CONTROL OF PANIC BEHAVIOR IN A NON IDENTICAL NETWORK COUPLED WITH A GEOGRAPHICAL MODEL

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Abstract
In this paper, our aim is to study the control of panic behavior in a non-identical network coupled with a model for human behaviors during catastrophic events, and to establish the link between control and synchronization of the network. We show how to model the domino effect in the case of a succession of disasters and exhibit a Hopf bifurcation. We explore the patterns emerging from basic two-nodes configurations and analyze the effect of the coupling strength on the bifurcations that occur in the subsequent dynamical system. We illustrate our qualitative results by a numerical simulation of a specific catastrophe, prepared with the collaboration of geographers.

Key words
Control, coupled network, synchronization, panic, bifurcation.

1 Introduction

The topic of synchronization in coupled networks has been widely studied in the last two decades. Different methods have been proposed to give a general treatment of synchronization [Belykh et al., 2005], [Pecora and Carroll, 1998], [Arenas et al., 2008]. Chaotic systems and bifurcations towards periodic orbits have been analyzed in [Aziz-Alaoui, 2006] and [Golubitsky and Stewart, 2006] respectively. The theory of normal forms has been used in [Rink and Sanders, 2015] to study general networks, among them non-identical networks, which we aim to study in this paper, by establishing how synchronization can be related to control.

Indeed, the PCR system (Panic-Control-Reflex) has been proposed in 2013 to better understand and predict human behaviors facing a catastrophic event. Observing a population affected by a brutal disaster, with a natural or industrial origin, we consider 3 different subgroups of behaviors, namely the *panic*, *reflex*, and *control* behaviors, in addition to the *daily* behavior [Verdière et al., 2014], [Provitolo et al., 2015], [Cantin et al., 2016]. The main question we are concerned with, is to control the level of panic. In order to model the geographical relief of the impacted zone, we consider a network coupled with non-identical instances of the PCR system: a subset of the nodes shall be coupled with a PCR system exhibiting a persistence of panic, while the rest of the nodes would be coupled with a PCR system which presents a return to daily behavior. We would like to study the effect of the topology on the bifurcation identified in the PCR system, and find out how the control of panic can be linked to the synchronization of the network by a lightened disposal of connections from panic nodes towards the other nodes.

This paper is organized as follows. In the next section, we present the PCR system, its basic properties, study the stability of the equilibrium points, and identify the parameter involved in the panic persistence phenomenon. Then, we show how to model a succession of disasters and prove that a Hopf bifurcation can occur in the system. After those preliminaries, we define non-identical PCR networks and explore patterns emerging from basic two-nodes configurations. We finish our paper with the study of a particular catastrophe, namely, the Mediterranean tsunami.

2 Panic-Control-Reflex system

The PCR system is given by the following adimensional system of ordinary differential equations:

$$\begin{cases} \dot{r} = & \gamma(t)q(1-r) - (B_1+B_2)r + F(r,\ c)rc \\ & +G(r,\ p)rp \\ \dot{c} = & B_1 r + C_1 p - C_2 c - F(r,\ c)rc \\ & +H(c,\ p)cp - \varphi(t)c(r+c+p+q) \\ \dot{p} = & B_2 r - C_1 p + C_2 c - G(r,\ p)rp \\ & -H(c,\ p)cp \\ \dot{q} = & -\gamma(t)q(1-r). \end{cases} \tag{1}$$

The unknown functions r, c, p, q are real valued functions, and denote respectively the densities of individuals in *reflex*, *control*, *panic* and *daily* behaviors, among a population concerned with the catastrophe. The functions γ and φ model respectively the beginning of the disaster, and the return to a daily behavior. They both satisfy the properties

$$\gamma(t) \geq 0, \quad \varphi(t) \geq 0, \quad \forall t \geq t_0, \tag{2}$$

$$\gamma(t) = \varphi(t) = 1, \quad \forall t \geq t_1, \tag{3}$$

for given $t_1 > t_0 \geq 0$. The evolution parameters $B_i > 0, C_i \geq 0, i \in \{1,\ 2\}$, model the behavioral changes of each individual, while the imitation functions F, G and H model the interaction phenomena that act in parallel. The next proposition summaries the qualitative results of the mathematical analysis.

Proposition 2.1. *For any initial condition* $(r_0,\ c_0,\ p_0,\ q_0) \in (\mathbb{R}^+)^4$*, the system (1) admits a unique global solution whose components are positive and bounded. If* $C_1 > 0$*,* $\mathcal{O}(0,\ 0,\ 0,\ 0)$ *is the only equilibrium point, and it is locally asymptotically stable. If* $C_1 = 0$*, the system presents a persistence of panic behavior, that is:*

$$\lim_{t\to+\infty} p(t) = \bar{p} > 0.$$

Remark 2.1. *The parameter* C_1 *models the behavioral evolution from panic to control. The latter proposition shows that the solution of the PCR system bifurcates towards a persistence of panic when* C_1 *approaches* 0. *This bifurcation has been analyzed in [Cantin et al., 2016] as a degeneracy case of a saddle-node bifurcation at infinity.*

3 Hopf bifurcation in the case of a succession of disasters

In this section, we show how to model a succession of disasters. To that aim, we consider the transitional phase of the PCR system, defined by

$$\gamma(t) = 1, \quad \varphi(t) = 0, \quad \forall t \in [t_1,\ t_2], \tag{4}$$

for given $t_2 > t_1 > t_0$. Consequently, we can reduce the system using the substitution $q = 1-r-c-p$. We simplify the equations by assuming that the imitation functions F, G and H are null, in order to clarify our purpose. Indeed, we will explain briefly how to proceed in the general case. In counterpart, we model the domino effect by adding a forcing term as follows:

$$\begin{cases} \dot{s} & = \lambda s + z - s(s^2+z^2) \\ \dot{z} & = -s + \lambda z - z(s^2+z^2) \\ \dot{r} & = (1-r-c-p)(1-r) - (B_1+B_2)r \\ & +s^2(c+p) \\ \dot{c} & = B_1 r + C_1 p - C_2 c - s^2 c \\ \dot{p} & = B_2 r - C_1 p + C_2 c - s^2 p. \end{cases} \tag{5}$$

This choice is motivated by the fact that the domino effect is generated by an external cause, and not by an inherent interaction of the behavioral subgroups of the affected population. The value of λ can be chosen to model various types of successions of disasters. The numerical results presented in the next section are analyzed with a geographical approach that comforts the model. The system (5) has a master-slave structure, and the two first equations correspond to the normal form of a Hopf bifurcation. The external domino effect provokes a return of individuals in control behavior or in panic behavior to a reflex behavior. It can easily be modified when considering that individuals in panic behavior are not subject to this domino effect. The next proposition guarantees that for $\lambda > 0$, the orbit of the whole system (5) is attracted to a limit cycle, whose projection in the $(r,\ c,\ p)$ space is shown in Figure 1.

Proposition 3.1. *For any value of the parameters* $B_1 > 0, B_2 > 0, C_1 \geq 0, C_2 > 0, \lambda \in \mathbb{R}$*, the system (5) admits a non trivial equilibrium point* $\mathcal{E}(0,\ 0,\ 0,\ c_0,\ p_0)$*. Furthermore, a supercritical Hopf bifurcation occurs at* $\mathcal{E}$ *in a neighborhood of* $\lambda = 0$*.*

Proof. The equilibrium points of system (5) are the so-

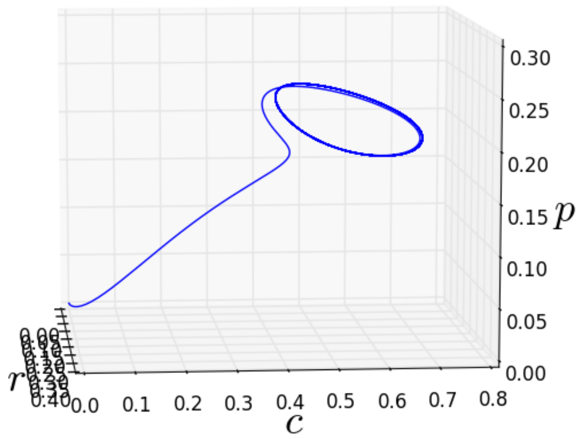

Figure 1. Domino effect in the PCR system. When taking into account a succession of disasters, the orbit is attracted to a limit cycle during the transitional phase.

lutions of the following system:

$$\begin{cases} \lambda s + z - s(s^2+z^2) = 0 \\ -s + \lambda z - z(s^2+z^2) = 0 \\ (1-r-c-p)(1-r) = (B_1+B_2)r - s^2(c+p) \\ B_1 r + C_1 p - C_2 c - s^2 c = 0 \\ B_2 r - C_1 p + C_2 c - s^2 p = 0. \end{cases} \tag{6}$$

The two first equations correspond to the equilibrium points of the normal form of a Hopf bifurcation, hence we obtain $s = z = 0$. Moreover, some basic computations show that $r = 0$, and

$$c = c_0 = \frac{C_1}{C_1+C_2}, \quad p = p_0 = \frac{C_2}{C_1+C_2}. \tag{7}$$

Then, we compute the Jacobian matrix $J(\lambda)$ at $\mathcal{E}(0, 0, 0, c_0, p_0)$.

$$J(\lambda) = \begin{pmatrix} \lambda & 1 & 0 & 0 & 0 \\ -1 & \lambda & 0 & 0 & 0 \\ 0 & 0 & -1-B_1-B_2 & -1 & -1 \\ 0 & 0 & B_1 & -C_2 & C_1 \\ 0 & 0 & B_2 & C_2 & -C_1 \end{pmatrix}. \tag{8}$$

$J(\lambda)$ admits 2 complex eigenvalues $\alpha(\lambda) \pm i\omega(\lambda)$, such that $\alpha(0) = 0$, $\frac{\partial \alpha}{\partial \lambda}(0) = 1 \neq 0$. The other eigenvalues are real and negative. Finally, we easily compute the first Lyapunov number c_1 [Hassard and Wan, 1978], [Kuznetsov, 2004] and check that $c_1 = -1$.

As mentioned previously, we can prove the above proposition in the general case, that is, when the imitation functions F, G and H are not supposed to be null. Indeed, it suffices to consider that the three following assumptions hold :

$$\begin{aligned} (H_1) \quad & \alpha_2 + \delta_2 < B_1 + B_2, \\ (H_2) \quad & a_1 + a_2 < B_1 + B_2, \\ (H_3) \quad & a_3 - a_4 < C_1 + C_2, \end{aligned}$$

where

$$\begin{aligned} a_1 &= F(0,\, c_0)c_0, \quad a_2 = G(0,\, p_0)p_0, \\ a_3 &= \frac{\partial H}{\partial c}(c_0,\, p_0)c_0p_0 + H(c_0,\, p_0)p_0, \\ a_4 &= \frac{\partial H}{\partial p}(c_0,\, p_0)c_0p_0 + H(c_0,\, p_0)c_0, \end{aligned}$$

and α_2, β_2 are parameters involved in the definition of F, G and H [Verdière et al., 2014].

4 Non identical coupled networks

In this section, we consider a graph whose nodes are coupled with non-identical instances of the PCR system. Its state equations can be rewritten

$$\dot{x} = f(t,\, x,\, C_1), \quad t \geq t_0, \quad x \in \mathbb{R}^4, \quad C_1 \geq 0, \tag{9}$$

where $x = (r,\, c,\, p,\, q)^T$, and $f(t,\, x,\, C_1)$ is defined by $f = (f_1,\, f_2,\, f_3,\, f_4)^T$ where

$$\begin{aligned} f_1(t,\, x,\, C_1) &= \gamma q(1-r) - (B_1+B_2)r \\ &\quad + F(r,\, c)rc + G(r,\, p)rp, \\ f_2(t,\, x,\, C_1) &= B_1 r + C_1 p - C_2 c - F(r,\, c)rc \\ &\quad + H(c,\, p)cp - \varphi c(r+c+p+q), \\ f_3(t,\, x,\, C_1) &= B_2 r - C_1 p + C_2 c \\ &\quad - G(r,\, p)rp - H(c,\, p)cp, \\ f_4(t,\, x,\, C_1) &= -\gamma q(1-r). \end{aligned}$$

Definition 4.1. *The nodes which are coupled with an instance of system (1) where $C_1 = 0$ will be called* panic nodes *or* nodes of type (1)*, while those which are coupled with an instance of system (1) where $C_1 > 0$ will be called* control nodes *or* nodes of type (2).

Next, we consider a network made of n nodes x_i, $1 \leq i \leq n$ of type (1), and m nodes $y_j(C_1^j)$, $1 \leq j \leq m$ of type (2), with $C_1^j > 0$, $1 \leq j \leq m$. The whole network system reads

$$\dot{X} = \Phi(t,\, X,\, C) + L\tilde{X}, \tag{10}$$

where the vectors X, $\tilde{X} \in \mathbb{R}^{4(n+m)}$ and $C \in \mathbb{R}^{n+m}$ are defined by

$$\begin{cases} X = (x_1,\, \dots,\, x_n,\, y_1,\, \dots,\, y_m)^T \\ \tilde{X} = (\mathcal{H}x_1,\, \dots,\, \mathcal{H}x_n,\, \mathcal{H}y_1,\, \dots,\, \mathcal{H}y_m)^T \\ C = \left(0,\, \dots,\, 0, C_1^1,\, \dots,\, C_1^m\right), \end{cases}$$

and Φ corresponds to the internal dynamic of each node, and is given by

$$\left(\left(f(t,\ x_i,\ 0)\right)_{1\leq i\leq n},\ \left(f(t,\ y_j,\ C_1^j)\right)_{1\leq j\leq m} \right)^T .$$

The matrix $\mathcal{H}$ determines which components are coupled:

$$\mathcal{H} = \begin{pmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,0 \end{pmatrix}. \tag{11}$$

The latter definition of $\mathcal{H}$ means that we consider that only individuals in a catastrophe behavior (reflex, control or panic, but not daily behavior) are concerned with migrations from one node to another. Finally, L is a zero column sum matrix that contains the coupling terms according to the topology of the network.

Proposition 4.1. *The compact set*

$$\Omega = \left\{ X \in (\mathbb{R}^+)^{4(n+m)},\ \sum_{i=1}^{4(n+m)} X_i \leq 1 \right\}$$

is an invariant region for the flow induced by the PCR network (10).

We skip the proof, since it is very similar to the proof of positiveness and boundedness of the solution of the PCR system (1), detailed in [Cantin et al., 2016].

Definition 4.2. *We will say that the PCR network (10) presents a global return to daily behavior if:*

$$\lim_{t\to+\infty} \|X(t)\|_{\mathbb{R}^{4(n+m)}} = 0. \tag{12}$$

Remark 4.1. *That network can be improved by considering additional quadratic couplings. This work will be presented in a forthcoming paper.*

4.1 Patterns emerging from two-nodes configurations

In this section, we study two-nodes PCR networks, considering a non symmetric coupling, and show which patterns emerge from those basic configurations.

We first consider a two-nodes PCR network with a panic node (x) connected to a control node (y) in a linear form. Such a network is given by the following system:

$$\begin{cases} \dot{x} = f(t,\ x,\ 0) - \eta\mathcal{H}x \\ \dot{y} = f(t,\ y,\ C_1) + \eta\mathcal{H}x, \end{cases} \tag{13}$$

where $x = (r_1,\ c_1,\ p_1,\ q_1)^T$, $y = (r_2,\ c_2,\ p_2,\ q_2)^T$, $C_1 > 0$, $\mathcal{H}$ is defined by (11), and $\eta > 0$ corresponds to the coupling strength.

Proposition 4.2. *The two nodes of system (13) synchronize, and the control node drives the panic node to its dynamic. Furthermore, the equilibrium $\mathcal{O} \in \mathbb{R}^8$ is locally asymptotically stable.*

Proof. We first look for the equilibrium points of node (x), which can be seen as a perturbation of a PCR system. Some basic computations lead to $r_1 = c_1 = p_1 = q_1 = 0$. We then write the 4 equations of node (x) as:

$$\dot{x} = Mx + N(t)x + \epsilon(t,\ x),$$

where M and $N(t)$ are two matrices of order 4 defined by:

$$M = \begin{pmatrix} -B_1 - B_2 - \eta & 0 & 0 & 1 \\ B_1 & -C_2 - \eta & 0 & 0 \\ B_2 & C_2 & -\eta & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{14}$$

$$N(t) = \begin{pmatrix} 0\,0\,0 & \gamma(t) - 1 \\ 0\,0\,0 & 0 \\ 0\,0\,0 & 0 \\ 0\,0\,0 & -\gamma(t) + 1 \end{pmatrix}, \tag{15}$$

and ϵ contains non linear terms:

$$\epsilon(t,\ x) = \begin{pmatrix} -\gamma(t)q_1r_1 + F(r_1,\ c_1)r_1c_1 + G(r_1,\ p_1) \\ -F(r_1,\ c_1)r_1c_1 + H(c_1,\ p_1)c_1p_1 \\ -G(r_1,\ p_1)r_1p_1 - H(c_1,\ p_1)c_1p_1 \\ \gamma(t)q_1r_1 \end{pmatrix}. \tag{16}$$

The eigenvalues of M are given by:

$$-B_1 - B_2 - \eta, \quad -C_2 - \eta, \quad -\eta, \quad -1, \tag{17}$$

thus they are negative, since $\eta > 0$. Furthermore, some basic algebraic computations, and property (3), guaranty that

$$\lim_{t\to+\infty} \|N(t)\| = 0, \tag{18}$$

$$\lim_{\|x\|\to 0} \frac{\|\epsilon(t,\ x)\|}{\|x\|} = 0, \text{uniformly in } t. \tag{19}$$

The Poincaré-Lyapunov theorem (see [Verhulst, 1996] for instance) guarantees that the equilibrium $(0,\ 0,\ 0,\ 0)$ is locally asymptotically stable. Finally, we look for the equilibrium points of node (y). Since $(r_1,\ c_1,\ p_1,\ q_1) = (0,\ 0,\ 0,\ 0)$ is the only equilibrium point of node (x), we obtain the equations corresponding to the equilibrium points of a PCR system (1) with $C_1 > 0$. Thus $(r_2,\ c_2,\ p_2,\ q_2) = (0,\ 0,\ 0,\ 0)$, and the only equilibrium point for the two-nodes network (13) is $0 \in \mathbb{R}^8$. Its stability follows from the stability of the equilibrium point $(0,\ 0,\ 0,\ 0)$ of node (x), combined with proposition (2.1).

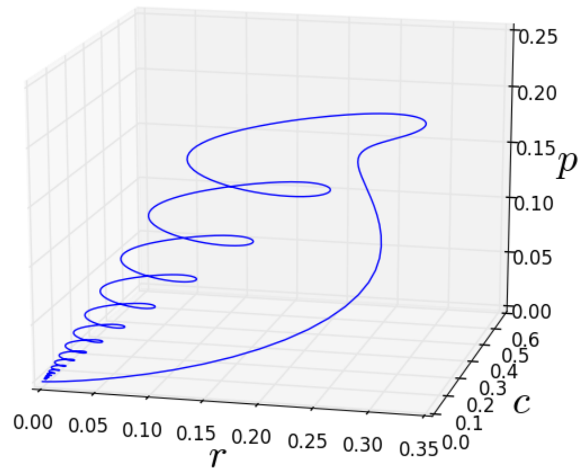

Figure 2. Phase portrait for a panic node connected to a control node in a two-nodes network with domino effect.

Remark 4.2. *This first proposition means that an evacuation of individuals in panic behavior from a panic node towards a control node, brings the whole network to a global return to daily behavior. In other words, the linear coupling affects the bifurcation on the panic node* (x)*, and makes the panic persistence vanish. Furthermore, such an evacuation is similarly efficient to empty the panic behavior in the case of domino effect (see Figure 2).*

We then look ahead to the inverse situation, when a control node (y) is connected towards a panic node (x):

$$\begin{cases} \dot{x} = f(t,\ x,\ 0) + \eta \mathcal{H} y \\ \dot{y} = f(t,\ y,\ C_1) - \eta \mathcal{H} y. \end{cases} \tag{20}$$

Proposition 4.3. *System (20) exhibits a persistence of panic on node* (x)*.*

Proof. We begin with the research of the equilibrium points of node (y), which is very similar to the previous proof, and leads to the uniqueness and local asymptotic stability of $(r_2,\ c_2,\ p_2,\ q_2) = (0,\ 0,\ 0,\ 0)$. It follows that the equilibrium points of node (x) correspond to the equilibrium points of a PCR system (1) with $C_1 = 0$, thus the persistence of panic in node (x).

Remark 4.3. *This second proposition shows that the PCR network associated with system (20) cannot present a global return to the daily behavior, and suggests that a displacement of individuals of a control node towards a panic node should be avoided, at the risk to worsen the panic persistence level.*

4.2 Numerical simulation of a tsunami on the Mediterranean coast

We end our paper with a numerical simulation corresponding to a particular type catastrophe, whose

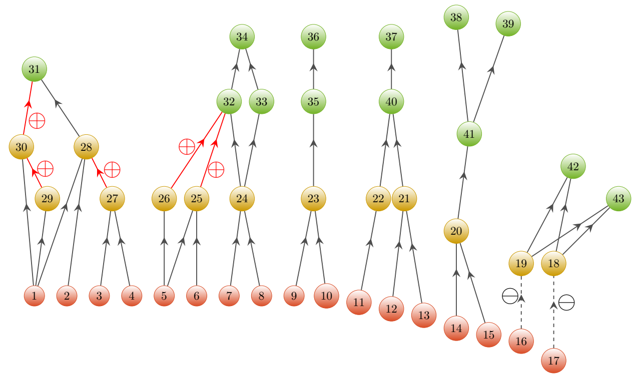

Figure 3. A PCR network corresponding to the geographical relief of a city on the Mediterranean coast.

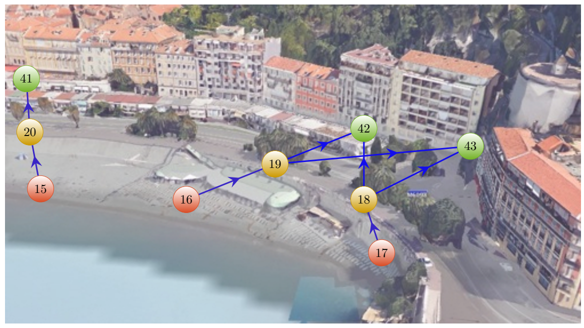

Figure 4. A view of the front coast in Nice, showing the evacuation paths in the case of a Mediterranean tsunami (adapted from *Google Earth*).

risk is clearly identified, due to the presence of the azurean sub-marine fracture, namely, the Mediterranean tsunami [Ioualalen et al., 2014]. The corresponding network has been proposed after a geographical analysis of the relief, and is depicted in Figures 3 and 4.

The nodes $1, 2, \ldots, 17$ correspond to different places on the beach, which are directly concerned with the risk of tsunami. The nodes $18, 19, \ldots 30$ model steps corridors connecting the beach to the *Promenade des Anglais*, which is a large street, approximately 5 meters over the beach places, and can be considered as protected for the water submersion risk. Finally, the nodes $31, 32, \ldots, 43$ represent places in the heart of the city. The value of the evolution parameter C_1 has been chosen accordingly to the nature of those nodes, and is shown with the other parameters values in Table 1. Meanwhile, the coupling strength has been fixed to 0.3. The main disposal of edges corresponds to the usual evacuation paths. We compare the solution of the network with two other different configurations, presented in Figure 3, obtained by adding 5 edges $(25, 32)$, $(26, 32)$, $(27, 28)$, $(29, 30)$ and $(30, 31)$, indicated by $\oplus$, chosen to improve the evacuation of nodes $1, \ldots, 6$, or by removing 2 edges $(17, 18)$ and $(16, 19)$, indicated by $\ominus$, chosen to experiment the effect of broken con-

Table 1. Parameters values for a PCR network corresponding to a Mediterranean tsunami.

Nodes	$1, \ldots, 17$	$18, \ldots 43$
B_1, B_2	0.5	0.5
$\mathbf{C_1}$	**0**	**0.3**
C_2	0.2	0.2
$\alpha_i, \delta_i, \mu_i, i = 1,\ 2$	0.1	0.1

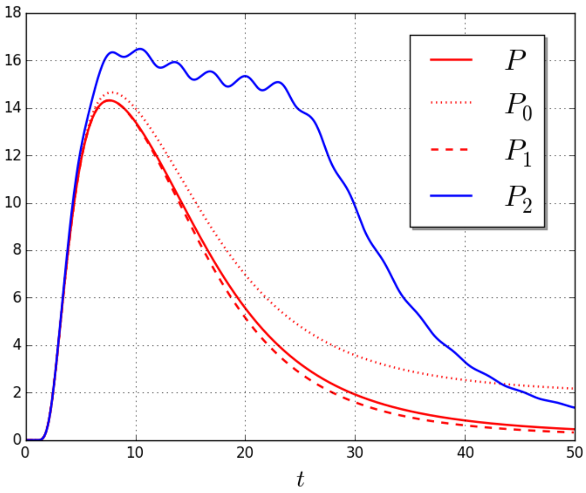

Figure 5. Numerical results for a tsunami on the Mediterranean coast. P corresponds to the population density in panic behavior for the network shown in Figure 3. P_0 shows the effect of 2 removed edges, P_1 the effect of 5 additional edges, and P_2 the effect of a succession of disasters.

nections. The total population density in panic behavior is denoted P for the main configuration, P_1 for the configuration with 5 additional edges, and P_0 for the configuration with 2 removed edges. Furthermore, we experiment a domino effect on the main configuration, and denote by P_2 the total population in panic behavior in that case. The Figure 5 shows the results of the numerical simulation. Those results are in agreement with the qualitative results of the previous section, and show the decisive control of the edges disposal on the panic persistence level in the network. A broken evacuation of a panic node generates an increase of the panic behavior, while an addition of connections towards refuge zones improves the fluidity of human displacements, and accelerates the return of the affected population to the daily behavior.

5 Perspectives

In a forthcoming paper, we shall demonstrate in the general case that the evacuation of every panic node by a oriented chain of linear edges towards a refuge zone, is a necessary and sufficient condition for the synchronization and the global stability of any PCR network.

Acknowledgements

We thank the anonymous referee for valuable suggestions which greatly improve the presentation of the paper. This work has been supported by the French government, through the UCA-JEDI Investments in the Future project managed by the National Research Agency (ANR) with the reference number ANR-15-IDEX-01.

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