\cdot), \omega(\cdot))\}\}; \qquad (23)$$

$$c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) =$$

$$= \beta(w^{(1)}(\tau), \hat{u}^{(e)}(\cdot), \hat{u}^{(1,e)}(\cdot)) =$$

$$= \min_{u^{(1)}(\cdot) \in \Psi_{\overline{\tau, \mathrm{T}}}(\hat{u}^{(e)}(\cdot))} \{$$

$$\beta(w^{(1)}(\tau), \hat{u}^{(e)}(\cdot), u^{(1)}(\cdot))\}. \qquad (24)$$

Here the functional α is defined by relations (13)–(18), and the functional β is defined by relations (15), and (16); τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, y_0^{(1)}\} = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S is constructed from the τ-position $w(\tau)$ of player P and defines, at instant τ, the realization of the phase vector of object I_1 on control level II of this two-level hierarchical control system for discrete-time dynamical system (1)–(8); the set $\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) = \{u^{(1,e)}(\cdot)\}\} \subseteq \Psi_{\overline{\tau, \mathrm{T}}}(\hat{u}^{(e)}(\cdot))$ of optimal program controls of player S on control level II of this two-level hierarchical control system for any fixed and admissible interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$; realization of minimax program control $\hat{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of player P on control level I of this two-level hierarchical control system for discrete-time dynamical system (1)–(8) can be found from the solution of Problem 1; the sets $\Omega_2(u^{(1,e)}(\cdot)) = \Omega(\overline{\tau, \mathrm{T}}, w(\tau), \hat{u}^{(e)}(\cdot), u^{(1,e)}(\cdot))$, and $\Omega_2(\hat{u}^{(1,e)}(\cdot)) = \Omega(\overline{\tau, \mathrm{T}}, w(\tau), \hat{u}^{(e)}(\cdot), \hat{u}^{(1,e)}(\cdot))$.

7 General solving scheme for the main approach problem

Thus, for every admissible and fixed time interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, and τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ of player P on control level I of this two-level hierarchical control system for discrete-time dynamical system (1)–(8), and the corresponding τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, y_0^{(1)}\} = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S on control level II of this two-level hierarchical control system we can consider the solutions of Problems 1–3 formulated above that together define a multistep problem of two-level hierarchical minimax program control over the terminal approach process with incomplete information for the discrete-time dynamical system (1)–(8).

Then the general scheme for realization of minimax program control over the terminal approach process that define the two-level hierarchical control system for the discrete-time dynamical system (1)–(8), for every fixed and admissible time interval $\overline{\tau, \mathrm{T}} \subseteq \overline{0, \mathrm{T}}$ $(\tau < \mathrm{T})$, realizations of τ-position $w(\tau) = \{\tau, y(\tau), y^{(1)}(\tau), Z(\tau)\} \in \hat{\mathbf{W}}(\tau)$ $(w(0) = \{0, y_0, y_0^{(1)}, Z_0\} = w_0 \in \hat{\mathbf{W}}_0)$ of player P on control level I, and the corresponding τ-position $w^{(1)}(\tau) = \{\tau, y^{(1)}(\tau)\} \in \hat{\mathbf{W}}^{(1)}(\tau)$ $(w^{(1)}(0) = \{0, y_0^{(1)}\} = w_0^{(1)} \in \hat{\mathbf{W}}_0^{(1)})$ of player S on control level II, can be represented as the following sequence of actions:

(1) for every fixed control $u(\cdot) \in \mathbf{U}(\overline{\tau, \mathrm{T}})$ of player P on control level I, construct with the solution of the corresponding Problem 1 the set $\mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))$ of optimal program controls of player S on control level II, and the number $c_\beta^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), u(\cdot))$, i.e., the optimal program result value for this player on control level II corresponding to control $u(\cdot)$ that satisfy relation (20);

(2) with the solution of Problem 2, which is based on the solution of Problem 1, construct the set $\mathbf{U}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of minimax program controls of player P on control level I, and the number $c_{\alpha}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$, the optimal guaranteed value of the result of a minimax program control of player P for the terminal approach process on control level I that satisfy relation (21);

(3) with the solution of Problem 2, which is based on the solution of Problem 1, and the problem defined by (22) construct the set $\hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of minimax program controls of player P on control level I;

(4) for any minimax program control $\hat{u}^{(e)}(\cdot) \in \hat{\mathbf{U}}^{(e)}(\overline{\tau, \mathrm{T}}, w(\tau))$ of player P on control level I, with the solution of Problem 3, which is based on the solutions of Problems 1 and 2, construct the set $\hat{\mathbf{U}}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot)) \subseteq \mathbf{U}^{(1,e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))$ of minimax program controls of player S on control level II, and the number $c_{\beta}^{(e)}(\overline{\tau, \mathrm{T}}, w^{(1)}(\tau), \hat{u}^{(e)}(\cdot))$, the optimal program control result value of player S for the approach process on control level II that correspond to control $\hat{u}^{(e)}(\cdot)$ and satisfy relations (23), and (24).

8 Conclusion

In conclusion we note that a concrete algorithm for realization of the minimax program terminal control over the approach process with two-level hierarchical control system for the discrete-time dynamical system (1)–(8) can be constructed with algorithms for solving minimax program terminal control problems with incomplete information from works [Shorikov, 1997], and [Shorikov, 2005].

Results of this paper can be used for computer simulation, design and construction of multilevel control systems for actual technical, robotics, economic, and other dynamical processes operating under deficit of information and uncertainty.

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