Dynamics and Control of an Autogiro Rotor with a Flexible Hub*

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Abstract: Mathematical model of an autorotating main rotor with flexible hub has been developed. Numerical solution of the model has been presented. Special features of the rotor dynamics have been investigated and discussed. Parametric studies of the A-002M main rotor have been performed. The results of the study have been used in designing the autogiro.

Keywords: autogiro rotor, flexible hub, nonlinear dynamics, robust control

1. INTRODUCTION

Modern world sees increased interest in autogiros, which will have numerous applications, including unmanned aerial platforms. The autogiro theory was first developed by *Glauert* and *Lock*, and than mainly advanced in 20-40th of the former century by *J.B. Weatley*, *F.J. Beiley*, *M.L. Mil'*, *N.I. Kamov*, *A.P. Proskuryakov*, *V.G. Tabachnikov et al.* Due to advancement of the helicopters, until recently the scope of research of main rotor dynamics was mainly restricted by investigation of flapping, whereas auto-rotation study was secondary. Some assumptions of the analytical models of auto-rotation applied earlier did not allow the designers to investigate auto-rotating rotors comprehensively. Our interest in autogiro dynamics was evoked by the A-002 aircraft, which was being developed by IRKUT Corporation, Kalmykov et al. (2002a,b). Previously, the autogiro main rotor models were developed for both steady-state (Polyntsev, 2003a) and unsteady (Polyntsev, 2003b) auto-rotation conditions. An analytical solution to the first model was found, and flapping stability issues considered (Polyntsev, 2003c). Nonlinear dynamics of the two-bladed main rotor with flexible blades was investigated for the cases of flapping-plane bending (Somov and Polyntsev, 2003) and coupled flapping and rotation-plane bending (Somov and Polyntsev, 2004). The results of these studies were used in developing an autogiro flight simulator, which showed good accordance of the predicted results with the experimental data (Polyntsev, 2005). Also, problems of main rotor aero-flexible stability and robust control were addressed (Belyash et al., 2005; Somov and Polyntsev, 2005). This work focuses on modeling the autogiro rotor (Fig. 1) with the flexible rotor head invented by *A. Tatarnikov* and *O. Polyntsev* (patent RU 2281885C1). The rotor consists of two blades attached to the hub, which incorporates three flapping hinges and flexible beams, restricting individual flapping of the blades, Fig. 2.

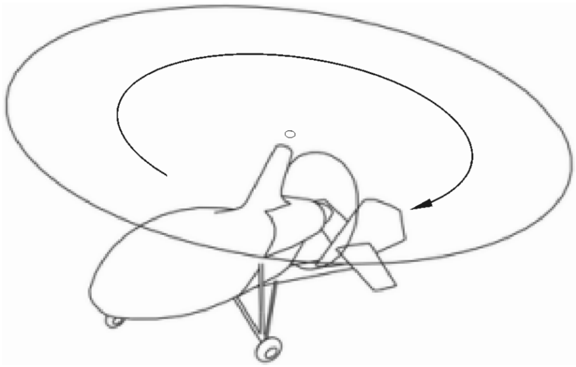

Fig. 1. The scheme of the autogiro

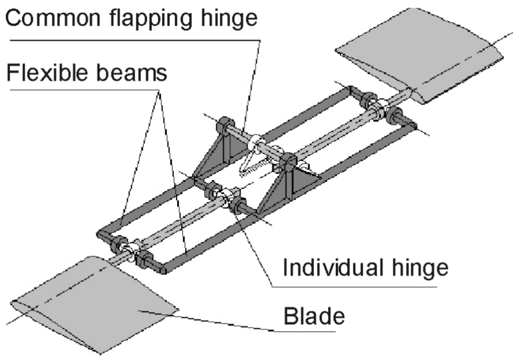

Fig. 2. The scheme of the flexible hub

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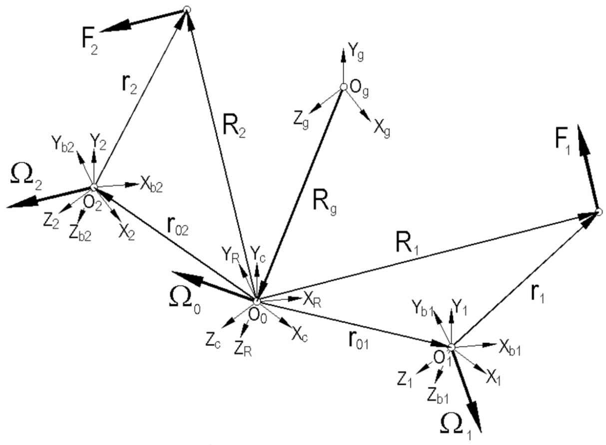

Fig. 3. Kinematic scheme

2. MODEL OF AN AUTOGIRO ROTOR WITH FLEXIBLE HUB

The following assumptions are made with respect to the model:

- blades are absolutely rigid;
- flexible oscillations of the beams in the rotation plane are negligible;
- rotor is in a steady-state auto-rotation regime.

Blade-element motion is considered according to the kinematic scheme in Fig. 3 with the notations: O_0 is center of the common flapping hinge; O_1 and O_2 are centers of individual hinges, and O_g is center of the fixed coordinate system (the Earth).

If restriction by the flexible beam is not considered, motion of the blade-element with distributed mass m_r and dx length is described by the following relation

$$m_r \ddot{\mathbf{R}}_{gx} = \mathbf{F}_s dx,$$

where s is number of the considered blade (hereafter, $s = 1$ or $s = 2$); $\mathbf{R}_{gx} = \mathbf{R}_g + \mathbf{R}_s$ is a radius-vector of a blade-element with respect to the fixed coordinate system; $\mathbf{F}_s$ is vector of external distributed forces (aerodynamic forces and gravity), and $(\dot{}) = \partial()/\partial t$ – standard notation.

If $\boldsymbol{\Omega}_0$ is angular rate of the rotor, and $\boldsymbol{\Omega}_s$ is angular rate of the blade s with respect to the hub, then the vector of absolute acceleration is written as

$$\ddot{\mathbf{R}}_{gx} = \dot{\boldsymbol{\Omega}}_0 \times \mathbf{R}_s + \dot{\boldsymbol{\Omega}}_s \times \mathbf{r}_s + \mathbf{W}_s,$$

where

$$\mathbf{W}_s = \boldsymbol{\Omega}_0 \times (\boldsymbol{\Omega}_0 \times (\mathbf{r}_{0s} + \mathbf{r}_s)) + (\boldsymbol{\Omega}_s + 2\boldsymbol{\Omega}_0) \times (\boldsymbol{\Omega}_s \times \mathbf{r}_s).$$

Reference frames applied to the model are shown in Fig. 4, and the matrixes are defined by the following expressions using the standard notations:

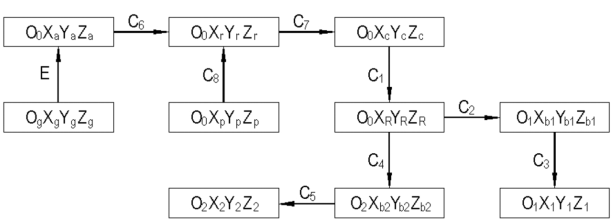

Fig. 4. Reference frames

$$\mathbf{C}_1 = [\beta_0]_3;\quad \mathbf{C}_2 = [\mathrm{a_c}]_3;\quad \mathbf{C}_3 = [\beta_1]_3;\quad \mathbf{C}_4 = \mathbf{C}_2[\pi]_2;$$
$$\mathbf{C}_5 = [\beta_2]_3;\ \mathbf{C}_6 = [\alpha_\mathrm{R}]_3;\ \mathbf{C}_7 = [-(\pi + \psi_\mathrm{L})]_2;\ \mathbf{C}_8 = [\delta_\mathrm{p}]_3;$$

$\mathbf{E}$ – identity matrix, where β_0 and β_1, β_2 are flapping angles in common and individual hinges, respectively; $\mathrm{a_c}$ is design rotor cone angle; α_R is rotor's angle of attack; ψ_L is azimuth of the first blade; δ_p is controlled pitch angle.

Since sum of the elementary moments amounts to zero on the three flapping hinges, by integrating one could obtain the equation of common hinge flapping

$$\int \{\mathbf{R}_1 \times (\mathbf{F}_1 - m_r \ddot{\mathbf{R}})_{g1}\}_{zc} dx - \{\mathbf{r}_{b1} \times \mathbf{F}_{y1}\}_{zc} + \int \{\mathbf{R}_2 \times (\mathbf{F}_2 - m_r \ddot{\mathbf{R}}_{g2}\}_{zc} dx - \{\mathbf{r}_{b2} \times \mathbf{F}_{y2}\}_{zc} = 0, \quad (1)$$

and two equations for the individual hinge flapping motions

$$\int \{\mathbf{r}_s \times (\mathbf{F}_s - m_r \ddot{\mathbf{R}}_s)\}_{zbs} dx + \{\mathbf{r}_{as} \times \mathbf{F}_{ys}\}_{zbs} = 0, \quad (2)$$

where integral

$$\int (\cdot) dx \equiv \int_{x_2}^{X} (\cdot) dx;$$

x_2 is a horizontal offset of a blade and X is blade length; $\mathbf{r}_{bs}$ and $\mathbf{r}_{as}$ are radius-vectors of the blade-beam junction points with respect to points O_0 and O_s, respectively; $\mathbf{F}_{ys}$ is vector of reaction of the beam s.

Forced flexible oscillations of the beam element with length dr are described by the relation

$$[EJy'']'' dr - [N_s y']'\ dr - F_x y''\ dr q_b + \ddot{y}\ dr = F_y, \quad (3)$$

where $(\cdot)' \equiv \partial(\cdot)/\partial x$ – standard notation; y is flapping-plane displacement; EJ is bending stiffness of the beam's section; N_s is a tensile force; F_x and F_y are components of the vector $\mathbf{F}_{ys}$ in the beam's coordinate system; q_b is distributed mass of the beam.

To solve the boundary problem (3) the *Bubnov – Galerkin* method is used as the method of given forms (Morozov et al., 1995). The beam's displacement is presented as

$$y(t, r) = \sum_{j=1}^{n} h^{(j)}(r)\ \zeta^{(j)}(t),$$

where $\zeta^{(j)}(t)$ and $h^{(j)}(r)$ are oscillation amplitude and function of natural mode shape for mode j, respectively.

The shape functions are estimated separately by finite element analysis. After transformations, one can achieve the following equation for any mode (symbol j is omitted hereafter):

$$F_{ys} h_\mathrm{L} = \zeta C_s + \ddot{\zeta} m_b.$$

Here

$$C_s = c_b + F_{xs} K_\mathrm{F}; m_b = \int q_b h^2 dr; K_\mathrm{F} = \int (h')^2 dr;$$
$$c_b = \int EJ(h'')^2\ dr + \int N_s (h')^2\ dr$$

with notation for the integral

$$\int (\cdot)\ dr = \int_0^{x_1} (\cdot)\ dr,$$

where x_1 is length of the cantilevered beam; h_L is value of the shape function in the point of the applied force.

Calculations show that for the projected application of this rotor hub only first bending mode is significant, and no resonant oscillations is expected for the higher-order modes in the operational regimes. Therefore, the solution of the problem (1) – (3) is simplified, taking into account a kinematic link between oscillations of the beams and flapping motions of the blades. Components of the angular rate vectors in the corresponding reference frames are presented as follows

$$\{\mathbf{\Omega}_0\}_c = [0,\ -\omega,\ \dot{\beta}_0]^{\mathrm{t}}; \{\mathbf{\Omega}_s\}_{bs} = [0,\ 0\ \dot{\beta}_s]^{\mathrm{t}},$$

where ω is rotor's angular rate. Hence, the final three equations describing three flapping motions are expressed as

$$\ddot{\beta}_0 = (M_{\mathrm{a}0} + M_{y0} - M_0)/J_{\mathrm{R}y};$$

$$\ddot{\beta}_s = (M_{\mathrm{a}s} + M_{us} - M_s)/J_{ys},$$

where $M_{\mathrm{a}0}$ and $M_{\mathrm{a}s}$ are moments of external forces; M_{y0} and M_{us} are moments caused by stiffness of the beams; $J_{\mathrm{R}y}$ and J_{ys} are time-dependant moments of inertia; M_0 and M_s are moments of inertial forces. The components are determined by the following relations:

$$M_0 = \sum_s M_{0s}; M_{0s} = \int m_r\{\mathbf{R}_s \times (\dot{\mathbf{\Omega}}_s \times \mathbf{r}_s + \mathbf{W}_s)\}_{zc}\ dx;$$

$$M_{\mathrm{a}0} = \sum_s \int m_r\{\mathbf{R}_s \times \mathbf{F}_s\}_{zc}\ dx; \quad M_{y0} = \sum_s l_s c_{0s};$$

$$l_1 = \{\mathbf{r}_{b1}\}_{xc}\cos(\mathrm{a_c} + \beta_0) + \{\mathbf{r}_{b1}\}_{yc}\sin(\mathrm{a_c} + \beta_0);$$

$$l_2 = \{\mathbf{r}_{b2}\}_{xc}\cos(\mathrm{a_c} - \beta_0) + \{\mathbf{r}_{b2}\}_{yc}\sin(\mathrm{a_c} - \beta_0);$$

$$c_{0s} = x_1[(c_b + N_{xs}K_{\mathrm{F}})\sin\beta_s + m_b(\ddot{\beta}_s\ \cos\beta_s - \dot{\beta}_s^2\ \sin\beta_s)];$$

$$M_{\mathrm{a}s} = \int\{\mathbf{r}_s \times \mathbf{F}_s\}_{zbs}; \quad M_{us} = c_s x_1 \cos\beta_s;$$

$$c_s = x_1(c_b + N_{ys}K_{\mathrm{F}} - m_b\dot{\beta}_s^2)\ \sin\beta_s;$$

$$N_{xs} = \int\{\mathbf{F}_s - m_r(\dot{\mathbf{\Omega}}_s \times \mathbf{r}_s + \mathbf{W}_s)\}_{xbs}\ dx;$$

$$N_{ys} = \int\{\mathbf{F}_s - m_r(\dot{\mathbf{\Omega}}_s \times \mathbf{r}_s + \mathbf{W}_s)\}_{ybs}\ dx;$$

$$J_{\mathrm{R}y} = J_{\mathrm{R}} - l_1 J_{01} + l_2 J_{02}; J_{\mathrm{R}} = \sum_s \int m_r(R_{sxc}^2 + R_{syc}^2)dx;$$

$$J_{0s} = x_1 S_{0s} K_{\mathrm{F}}\ \sin\beta_s; \quad S_{0s} = \int m_r\{\mathbf{R}_s\}_{ybs}\ dx;$$

$$J_{ys} = J_b + x_1 S_s \cos\beta_s; S_s = x_1(m_b\cos\beta_s + S_b K_{\mathrm{F}}\sin^2\beta_s);$$

$$J_b = \int m_r x^2 dx; \quad S_b = \int m_r x dx.$$

Final equation required to complete the model describes steady-state auto-rotation as $\dot{\psi}_{\mathrm{L}} = \omega_r$.

3. DYNAMICS OF AN AUTOGIRO ROTOR WITH FLEXIBLE HUB

The system of differential equations is solved by means of the *Runge – Kutta* method. Nonlinearity of the system results from nonlinear dependencies of aerodynamic forces upon local attack angles, *Mach* and *Reynolds* numbers, and flexible deformations of the beams; appearance of effects of flow non-stationarity; non-uniformity of distribution of mass and stiffness etc. To add, the blades are under influence of non-symmetrical air stream.

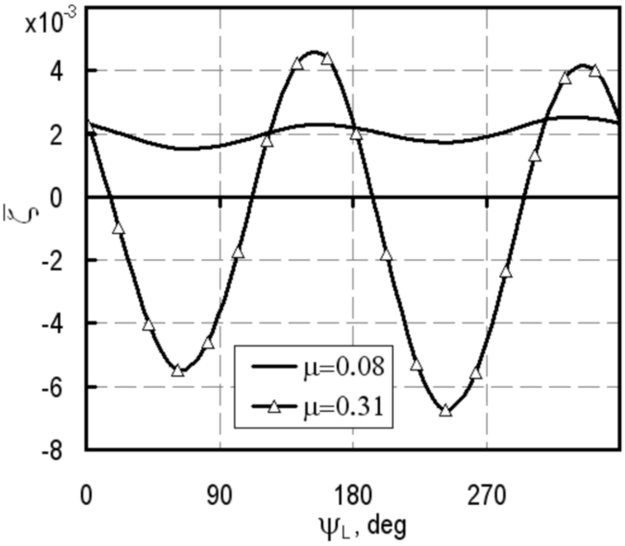

Fig. 5. Deformation coefficient vs. azimuth

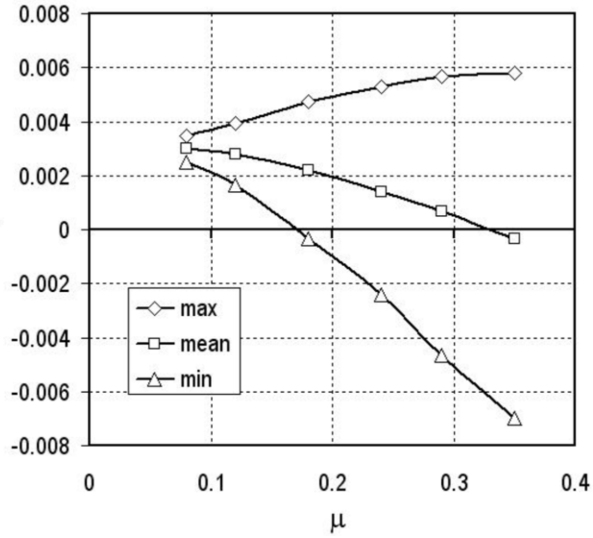

Fig. 6. Deformation coefficient vs. advance ratio

It is well known that aerodynamics of a main rotor is a separate complicated problem (Mil' et al., 1966; Boyd et al., 2002). Thus, for the purpose of our study we apply the modified classic blade-element theory (Polyntsev, 2004).

Input parameters for the model are calculated separately using the analytical solution of the auto-rotating rotor (Polyntsev, 2003c).

In order to illustrate the special features of the rotor dynamics, calculations were performed for the A-002M autogiro developed by IRKUT Corporation. Fig. 5 presents deformation coefficient $\bar{\zeta} = \zeta/x_1$ of a blade versus azimuth and rotor's tip-speed ratio μ. It is seen that the beams oscillate with doubled auto-rotation frequency.

Fig. 6 illustrates that oscillation magnitudes go up with the advance ratio μ value. Parametric studies of the beam stiffness influence on the rotor dynamics revealed that by changing the stiffness it was possible to minimize oscillations magnitudes, and, therefore, loads and vehicle vibrations.

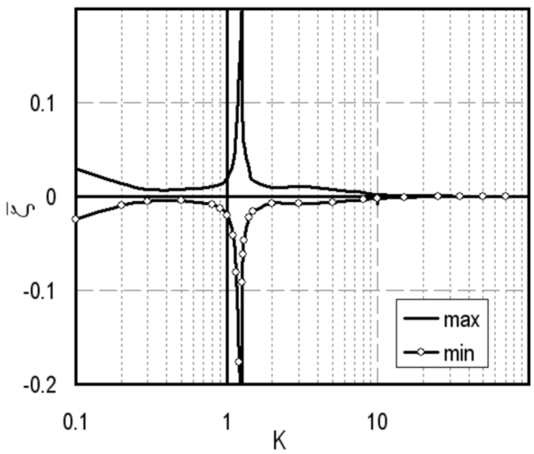

Fig. 7. Deformation coefficient vs. K-ratio

Fig. 7 shows deformation coefficient values versus K, where K is ratio of parametric and actual bending stiffness of the A-002M hub beams.

The other finding indicated significance of avoidance the resonant oscillations coming from inadequate stiffness and leading to increased vibrations.

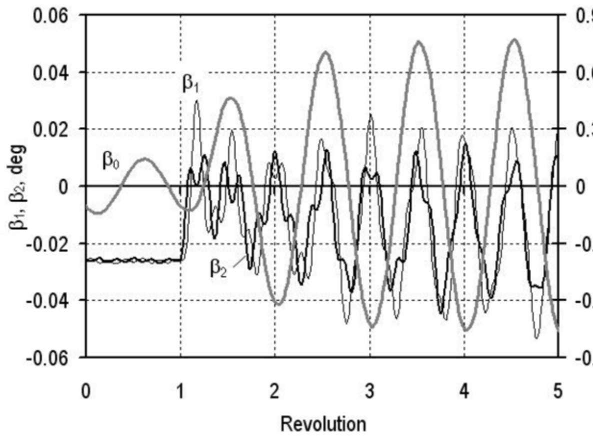

Fig. 8. Wind gust dynamics

Fig. 8 shows transient behavior of the blades after the sudden gust of wind. It is seen that stabilization of the flapping motions occurs in four-five revolutions.

4. ROBUST CONTROL OF AN AUTOGIRO

Applied general approach to synthesis of *nonlinear* control system (NCS) with a partial measurement of its state is presented, moreover the method of *vector Lyapunov functions* (VLF), which has a strong mathematical basis for analysis of stability of various nonlinear interconnected systems with the *discontinuous right-hand side*, is used in cooperation with the *exact feedback linearization* (EFL) technique. Let there be given a nonlinear controlled object

$$D^{+}\mathrm{x}(t)=\mathcal{F}(\mathrm{x}(t),\mathrm{u});\quad \mathrm{x}(t_0)=\mathrm{x}_0;\ t\in \mathrm{T}_{t_0},$$

where $\mathrm{x}(t)\in\mathcal{H}\subset\mathbb{R}^n$ is a state vector with an initial condition $\mathrm{x}_0\in\mathcal{H}_0\subseteq\mathcal{H}$; $\mathrm{u}=\{u_j\}\in\mathrm{U}\subset\mathbb{R}^r$ is a control vector. Let some *vector norms* $\rho(\mathrm{x})\in\overline{\mathbb{R}}_+^l$ and $\rho^0(\mathrm{x}_0)\in\overline{\mathbb{R}}_+^{l_0}$ also be *given*. For any control law (CL) $\mathrm{u}=\mathcal{U}(\mathrm{x})$ the closed-loop system has the form

$$D^{+}\mathrm{x}(t)=\mathcal{X}(t,\mathrm{x});\quad \mathrm{x}(t_0)=\mathrm{x}_0, \qquad (4)$$

where $\mathcal{X}(t,\mathrm{x})=\mathcal{F}(\mathrm{x},\mathcal{U}(\mathrm{x}))$, $\mathcal{X}:\mathrm{T}_{t_0}\times\mathcal{H}\to\mathcal{H}$ is a *discontinuous* operator. Assuming the existence and the non-local continuability of the *right-sided* solution $x(t)\equiv\mathrm{x}(t_0,\mathrm{x}_0;t)$ of the system (4) for its *extended definition* in the aspect of physics, the most important dynamic property is obtained, that is $\rho\rho^0$-*exponential invariance* of the solution $x(t)=0$ under the *desired* $\gamma\in\overline{\mathbb{R}}_+^l$:

$$(\exists\alpha\in\mathbb{R}_+)\,(\exists\mathcal{B}\in\overline{\mathrm{B}}_+^{l\times l_0})\,(\exists\delta\in\mathbb{R}_+^{l_0})\,(\forall\rho^0(\mathrm{x}_0)<\delta)$$
$$\rho(x(t))\le\gamma+\mathcal{B}\,\rho^0(\mathrm{x}_0)\,\exp(-\alpha(t-t_0))\quad\forall t\in\mathrm{T}_{t_0}.$$

For the VLF $\upsilon:\mathcal{H}\to\overline{\mathbb{R}}_+^k$ with components $\upsilon^s(\mathrm{x})\ge0$, $\upsilon^s(0)=0$, $s=1:k$ and the norm $\|\upsilon(\mathrm{x})\|=\max\{\upsilon^s(\mathrm{x})\}$, defined are scalar function $\overline{\upsilon}(\mathrm{x})=\max\{\upsilon^s(\mathrm{x}),s=1:l_k,1\le l_k\le k\}$ and a υ *upper right derivative* with respect to (4):

$$\overline{\upsilon}\,'(\mathrm{x})\equiv\overline{\lim_{\delta t\to0+}}\;(\upsilon(\mathrm{x}+\delta t\,\mathcal{X}(t,\mathrm{x}))-\upsilon(\mathrm{x}))/\delta t.$$

Theorem (Somov et al., 2007). *Let there exist the VLF* υ*, so that*:

1) $(\exists\mathrm{a}\in\mathbb{R}_+^l)\ (\forall\mathrm{x}\in\mathcal{H})\qquad \rho(\mathrm{x})\le\mathrm{a}\cdot\overline{\upsilon}(\mathrm{x})$;
2) $(\exists\mathrm{b}\in\mathbb{R}_+^{l_0})\ (\forall\mathrm{x}_0\in\mathcal{H}_0)\ \|\upsilon(\mathrm{x}_0)\|\le\langle\mathrm{b},\rho^0(\mathrm{x}_0)\rangle$;
3) $\exists\gamma_c\in\overline{\mathbb{R}}_+^k$, *a function* $\varphi_\gamma(\cdot)$ *exists for* $\gamma_c\le\varphi_\gamma(\mathrm{a},\gamma)$;
4) $\forall\ (t,\mathrm{x})\in(\mathrm{T}_{t_0}\times\mathcal{H})$ *the conditions are satisfied*:
   a) $\overline{\upsilon}'_\gamma(\mathrm{x})\dot{\le}\mathrm{f}_c(t,\upsilon_\gamma(\mathrm{x}))\equiv\mathrm{P}\upsilon_\gamma(\mathrm{x})+\tilde{\mathrm{f}}_c(t,\upsilon_\gamma(\mathrm{x}))$;
   b) Hurwitz condition for positive matrix P;
   c) Ważewski condition on quasi-monotonicity for the function $\tilde{\mathrm{f}}_c(t,\mathrm{y})$;
   d) Carateodory condition for the function $\tilde{\mathrm{f}}_c(t,\mathrm{y})$*, bounded in each domain* $\Omega_c^r=(\mathrm{T}_{t_0}\times\mathcal{S}_c^r)$*, where* $r>0$ *and* $\mathcal{S}_c^r=\{\mathrm{y}\in\mathbb{R}^k:\|\mathrm{y}\|_{\mathrm{E}}<r\}$;
   e) $(\tilde{\mathrm{f}}_c(t,\mathrm{y})/\|\mathrm{y}\|)\ \overset{t\in\mathrm{T}_{t_0}}{\Longrightarrow}\ 0$ *for* $\mathrm{y}\to0$ *uniformly with respect to time* $t\in\mathrm{T}_{t_0}$,

where $\upsilon_\gamma=\upsilon-\gamma_c$*. Then solution* $x(t)=0$ *of the system* (4) *is* $\rho\rho^0$*-exponential invariant and the matrix* $\mathcal{B}$ *has the form* $\mathcal{B}=c\cdot\mathrm{ab}^{\mathrm{t}}$ *with* $c\in\mathbb{R}_+$.

There is such an important problem: by what approach is it possible to create *constructive* techniques for constructing the VLF $\upsilon(\mathrm{x})$ and *simultaneous* synthesis of a nonlinear control law $\mathrm{u}=\mathcal{U}(\mathrm{x})$ for the close-loop system (4) with given vector norms $\rho(\mathrm{x})$ and $\rho^0(\mathrm{x}_0)$? Recently, a pithy technique on constructing VLF at such synthesis has been elaborated. This method is based on a *nonlinear transformation* of the NCS model and solving the problem in two stages.

In stage 1, the right side $\mathcal{F}(\cdot)$ in (4) is transformed as $\mathcal{F}(\cdot)=\mathrm{f}(\mathrm{x})+\mathrm{G}(\mathrm{x})\,\mathrm{u}+\tilde{\mathcal{F}}(t,\mathrm{x}(t),\mathrm{u})$, some *principal variables* in a state vector $\mathrm{x}\in\tilde{\mathcal{H}}\subset\mathbb{R}^{\tilde{n}}\subseteq\mathbb{R}^n$ with $\tilde{n}\le n$, $\mathrm{x}_0\in\tilde{\mathcal{H}}_0\subseteq\tilde{\mathcal{H}}$ are selected and a *simplified nonlinear model* of the object (4) is presented in the form of an affine *quite smooth* nonlinear control system

$$\dot{\mathrm{x}}=\mathrm{F}(\mathrm{x},\mathrm{u})\equiv\mathrm{f}(\mathrm{x})+\mathrm{G}(\mathrm{x})\mathrm{u}\equiv\mathrm{f}(\mathrm{x})+\sum\mathrm{g}_j(\mathrm{x})u_j,$$

which is structurally synthesized by the EFL technique. In this aspect, based on the structural analysis of *given* vector norms $\rho(\mathrm{x})$ and $\rho^0(\mathrm{x})$, and also vector-functions $\mathrm{f}(\mathrm{x})$ and $\mathrm{g}_j(\mathrm{x})$, the *output vector-function* $\mathrm{h}(\mathrm{x})=\{\mathrm{h}_i(\mathrm{x})\}$ is carefully selected. Furthermore, the nonlinear invertible (one-to-one) coordinate transformation $z = \Phi(x)\ \forall x \in \mathcal{S}_h \subseteq \tilde{\mathcal{H}}$ with $\Phi(0) = 0$ is analytically obtained with *simultaneous* constructing the VLF. Finally, bilateral component-wise inequalities for the vectors $x, z, \upsilon(x), \rho(x), \rho^0(x_0)$ are derived, it is most desirable to obtain the *explicit* form for the nonlinear transformation $x = \Psi(z)$, inverse with respect to $z = \Phi(x)$, and the VLF aggregation procedure is carried out with analysis of proximity for a singular directions in the *Jacobian* $[\partial F(x, \mathcal{U}(x))/\partial x]$.

In stage 2, the problem of nonlinear CL synthesis for the *complete model* of the NCS (4), taking rejected coordinates, nonlinearities and restrictions on control, into account is solved by the VLF-method. If a forming control is digital, a measurement the model's state is discrete and incomplete, then a simplified nonlinear discrete object's model is obtained by *Teylor-Lie* series, a *nonlinear* digital CL is formed and its parametric synthesis is carried out with a simultaneously construct a *discrete sub-vector* VLF.

Principal problems on a robust stabilization of the autogiro flexible rotor were studied. This research was carried out in association with ground physical experiments and identification of the main rotor parameters during full-scale test flights.

Obtained results on onboard signal processing by multiple discrete filtering, on guidance and nonlinear robust digital control applied for the aircraft will presented in a final variant of this paper.

5. PRACTICAL REALIZATION

Based on the results of main rotor dynamics research, the authors developed several simulation software products (Kalmykov et al., 2002a,b; Polyntsev, 2005), which allow engineers to investigate spatial motion of autogiros in various flight regimes, including take-off, climbing, descending, level flight, and transient manoeuvres.

Dynamic loads on the vehicle structure could be investigated using these programs as well. Simulation results give opportunity to optimize flight regimes with a view to improving aircraft dynamical features and raising in-flight safety.

In order to verify the developed engineering techniques three autogiros were designed and built: two-seated in 1997 and two three-seated in 2000 and 2005. The autogiros were equipped with flight recorders to obtain data from the strain-gage systems to define actual loads on the rotor pylon and mechanical control system. Good accordance of the predicted parameters with their experimental values was observed. Some details will presented in a final variant of this paper.

The results were used in designing the main rotor for the A-002 and A-002M autogiros. These aircraft were flight tested and presented at the Moscow Aero-Space Exhibitions MAKS 2001-2007, see Fig. 9.

Fig. 9. The A-002M autogiro in flight

6. CONCLUSIONS

Mathematical model of a two-bladed main rotor with flexible hub has been developed to describe three fundamental flapping-plane motions of the blades. Major special features of the rotor dynamics in the flapping plane have been studied and discussed. The mathematical model has been implemented in the A-002M autogiro design processes.

Principal problems of robust control and stabilization of the autogiro flexible rotor have been studied and associated both with ground physical experimental research and natural flights.

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