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A NOVEL ADAPTIVE UNSCENTED KALMAN FILTER FOR PICO SATELLITE ATTITUDE ESTIMATION

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Abstract

Unscented Kalman Filter (UKF) is a filtering algorithm which gives sufficiently good estimation results for estimation problems of nonlinear systems even in case of high nonlinearity. However, in case of system uncertainty UKF becomes to be inaccurate and diverges by time. In other words, if any change occurs in the process noise covariance, which is known as *a priori*, filter fails. This study, introduces a novel Adaptive Unscented Kalman Filter (AUKF) algorithm based on the correction of process noise covariance for the case of mismatches with the model. By the use of a newly adaptation scheme for the conventional UKF algorithm, change in the noise covariance is detected and corrected. Differently from the most of the existing adaptive UKF algorithms, covariance is not updated at each step; it has been only corrected when the fault occurs and that brings about a noteworthy reduction in the computational burden. Proposed algorithm is tested as a part of the attitude estimation algorithm of a pico satellite, a satellite type for which computational convenience is necessary because of the design limitations.

Key words
adaptive systems; filtering & identification; nonlinear dynamics.

1 Introduction

Unscented Kalman Filter (UKF) algorithm is a considerably new filtering method which was proposed for nonlinear systems. Since it has many advantages against well known Extended Kalman Filter, especially in case of high nonlinearity (Julier *et al.*, 1995), it has been preferred in lately methods presented for solving problems in the area of control, guidance, signal processing etc.

The basic of UKF is the fact that; the approximation of a nonlinear distribution is easier than the approximation of a nonlinear function or transformation (Julier *et al.*, 2000). UKF introduces sigma points to catch higher order statistic of the system and by securing higher order information of the system, it satisfies both better estimation accuracy and convergence characteristic (Soken and Hajiyev, 2009).

As a spacecraft attitude estimation algorithm, UKF has many implementation examples in literature. In (Crassidis and Markley, 2003) it is used as a state estimator, while both the states and the parameters of the satellite are estimated by UKF in (Dyke *et al.*, 2004; Sekhavat *et al.*, 2007). Moreover, in (Fisher and Vadali, 2008) UKF is used as a part of the attitude control scheme of multibody satellites.

Despite its recent popularity, conventional UKF algorithm has no capability to adapt itself to the changing conditions. One of the problems which may be considered within this perspective is the changes in process noise covariance which is generally known as *a priori*.

In literature there are several methods to adapt linear Kalman filter. As an example, Multiple Model Based Adaptive Estimation (MMAE), Innovation Based Adaptive Estimation (IAE) or Residual Based Adaptive Estimation (RAE) are some of known methods which are used in such situations (Hide *et al.*, 2004). Nonetheless another concept is to scale the noise covariance matrix by multiplying it with a time dependent variable. One of the methods for constructing such algorithm is to use a scale factor as a multiplier to the process or measurement noise covariance matrices (Hajiyev and Soken, 2009; Geng and Wang, 2008). This kind of gain correction based algorithms can be both used when the information about the dynamic or measurement process is absent (Kim *et al.*, 2006). Per contra, these algorithms are generally appropriate for linear Kalman filter and cannot be used for the process noise adaptation process of the UKF.

It is also possible to meet adaptive unscented Kalman filter (AUKF) applications in literature. In (Han *et al.*, 2009), two distinct methods are described. In first method MIT rule is used to derive the adaptive law and a cost function is defined in order to minimize the difference between the filter computed covariance and the actual innovation covariance. However, presented algorithm requires calculation of partial derivatives and that introduces a relative large computational burden as it is also stated by author himself. In second method, two UKFs are run in parallel within master and slave filter manner. Its computational demands may be relatively lower than the first method but still utilizing two filters means a necessity for a high processing capability and that is not usually possible for onboard small satellite applications. Hence, they may be hard to imply for a pico satellite which has a limited computer processor capacity. Nonetheless in (Liu and Lu, 2009; Shi *et al.*, 2009) Saga-Husa noise statistics estimator is integrated with UKF in order to make up an AUKF. Although it may be possible to have satisfactory results for target tracking problem, this method has an unstability issue; when noise covariance loses its semi-positive definiteness filter diverges.

UKF may be also built adaptively by using fuzzy logic based techniques. In (Jwo and Tseng, 2009) fuzzy logic adaptive system aids the interacting multiple models and by switching between filters suitable value for the process noise covariance can be determined. As a disadvantage such method also requires more than one filter running simultaneously. Besides the essences of these kinds of fuzzy methods are human experience and heuristic information; in out of experience cases they may not work.

This study, introduces a novel Adaptive Unscented Kalman Filter algorithm based on the correction of process noise covariance for the case of possible mismatches with the model known as *a priori*. By the use of a newly adaptation scheme for the conventional UKF algorithm, change in the noise covariance is detected and corrected. Differently from the most of the existing adaptive UKF algorithms, covariance is not updated at each step; it has been only corrected when the fault occurs and that brings about a noteworthy reduction in the computational burden. Proposed algorithm is tested as a part of the attitude estimation algorithm of a pico satellite, a satellite type for which computational convenience is necessary because of the design limitations.

2 Satellite Mathematical Model

If the kinematics of the pico satellite is derived in the base of Euler angles, then the mathematical model can be expressed with a 6 dimensional system vector which is made of attitude Euler angles (ϕ is the roll angle about x axis; θ is the pitch angle about y axis; ψ is the yaw angle about z axis) vector and the body angular rate vector with respect to the inertial axis frame. Hence,

$$\bar{x} = \begin{bmatrix} \phi & \theta & \psi & \omega_x & \omega_y & \omega_z \end{bmatrix}^T , \qquad (1)$$

Also for consistency with the further explanations, the body angular rate vector with respect to the inertial axis frame should be stated separately as; $\bar{\omega}_{BI} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$, where $\bar{\omega}_{BI}$ is the angular velocity vector of the body frame with respect to the inertial frame. Besides, dynamic equations of the satellite can be derived by the use of the angular momentum conservation law;

$$J_x \frac{d\omega_x}{dt} = N_x + \left(J_y - J_z\right)\omega_y\omega_z , \qquad (2)$$

$$J_y \frac{d\omega_y}{dt} = N_y + \left(J_z - J_x\right)\omega_z\omega_x , \qquad (3)$$

$$J_z \frac{d\omega_z}{dt} = N_z + \left(J_x - J_y\right)\omega_x\omega_y , \qquad (4)$$

where J_x, J_y and J_z are the principal moments of inertia and N_x, N_y and N_z are the terms of the external torque affecting the satellite. For a Low Earth Orbit (LEO) pico satellite as in case, gravity gradient torque should be taken into consideration while the other disturbance torque terms such as aerodynamic torques, magnetic disturbance torques and torques caused by the solar radiation pressure may be neglected (Sekhavat *et al.*, 2007). If only the gravity gradient torque is put into account for the satellite, these torque terms in eq.(2-4) can be written as

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = -3\frac{\mu}{r_0^3}\begin{bmatrix} \left(J_y - J_z\right)A_{23}A_{33} \\ \left(J_z - J_x\right)A_{13}A_{33} \\ \left(J_x - J_y\right)A_{13}A_{23} \end{bmatrix} . \qquad (5)$$

Here μ is the gravitational constant, r_0 is the distance between the centre of mass of the satellite and the Earth and A_{ij} represents the corresponding element of the direction cosine matrix of;

$$A = \begin{bmatrix} c(\psi)c(\theta)+s(\phi)s(\theta)s(\psi) & c(\phi)s(\psi) & -c(\psi)s(\theta)+s(\phi)c(\theta)s(\psi) \\ -s(\psi)c(\theta)+s(\phi)s(\theta)c(\psi) & c(\phi)c(\psi) & s(\psi)s(\theta)+s(\phi)c(\theta)c(\psi) \\ c(\phi)s(\theta) & -s(\phi) & c(\phi)c(\theta) \end{bmatrix}. \qquad (6)$$

In matrix A, $c(\cdot)$ and $s(\cdot)$ are the cosine and sine functions successively. Kinematic equations of motion of the pico satellite with the Euler angles can be given as (Wertz, 1988);

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} c(\psi) & -s(\psi) & 0 \\ s(\psi)\sec(\phi) & c(\psi)\sec(\phi) & 0 \\ s(\psi)t(\phi) & c(\psi)t(\phi) & 1 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}. \qquad (7)$$

Here $t(\cdot)$ and $\sec(\cdot)$ stand for tangent and secant functions respectively. Moreover p, q and r are the components of $\bar{\omega}_{BR}$ vector which indicates the angular velocity of the body frame with respect to the reference frame. $\bar{\omega}_{BI}$ and $\bar{\omega}_{BR}$ can be related via,

$$\bar{\omega}_{BR} = \bar{\omega}_{BI} + A\begin{bmatrix} 0 & -\omega_0 & 0 \end{bmatrix}^T . \tag{8}$$

where ω_0 denotes the angular velocity of the orbit with respect to the inertial frame, found as $\omega_0 = \left(\mu / r_0^3\right)^{1/2}$.

3 Measurement Sensor Model

Pico satellite has three axis magnetometer (TAM) and three rate gyros onboard for attitude measurements. Models of these sensors are given below:

3.1 The Magnetometer Model

As the satellite navigates along its orbit, magnetic field vector differs in a relevant way with the orbital parameters. If those parameters are known, then, magnetic field tensor vector that affects satellite can be shown as a function of time analytically (Sekhavat *et al.*, 2006). Note that, these terms are obtained in the orbit reference frame.

$$H_1(t) = \frac{M_e}{r_0^3}\left\{\cos(\omega_0 t)\left[\cos(\varepsilon)\sin(i) - \sin(\varepsilon)\cos(i)\cos(\omega_e t)\right] - \sin(\omega_0 t)\sin(\varepsilon)\sin(\omega_e t)\right\} \tag{9}$$

$$H_2(t) = -\frac{M_e}{r_0^3}\left[\cos(\varepsilon)\cos(i) + \sin(\varepsilon)\sin(i)\cos(\omega_e t)\right], \tag{10}$$

$$H_3(t) = \frac{2M_e}{r_0^3}\left\{\sin(\omega_0 t)\left[\cos(\varepsilon)\sin(i) - \sin(\varepsilon)\cos(i)\cos(\omega_e t)\right] - 2\sin(\omega_0 t)\sin(\varepsilon)\sin(\omega_e t)\right\} \tag{11}$$

Here

- $M_e = 7.943 \ x \ 10^{15}$ *Wb.m*; the magnetic dipole moment of the Earth,
- $\mu = 3.98601 \ x \ 10^{14} \ m^3 / s^2$; the Earth Gravitational constant,
- $i = 97^\circ$; the orbit inclination,
- $\omega_e = 7.29 \ x \ 10^{-5} \ rad/s$; the spin rate of the Earth,
- $\varepsilon = 11.7^\circ$; the magnetic dipole tilt,
- $r_0 = 6{,}928{,}140 \ m$; the distance between the centre of mass of the satellite and the Earth.

Three onboard magnetometers of pico satellite measures the components of the magnetic field vector in the body frame. Therefore for the measurement model, which characterizes the measurements in the body frame, gained magnetic field terms must be transformed by the use of direction cosine matrix, A. Overall measurement model may be given as;

$$\begin{bmatrix} H_x(\phi,\theta,\psi,t) \\ H_y(\phi,\theta,\psi,t) \\ H_z(\phi,\theta,\psi,t) \end{bmatrix} = A\begin{bmatrix} H_1(t) \\ H_2(t) \\ H_3(t) \end{bmatrix} + \eta_1 , \tag{12}$$

where, $H_1(t)$, $H_2(t)$ and $H_3(t)$ represent the Earth magnetic field vector components in the orbit frame as a function of time, and $H_x(\phi,\theta,\psi,t)$, $H_y(\phi,\theta,\psi,t)$ and $H_z(\phi,\theta,\psi,t)$ show the measured Earth magnetic field vector components in body frame as a function of time and varying Euler angles. Furthermore, η_1 is the zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_m^2\delta_{kj} . \tag{13}$$

I_{3x3} is the identity matrix with the dimension of 3×3, σ_m is the standard deviation of each magnetometer error and δ_{kj} is the Kronecker symbol.

3.2 The Rate Gyro Model

Three rate gyros are aligned through three axes, orthogonally to each other and they supply directly the angular rates of the body frame with respect to the inertial frame. Hence the model for rate gyros can be given as;

$$\bar{\omega}_{BI,meas} = \bar{\omega}_{BI} + \eta_2 . \tag{14}$$

where, $\bar{\omega}_{BI,meas}$ is the measured angular rates of the satellite, and η_2 is the zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{2k}\eta_{2j}^T\right] = I_{3x3}\sigma_v^2\delta_{kj} , \tag{15}$$

Here, σ_v is the standard deviation of each rate gyro random error.

4 Adaptive Unscented Kalman Filter

In case of normal operation, where the model for the process noise covariance matches with the real values, UKF works correctly. However, when a change occurs in the noise covariance, the filter fails and the estimation outputs become faulty.

Hence, an adaptive algorithm must be introduced so as the filter adapts itself to the changing conditions for the process noise covariance and corrects estimations without affecting good estimation characteristic of the remaining process.

4.1 Unscented Kalman Filter

In order to utilize Kalman filter for nonlinear systems without any linearization step, the unscented transform and so Unscented Kalman Filter is one of the techniques. UKF uses the unscented transform, a deterministic sampling technique, to determine a minimal set of sample points (or sigma points) from the *a priori* mean and covariance of the state. Then, these sigma points go through nonlinear transformation. The *posterior* mean and the covariance are obtained from these transformed sigma points (Julier *et al.*, 1995).

As it is stated, UKF procedure begins with the determination of $2n+1$ sigma points with a mean of $\hat{x}(k|k)$ and a covariance of $P(k|k)$. For an n dimensional state vector, these sigma points are obtained by

$$x_0(k|k) = \hat{x}(k|k) \tag{16}$$

$$x_l(k|k) = \hat{x}(k|k) + \left(\sqrt{(n+\kappa)P(k|k)}\right)_l \tag{17}$$

$$x_{l+n}(k|k) = \hat{x}(k|k) - \left(\sqrt{(n+\kappa)P(k|k)}\right)_l, \tag{18}$$

where, $x_0(k|k)$, $x_l(k|k)$ and $x_{l+n}(k|k)$ are sigma points, n is the state number and κ is the scaling parameter which is used for fine tuning and the heuristic is to chose that parameter as $n+\kappa=3$ (Julier *et al.*, 1995). Also, l is given as $l=1\ldots n$.

Next step of the UKF process is transforming each sigma point by the use of system dynamics,

$$x_i(k+1|k) = f\left[x_i(k|k), k\right]. \quad i = 0\ldots 2n \tag{19}$$

Then these transformed values are utilized for gaining the predicted mean and the covariance (Crassidis and Markley, 2003; Soken and Hajiyev 2009).

$$\hat{x}(k+1|k) = \frac{1}{n+\kappa}\left\{\kappa x_0(k+1|k) + \frac{1}{2}\sum_{i=1}^{2n} x_i(k+1|k)\right\}, \tag{20}$$

$$P(k+1|k) = \frac{1}{n+\kappa}\left\{\kappa\left[x_0(k+1|k) - \hat{x}(k+1|k)\right]\left[x_0(k+1|k) - \hat{x}(k+1|k)\right]^T \right.$$
$$\left. + \frac{1}{2}\sum_{i=1}^{2n}\left[x_i(k+1|k) - \hat{x}(k+1|k)\right]\left[x_i(k+1|k) - \hat{x}(k+1|k)\right]^T\right\} + Q(k). \tag{21}$$

Here, $\hat{x}(k+1|k)$ is the predicted mean, $P(k+1|k)$ is the predicted covariance and $Q(k)$ is the process noise covariance matrix.

Nonetheless, predicted observation vector is,

$$\hat{y}(k+1|k) = \frac{1}{n+\kappa}\left\{\kappa y_0(k+1|k) + \frac{1}{2}\sum_{i=1}^{2n} y_i(k+1|k)\right\}, \tag{22}$$

where,

$$y_i(k+1|k) = h\left[x_i(k+1|k), k\right]. \quad i = 0\ldots 2n \tag{23}$$

After that, observation covariance matrix is determined as,

$$P_{yy}(k+1|k) = \frac{1}{n+\kappa}\left\{\kappa\left[y_0(k+1|k) - \hat{y}(k+1|k)\right]\left[y_0(k+1|k) - \hat{y}(k+1|k)\right]^T \right.$$
$$\left. + \frac{1}{2}\sum_{i=1}^{2n}\left[y_i(k+1|k) - \hat{y}(k+1|k)\right]\left[y_i(k+1|k) - \hat{y}(k+1|k)\right]^T\right\}, \tag{24}$$

where innovation covariance is

$$P_{vv}(k+1|k) = P_{yy}(k+1|k) + R(k+1) \tag{25}$$

Here $R(k+1)$ is the measurement noise covariance matrix. On the other hand, the cross correlation matrix can be obtained as,

$$P_{xy}(k+1|k) = \frac{1}{n+\kappa}\left\{\kappa\left[x_0(k+1|k) - \hat{x}(k+1|k)\right]\left[y_0(k+1|k) - \hat{y}(k+1|k)\right]^T \right.$$
$$\left. + \frac{1}{2}\sum_{i=1}^{2n}\left[x_i(k+1|k) - \hat{x}(k+1|k)\right]\left[y_i(k+1|k) - \hat{y}(k+1|k)\right]^T\right\}. \tag{26}$$

Following part is the update phase of UKF algorithm. At that phase, first by using incoming measurements, $y(k+1)$, innovation sequence is found as

$$e(k+1) = y(k+1) - \hat{y}(k+1|k), \tag{27}$$

and then Kalman gain is computed via equation of,

$$K(k+1) = P_{xy}(k+1|k)P_{vv}^{-1}(k+1|k). \tag{28}$$

At last, updated states and covariance matrix are determined by,

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)e(k+1), \tag{29}$$

$$P(k+1|k+1) = P(k+1|k) - K(k+1)P_{vv}(k+1|k)K^T(k+1). \tag{30}$$

Here, $\hat{x}(k+1|k+1)$ is the estimated state vector and $P(k+1|k+1)$ is the estimated covariance matrix.

4.2 Adaptive UKF Algorithm

Adaptive algorithm performs the correction only when the real values of the process noise covariance does not match with the model used in the synthesis of the filter. Otherwise the filter works with regular algorithm (19)-(30) in an optimal way. Adaptation occurs as a change in the predicted covariance. First, let us rewrite (21) as;

$$P(k+1|k) = P^*(k+1|k) + Q(k) \tag{31}$$

So $P^*(k+1|k)$ is the predicted covariance without the additive process noise. In order to adapt the covariance an adaptive factor is put into the procedure;

$$P(k+1|k) = P^*(k+1|k) + \Lambda(k)Q(k) \tag{32}$$

where $\Lambda(k)$ is the adaptive factor calculated in the base of residual for the state vector estimation, $\tilde{e}(k+1)$, which may be defined as;

$$\tilde{e}(k+1) = y(k+1) - H(k+1)\hat{x}(k+1|k+1), \tag{33}$$

where $H(k+1)$ is the measurement matrix. Nonetheless covariance matrix of the residue (33) can be written as (Mohamed and Schwarz, 1999),

$$P_{\tilde{e}}\left(k+1\right) = R\left(k+1\right) - H(k+1)P\left(k+1\middle|k+1\right)H(k+1)^T \quad (34)$$

The gain matrix is changed when the condition of

$$tr\left[\tilde{e}\left(k+1\right)\tilde{e}^T\left(k+1\right)\right] \geq tr\left[R\left(k+1\right)\right] - tr\left[H(k+1)P\left(k+1\middle|k+1\right)H(k+1)^T\right] \quad (35)$$

is the point at issue. Here $tr(\cdot)$ is the trace of the related matrix. Left hand side of (35) represents the real filtration error while the right hand side is the accuracy of the residue known as a result of priori information. When the estimated observation vector $H(k+1)\hat{x}\left(k+1\middle|k+1\right)$ is reasonably different from measurement vector, $y\left(k+1\right)$, because of any change that occurs in $Q(k)$ real filtration error exceeds the theoretical one. Hence, the process noise covariance must be fixed hereafter by the use of defined adaptive factor $\Lambda(k)$. In order to calculate the measurement adaptive factor equality of

$$tr\left[\tilde{e}\left(k+1\right)\tilde{e}^T\left(k+1\right)\right] = tr\left[R\left(k+1\right)\right] - tr\left[H(k+1)P\left(k+1\middle|k+1\right)H(k+1)^T\right] \quad (36)$$

is used. If we replace $P\left(k+1\middle|k+1\right)$ with its definition (30), then;

$$tr\left[\tilde{e}\left(k+1\right)\tilde{e}^T\left(k+1\right)\right] = tr\left[R\left(k+1\right)\right] - tr\left\{H(k+1)\left[P\left(k+1\middle|k\right) - K\left(k+1\right)P_{vv}\left(k+1\middle|k\right)K^T\left(k+1\right)\right]H(k+1)^T\right\} \quad (37)$$

After that we should put (32) into (37) and;

$$tr\left[\tilde{e}\left(k+1\right)\tilde{e}^T\left(k+1\right)\right] = tr\left[R\left(k+1\right)\right] - tr\left[H(k+1)P^*\left(k+1\middle|k\right)H(k+1)^T\right] - \Lambda(k)tr\left[H(k+1)Q(k)H(k+1)^T\right] + tr\left[H(k+1)K\left(k+1\right)P_{vv}\left(k+1\middle|k\right)K^T\left(k+1\right)H(k+1)^T\right] \quad (38)$$

Finally if the knowledge of

$$tr\left[\tilde{e}\left(k+1\right)\tilde{e}^T\left(k+1\right)\right] = \tilde{e}^T\left(k+1\right)\tilde{e}\left(k+1\right) \quad (39)$$

is taken into consideration then the adaptive factor can be found as (note that discretization indices are not written for sake of readability)

$$\Lambda = \frac{tr\left[R\right] - tr\left[HP^*H^T\right] + tr\left[HKP_{vv}K^TH^T\right] - \tilde{e}^T\tilde{e}}{tr\left[HQH^T\right]} \quad (40)$$

On the other hand, as a main difference from the existing AUKF algorithms process noise covariance matrix is not updated for whole the estimation procedure; adaptive algorithm is used only in case of changes and in all other cases procedure is run optimally with regular Unscented Kalman filter. Checkout is satisfied via a kind of statistical information. In order to achieve that, following two hypotheses may be introduced:

- γ_o ; the system is normally operating
- γ_1 ; there is a malfunction in the estimation system.

Failure detection is realized by the use of following statistical function,

$$\beta(k) = \tilde{e}^T\left(k+1\right)\left[R\left(k+1\right) - H(k+1)P\left(k+1\middle|k+1\right)H(k+1)^T\right]^{-1}\tilde{e}\left(k+1\right) \quad (41)$$

This statistical function has χ^2 distribution with s degree of freedom where s is the dimension of the residual vector $\tilde{e}\left(k+1\right)$.

If the level of significance, α, is selected as,

$$P\left\{\chi^2 > \chi^2_{\alpha,s}\right\} = \alpha; \qquad 0 < \alpha < 1, \quad (42)$$

the threshold value, $\chi^2_{\alpha,s}$ can be determined. Hence, when the hypothesis γ_1 is correct, the statistical value of $\beta(k)$ will be greater than the threshold value $\chi^2_{\alpha,s}$, i.e.:

$$\gamma_0 : \beta\left(k\right) \leq \chi^2_{\alpha,s} \qquad \forall k$$
$$\gamma_1 : \beta\left(k\right) > \chi^2_{\alpha,s} \qquad \exists k \quad . \quad (43)$$

5 Simulations

Simulations are realized for 1000 seconds with a sampling time of $\Delta t = 0.1\sec$. For the used pico satellite model the inertia matrix is taken as;

$$J = \begin{bmatrix} 2.1x10^{-3} & 0 & 0 \\ 0 & 2.0x10^{-3} & 0 \\ 0 & 0 & 1.9x10^{-3} \end{bmatrix} kg.m^2$$

Nonetheless the orbit of the satellite is a circular orbit with an altitude of $r = 550km$. Other orbit parameters are same as it is presented in the section for the Magnetometer Model (Section 3.1).

For magnetometer measurements, sensor noise is characterized by zero mean Gaussian white noise with a standard deviation of $\sigma_m = 200nT$. Besides, rate gyro random error is taken as $\sigma_{vg} = 1\times10^{-3}[\deg/\sqrt{h}]$. That corresponds to $\sigma_v = 9.19\times10^{-7} rad/s$ for a sampling frequency of 10Hz.

During simulations, for testing AUKF algorithm, process noise covariance matrix is changed at 600th second, thus it mismatches with the model known as *a priori*. This abrupt change is simply formed by multiplying covariance with a constant.

Besides, in case of measurement faults, simulation is also achieved for optimal UKF in order to compare results with AUKF and understand efficiency of the adaptive algorithm in a better way. For robust Kalman filters $\chi^2_{\alpha,s}$ is taken as 12.592 and this value comes from chi-square distribution when the degree of freedom is 6 and the reliability level is 95%.

First part of figures gives UKF and AUKF parameter estimation results and the actual values in a comparing way. Second part of the figures shows the error of the estimation process based on the actual estimation values of the satellite. The last part indicates the variance of the estimation.

As it is apparent form Fig. 1, regular UKF fails at estimating the attitude parameters when the change occurs. Besides, the effect of the change continues for at least 400 seconds and at that period filter cannot estimate the parameters in an accurate way. On the other hand as given in Fig.2, AUKF is not affected from the change in the process noise covariance and it satisfies its good estimation characteristic for the whole process.

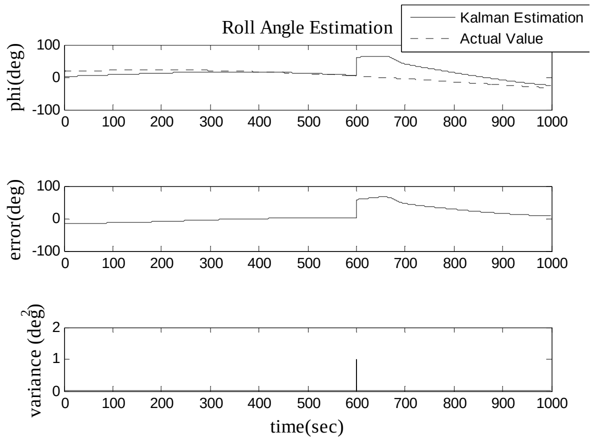

Figure 1. Roll angle estimation by regular UKF in case of change in the process noise covariance

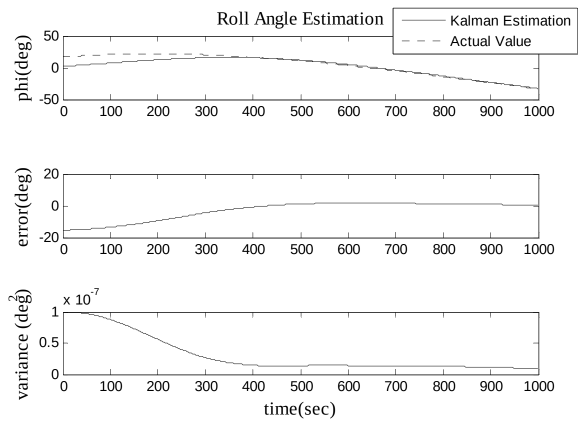

Figure 2. Roll angle estimation by AUKF in case of change in the process noise covariance

Furthermore, so as to unfurl the difference between the estimation characteristic in case of change in the process noise covariance, absolute values of error at two distinct time steps are tabulated below (Table 1). As it may be also interpreted from table, AUKF provides better performance than the regular UKF.

Par.	**Abs. Values of Error for Regular UKF**		**Abs. Values of Error for AUKF**	
	600 s.	900 s.	600 s.	900 s.
φ(deg)	1,8915	15,518	1,6595	0,7798
θ(deg)	38,830	0,6788	1,1083	0,1337
ψ(deg)	6,7968	3,5878	3,7717	1,3146
ω_x (deg/s)	1,5488 392e-5	3,3969 015e-5	1,5991 435e-5	1,1687 905e-5
ω_y (deg/s)	1,5488 324e-5	4,2829 485e-5	1,5992 616e-5	9,3625 491e-6
ω_z (deg/s)	1,5488 398e-5	2,5802 310e-5	1,5976 755e-5	1,5694 421e-5

Table 1. Absolute values of error for regular UKF and AUKF in case of change in the process noise covariance.

6 Conclusions

High possibility of any kind of unexpected events in space environment makes satellites vulnerable vehicles in point of view of attitude determination system. Results of this study show that it is not possible to get precise estimation results by optimal regular UKF if the model for the process noise covariance mismatches with the real value as a result of change occurred somehow. On the other hand, presented AUKF algorithm is not affected from those changes and secures its good estimation characteristic all the time.

Differently from the most of the existing adaptive UKF algorithms, covariance is not updated at each step; it has been only corrected when the fault occurs and that brings about a noteworthy reduction in the computational burden. This makes the proposed algorithm appropriate for pico satellites, especially if their limited onboard processing capacity is taken into account.

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