Hierarchical Equilibrium in Two-level Aircraft Guidance-stabilization System

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Abstract: In the article the new concept of hierarchical equilibrium in two-level aircraft guidance-stabilization system for inter-level interaction optimization on the basis of the generalization of Stackelberg's strategy for hierarchical differential games is formulated. The sufficient conditions of control law (strategy) optimality of the level-subsystems of the multi-object multi-criteria systems (MMS) are obtained for two-level control system. The approximate iteration procedure of obtaining the hierarchical equilibrium on the basis of the program-corrected control law (PCCL, strategy) of both MMS-subsystem levels is proposed. The block diagram of the two-level aircraft guidance-stabilization system is formed, in which an application of obtained procedure has the practical value.

Keywords: Multilevel control systems, hierarchical equilibrium, guidance, stabilization, aircraft control.

1. ARCHITECTURE AND MODEL OF MULTILEVEL SYSTEM

Multilevel systems are one of the classes of structurally and functionally complicated systems. Analysis and development of such kind of systems are the actual problems of system analysis within control theory. As is known (Plotnikov and Zverev, 1994), the typical structural-functional form of multi-level control system is the four-level form with the following set of levels: decision making – coordination – control optimization – regulation (Fig. 1), which is created over the structurally complex object consisted of the several connected subsystems (or tasks) or over the several connected objects. Meanwhile each of the levels of connected subsystems-tasks jointly with complex object is the multi-object multi-criteria system (MMS) (Voronov, 2001). MMS inherently is the structured task of the level and presents the set of equivalent requirements at the level with subsystems of their realization. These requirements form influences on subsystem of a subordinate level with known lead right.

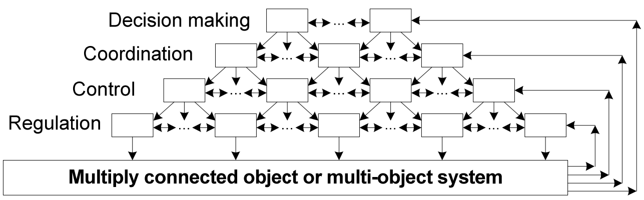

Fig. 1. Alternate functional structure of the multilevel control system.

An example of the part of such kind of multilevel control system and practically useful model for analysis is dual-channel pilotless vehicle control system (PVCS) (Fig. 2).

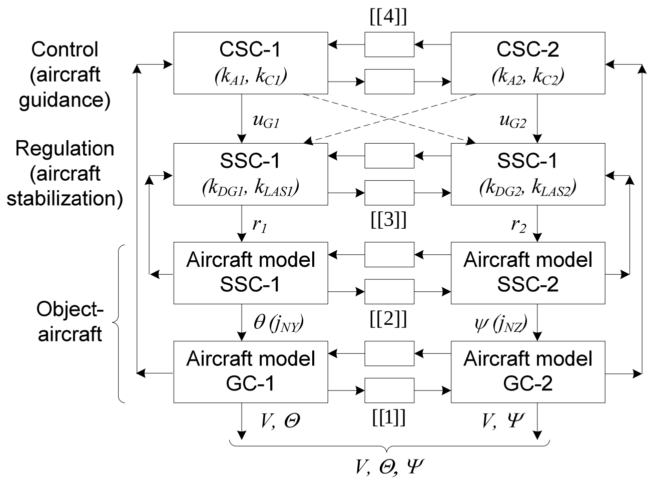

Fig. 2. Two-level guidance-stabilization model of dual-channel PVCS.

According to (Lebedev and Karabanov, 1965) terminology, CSC is aircraft control system channel; SSC is aircraft stabilization system channel; GC is aircraft guidance channel; u_G is aircraft guidance signal; k_A is coefficient of amplification of command production block (control parameter); k_C is transfer coefficient of target coordinator (control parameter); k_{DG} is transfer coefficient of differentiating gyroscope (control parameter); k_{LAS} is transfer coefficient of linear accelerometer sensor (control parameter); r is regulating influence on elevation and yaw rudders of aircraft.

Mathematical model of aircraft dynamics is presented by model of angular (rotary) motion around center of mass by the use of pitch angle θ and yaw angle ψ with according influences of aero-dynamic normal acceleration control (j_N) in center of mass velocity direction in vertical plane by trajectory tilt angle (Θ) control channel and in horizontal plane by trajectory rotation angle (Ψ) (Fig. 3).

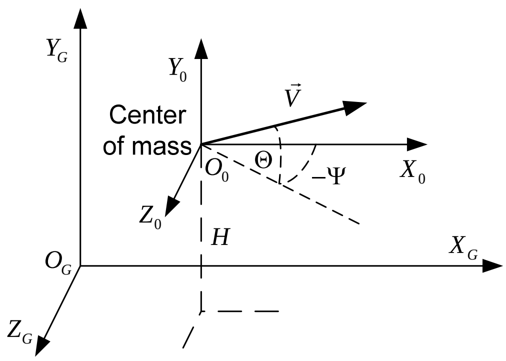

Fig. 3. PVCS state vector (V, Θ, Ψ).

Center of mass state vector (V, Θ, Ψ) in linearized variant of presented system is changed to vector $(\Delta V, \Delta\Theta, \Delta\Psi)$, where the dynamic values are aircraft deviations from the reference dynamic (V_0, Θ_0, Ψ_0).

Blocks with the sign $[[i]], i=\overline{1,4}$ are cross-connections in aircraft center of mass translational motion dynamic, angular (rotary) motion around aircraft center of mass dynamic, between SSC regulators and aircraft three-dimensional guidance accordingly. In typical situation the link between aircraft rotary and translational motion channels is considerable (Lebedev and Karabanov, 1965).

To illustrate the dynamic definition let's examine block diagram of dual-channel definition of linearized stabilization system in block SSC – mathematical model of aircraft angular motion – cross-connection [[2]] (Fig. 4).

a_{ij}, b_{ij} are aero-dynamic coefficients; θ_0, γ_0 are elements of reference trajectory.

Aircraft transfer function (TF):

$$W_{\delta_V}^{\dot{\theta}} = \frac{k(T_1 s+1)}{T^2 s^2 + sT\xi s+1}; \; W_{\delta_H}^{\dot{\psi}} = \frac{k(T_2 s+1)}{T^2 s^2 + sT\xi s+1};$$

$$W_{\dot{\theta}}^{j_{N_Y}} = \frac{\nu}{T_1 s+1}; \; W_{\dot{\psi}}^{j_{N_Z}} = \frac{\nu}{T_2 s+1}.$$

Elevation rudder TF:

$$W_{ER} = \frac{k_{ER}}{T_{ER} s+1}.$$

Angular velocity sensor (differentiating gyroscope) TF:

$$W_{DG\,1,2} = k_{DG\,1,2}.$$

Linear accelerometer sensor TF:

$$W_{LAS\,1,2} = k_{LAS\,1,2}.$$

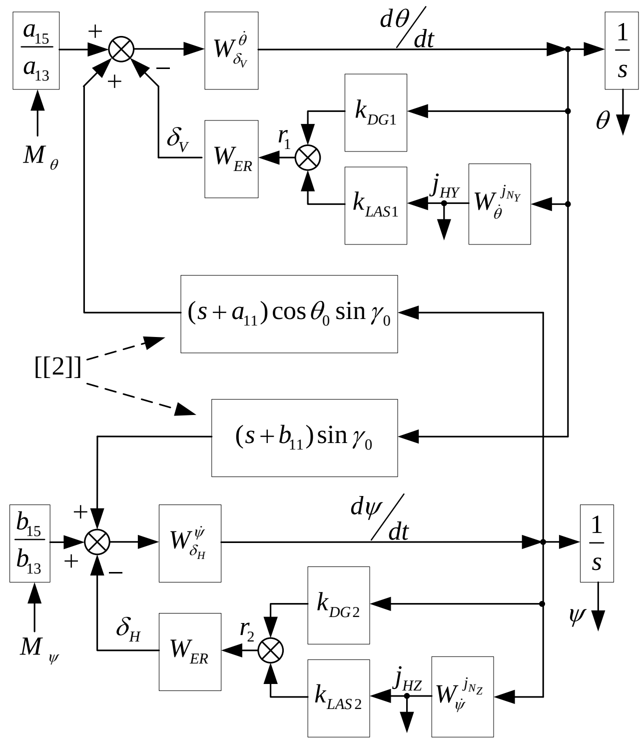

Fig. 4. Dual-channel stabilization system block diagram.

2. DEFINITION AND STRUCTURAL PROPERTIES OF HIERARCHICAL EQUILIBRIUM IN MULTILEVEL CONTROL SYSTEM WITH GENERALIZATION OF STACKELBERG'S STRATEGY

Solution of this problem develops optimization methods of MMS control on the basis of stable-effective compromises (Voronov, 2001). These methods form at the joint of game theory and control theory. The main purpose of optimization methods of MMS control is connected with attaining of efficiency balance in the MMS structure (level of hierarchical system) in different conditions of initial structural disagreement and different conflict games: antagonism, coalition conflict, coalition-free conflict, cooperative interaction.

In this article approaches of hierarchical differential games (HDG) class are developed and applied.

Without restriction of generality let's consider two-level HDG with the "lead right" of the upper level. As opposed to known results (Wisebord and Zhukovsky, 1980) and according to structural requirements of multilevel control system the upper level is structurized MMS with initial structural disagreement (Fig. 5).

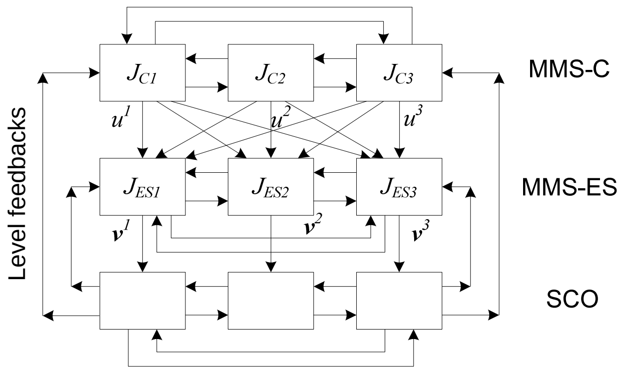

Fig. 5. Block diagram of two-level three-subsystem HDG. Upper level: MMS – Center (MMS-C), lower level: MMS – Executive system (MMS-ES) and Structurally complex object (SCO).

Figure 5 contains traditional designations of two-stage differential game of the Center and Executive system (Wisebord and Zhukovsky, 1980). But according to two-level structure of control-regulation (Fig. 2) the upper level may have the sense of MMS. Thus, in this article comes in generalization of two-stage HDG, in which the upper level is either structurally-functionally distributed Center with elements of initial structural disagreement, what may come in decision making – control problem, constituent particular two-level variant instead of multilevel control system (Fig. 1) or functional level, e.g. in control system in the form of MMS which assigned work of the neighbouring lower level (Fig. 2). This generalization apparently lets move up to more general class of problems (with greater number of levels) and general problem of obtaining of multilevel equilibrium on the basis of combination of developed in this work two-level equilibrium technologies can be considered.

SCO has following mathematical model

$$\dot{\mathbf{x}} = f(t,\ \mathbf{x},\ \mathbf{v},\ \mathbf{u}),\ \mathbf{x}(t_0) = \mathbf{x}_0,\ \mathbf{x} \in \mathbf{E}^n , \quad (1)$$

where $\mathbf{v}$ is executive control with distributed execution (Fig. 5)

$$\mathbf{v} = \left(\mathbf{v}^1,\ \mathbf{v}^2,\ \mathbf{v}^3\right),\ \dim v^i = m_i,\ i = 1,2,3,\ \mathbf{v}^i \in \mathbf{V}_i \subset \mathbf{E}^{m_i}, \\ \dim \mathbf{v} = m = \sum_{i=1}^{3} m_i,\ \mathbf{v} \in \mathbf{V} = \mathbf{V}_1 \times \mathbf{V}_2 \times \mathbf{V}_3 \subset \mathbf{E}^m. \quad (2)$$

Control-coordination of MMS-C

$$\mathbf{u} = \left(\mathbf{u}^1,\ \mathbf{u}^2,\ \mathbf{u}^3\right),\ \dim \mathbf{u}^l = k_l \geq 3,\ \mathbf{u}^l \in \mathbf{U}_l \subset \mathbf{E}^{k_l}, \\ \dim \mathbf{u} = k = \sum_{l} k_l,\ \mathbf{u} \in \mathbf{U} = \mathbf{U}_1 \times \mathbf{U}_2 \times \mathbf{U}_3 \subset \mathbf{E}^k. \quad (3)$$

For distributed coordination $\mathbf{u}^l$ is connected with one of subsystem of MMS-ES (Fig. 2) then the inequality in (3) may not be satisfied.

Structurally and functionally connected problems MMS-C and MMS-ES are characterized by according payoff function

$$\mathbf{J}_{C\,l} = J_{C\,l}(\mathbf{v},\ \mathbf{u}),\ l = 1,\ 2,\ 3, \quad (4)$$

$$\mathbf{J}_{ES\,i} = J_{ES\,i}(\mathbf{v},\ \mathbf{u}),\ i = 1,\ 2,\ 3. \quad (5)$$

The general structure of efficiency indexes (4) and (5) is

$$J_{ji} = \Phi_{ji}(\mathbf{x},\ t_k) + \int_{t_0}^{t_k} f_{ji}(t,\ \mathbf{x},\ \mathbf{u},\ \mathbf{v})dt,\ i = 1,\ 2,\ 3;\ j = (C,\ ES) \quad (6)$$

Properties of the second member of equation (1), sets $\mathbf{U}$, $\mathbf{V}$ and efficiency indexes (6) will be considered further.

Definition 1 (HEHDG). Hierarchical equilibrium of HDG with the "lead right" of the upper level by the pair-wise interaction of the levels is the set of interconnected equilibrium situations of the HDG level sets by fixed conflict game in MMS levels.

Special case 1 Equilibrium of HDG (EHDG). Two-level HDG with one task-subsystem included in MMS-C takes place.

Remark 1. Problem definition of Stackelberg's EHDG (SEHDG) determination as well as sufficient and necessary conditions of the equilibrium SEHDG control-strategy synthesis on the set of conflict games in MMS-ES are given in (Wisebord and Zhukovsky, 1980). Procedure of numerical computation of SEHDG on the basis of genetic approaches is given in (Serov, 2006a).

Special case 2. Vector equilibrium of HDG (VEHDG). Two-level HDG with vector of indexes of the Center takes place.

Remark 2. Problem definition of Stackelber's and Serov's VEHDG (S-SVEHDG) determination as well as solution procedure on the basis of developed genetic algorithm of multi-criteria optimization (GAMO) are given in (Serov, 2006b, 2007).

Definition 2. Stackelberg's HEHDG (SHEHDG). Structural properties of Stackelberg's hierarchical equilibrium of two-level HDG with generalization of Stackelberg's strategy form tree-step procedure.

At first step according to the "lead right" MMS-C informs MMS-ES its coordination in the form of law-strategy $\mathbf{u}(t,\ \mathbf{x}) \in \mathbf{U}$ for each position of the set $\{t,\ \mathbf{x}\}$ (Wisebord and Zhukovsky, 1980) or in the form of program-corrected control law-strategy (PCCL) (Voronov, 2001) for finite set $\{t_i,\ \mathbf{x}(t_i),\ t_0 < t_1 < ... < t_i < ... < t_K = T\}$ or program control $\mathbf{u}(t)$ for each $t \in [t_0,\ t_K]$ or vector-parametric set $\mathbf{q} \in \mathbf{Q}$.

At second step on the level of MMS-ES mapping $\mathbf{R} : \mathbf{U} \to \mathbf{V}$ is formed, for every fixed $\mathbf{u} \in \mathbf{U}$

$$\max_{\mathbf{v} \in \mathbf{V}} \varphi_{ES}\left(J_{ES\,1}(\mathbf{u},\ \mathbf{v}),...,J_{ES\,3}(\mathbf{u},\ \mathbf{v})\right) = \\ = \varphi_{ES}\left(J_{ES\,1}(\mathbf{u},\ \mathbf{Ru}),...,J_{ES\,3}(\mathbf{u},\ \mathbf{Ru})\right). \quad (7)$$

Specific format of the function φ_{ES} (Wisebord and Zhukovsky, 1980) is defined on the set of conflict games of MMS-ES subsystems (antagonism, coalition conflict, coalition-free conflict or cooperative interaction). Conflict game can also be formed by MMS-C or chosen by MMS-ES itself according to the sense of the problem domain. The first variant is the independent research problem within HDG.

At third step MMS-C selects solution

$$\max_{\mathbf{u}\in\mathbf{U}} \varphi_C\left(J_{C1}(\mathbf{u},\,\mathbf{Ru}),...,J_{C3}(\mathbf{u},\,\mathbf{Ru})\right)= \\ =\varphi_C\left(J_{C1}(\mathbf{u}^o,\,\mathbf{Ru}^o),...,J_{C3}(\mathbf{u}^o,\,\mathbf{Ru}^o)\right). \tag{8}$$

Third step develops Stackelberg's strategy and generalizes SEHDG. Specific format of the function φ_C is defined on the set of conflict games of MMS-C subsystems. Type of conflict (initial disagreement) is determined by the problem domain or from the more general goal properties of MMS-C. Finally by analogy with (Wisebord and Zhukovsky, 1980) the set of $\left\{\mathbf{u}^r,\,\mathbf{Ru}^r\right\}$ is defined as *Stackelberg's hierarchical equilibrium of HDG (SHEHDG).*

If $\mathbf{u}^r$ and its mapping $\mathbf{v}=\mathbf{Ru}^r$ is not a unique solution, then this circumstance lets introduce and analyse the hierarchical stable-effective compromise (HSTEC) on the set of $\left\{\mathbf{u}^r,\,\mathbf{Ru}^r\right\}$. This task is the independent research problem.

Practically hierarchical efficiency balancing characterizes the optimal interaction of efficiently balanced modes of operation of the multilevel system under conditions of natural coordinating influence of the upper level (the "lead right"). The notion of SHEHDG is potentially extendable for multilevel system and for the greater number of subsystems within the level.

3. GENERATION PROCEDURE OF SHEHDG IN COALITION-FREE VARIANT OF MMS-LEVELS BALANCING

Without restriction of possibility of game conflict change in MMS-levels the typical variant of optimal levels interaction for coalition-free efficiency balancing of MMS-levels is considered.

By analogy with (Wisebord and Zhukovsky, 1980) form the mapping

$$\mathbf{Ru}\left\|\nu^i\right.=\begin{cases}\mathbf{v}^1,\,\mathbf{R}_2\mathbf{u},\,\mathbf{R}_3\mathbf{u}, & at\ i=1,\\ \mathbf{R}_1\mathbf{u},\,\mathbf{v}^2,\,\mathbf{R}_3\mathbf{u}, & at\ i=2,\\ \mathbf{R}_1\mathbf{u},\,\mathbf{R}_2\mathbf{u},\,\mathbf{v}^3, & at\ i=3;\end{cases} \tag{9}$$

$$\mathbf{Ru}=\left(\mathbf{R}_1\mathbf{u},\,\mathbf{R}_2\mathbf{u},\,\mathbf{R}_3\mathbf{u}\right),\ \mathbf{u}=\left(\mathbf{u}^1,\,\mathbf{u}^2,\,\mathbf{u}^3\right). \tag{10}$$

According to second step of SHEHDG obtaining (7) with condition that φ_{ES} realizes operation of coalition-free conflict situation it is formed three mappings $\mathbf{R}_i:\mathbf{U}\rightarrow\mathbf{v}^i,\ i=1,2,3$ at the level of MMS-ES

$$J_{ES\,i}\left(\mathbf{u},\,\mathbf{Ru}\right)=\max_{\mathbf{v}^i\in\mathbf{V}_i} J_{ES\,i}\left(\mathbf{u},\,\mathbf{Ru}\left\|\mathbf{v}^i\right.\right),\ i=1,2,3. \tag{11}$$

In this case equation (10) realizes equilibrium solution with superscript r for fixed acceptable coordination $\mathbf{u}$

$$\mathbf{R}\left(\mathbf{u}\right)=\mathbf{v}^r=\left(\mathbf{v}^{1r}=\mathbf{R}_1\mathbf{u},\,\mathbf{v}^{2r}=\mathbf{R}_2\mathbf{u},\,\mathbf{v}^{3r}=\mathbf{R}_3\mathbf{u}\right). \tag{12}$$

Next according to third step (8) φ_C is formed

$$J_{C\,l}\left(\mathbf{u}^r,\,\mathbf{Ru}^r\right)=\max_{\mathbf{u}^l} J\left(\mathbf{u}\left\|\mathbf{u}^l\right.,\,\mathbf{R}\left(\mathbf{u}\left\|\mathbf{u}^l\right.\right)\right)= \\ =\max_{\mathbf{u}^l} J\left(\mathbf{u}\left\|\mathbf{u}^l\right.,\,\mathbf{v}^r\left(\mathbf{u}\left\|\mathbf{u}^l\right.\right)\right),\ l=\overline{1,3}, \tag{13}$$

where

$$\mathbf{u}^r=\left(\mathbf{u}^{1r},\,\mathbf{u}^{2r},\,\mathbf{u}^{3r}\right), \tag{14}$$

$$\mathbf{u}^r\left\|\mathbf{u}^l\right.=\begin{cases}\mathbf{u}^1,\,\mathbf{u}^{2r},\,\mathbf{u}^{3r}, & at\ l=1;\\ \mathbf{u}^{1r},\,\mathbf{u}^2,\,\mathbf{u}^{3r}, & at\ l=2;\\ \mathbf{u}^{1r},\,\mathbf{u}^{2r},\,\mathbf{u}^3, & at\ l=3.\end{cases} \tag{15}$$

It is obviously that obtaining of $\mathbf{v}^r\left(\mathbf{u}\right)$ in equation (13) demands the solution of the problem (9) – (15) in the form of synthesis problem or results in iteration problem of approximate optimization with the use of parametrized PCCL (Voronov, 2001). Next the general sufficient conditions of optimality in the problem of strategies synthesis are formed and the structure of the iteration algorithm of the obtaining of parametrized PCCL at the PCCL program time step is considered.

4. SUFFICIENT CONDITIONS OF HIERARCHICAL EQUILIBRIUM (SHEHDG) IN TWO-LEVEL SYSTEM

For forming of sufficient conditions let's use Stalford's results (Wisebord and Zhukovsky, 1980).

Function $f\left(...\right)$ in equation (1) is defined on $\mathbf{E}^n\times\mathbf{E}^m\times\mathbf{E}^k$ with values in $\mathbf{E}^n$. In the general case the set of probable states of the system $\mathbf{X}\subset\mathbf{E}^n$ is defined, notably the constraint $\mathbf{x}\in\mathbf{X}$ is set. Acceptable strategies $\mathbf{v}\left(t,\,\mathbf{x}\right)$ and $\mathbf{u}\left(t,\,\mathbf{x}\right)$ are obeyed the following conditions (Wisebord and Zhukovsky, 1980):

1) For every set of $\mathbf{u}\left(t,\,\mathbf{x}\right),\,\mathbf{v}\left(t,\,\mathbf{x}\right)$ exists unique solution of absolute continuous solution $\mathbf{x}(t)$ of the system (1).

2) $\mathbf{v}^i=\mathbf{v}^i\left(t,\,\mathbf{x}\right)$ and $\mathbf{u}^l=\mathbf{u}^l\left(t,\,\mathbf{x}\right)$, $i,\,l=\overline{1,3}$ are the members of the set of Borel's measureable functions (piecewise continuous functions with finite set of points of break of the first kind) with values in $\mathbf{E}^{m_i}$ and $\mathbf{E}^{k_l}$ respectively.

3) $\mathbf{v}^i\in\mathbf{V}_i,\,\mathbf{u}^l\in\mathbf{U}_l$ for every $t\in\left[t_0,\,t_K\right]$, $i,\,l=1,\,2,\,3$ and $\mathbf{V}_i\left(t,\,\mathbf{x}\right),\,\mathbf{U}_l\left(t,\,\mathbf{x}\right)$ are multiple valued functions, which for each time point $t\in\left[t_0,\,t_K\right]$ and

each $\mathbf{x} \in \overline{\mathbf{X}}$ ($\overline{\mathbf{X}}$ is enclosure of the set $\mathbf{X}$) are assigned the certain subset of the spaces $\mathbf{E}^{m_i}$ and $\mathbf{E}^{k_l}$ respectively.

Remark 3. If function $f(...)$ in system (1) is continuously differentiable and sets $\mathbf{V}_i$, $\mathbf{U}_l$, $i, l = 1, 2, 3$ are compact sets in $\mathbf{E}^{m_i}$ and $\mathbf{E}^{k_l}$ respectively, then $\overline{\mathbf{X}}$ always exists (Voronov, 2001), therefore there no need to input $\overline{\mathbf{X}}$ (only if this is needed by practical problem).

For the class of acceptable strategies it is supposed that functions Φ_{ji} and f_{ji} are piecewise continuous by arguments.

There are introduced some topological definitions for forming the sufficient conditions of SHEHDG (Wisebord and Zhukovsky, 1980; Voronov *et al.*, 2008).

Statement 1 (Voronov *et al.*, 2008). If exist

1) functions $V_{iu}(t, \mathbf{x}), i = \overline{1,3}$ of a class C^1 by D_u and one-valued functions $v^i(t, \mathbf{x}, \mathbf{u}), i = \overline{1,3}$ for every fixed $\mathbf{u} \in U$ and separation of the set $\mathbf{X}$ such as

$$v^i(t, \mathbf{x}, \mathbf{u}(t, \mathbf{x})) = \mathbf{R}_i(\mathbf{u}) \in V_i,$$

$$H_{iu}(t, \mathbf{x}, \mathbf{u}, \mathbf{v}(t, \mathbf{x}, \mathbf{u})) = \max_{\mathbf{v}^i \in V_i \subset E^{m_i}} H_{iu}(t, \mathbf{x}, \mathbf{u}, \mathbf{v}(t, \mathbf{x}, \mathbf{u}) \| \mathbf{v}^i)$$

$$H_{iu}(t, \mathbf{x}, \mathbf{u}(t, \mathbf{x}), \mathbf{R}(\mathbf{u})) = 0, \; i = \overline{1,3}$$

for every $\mathbf{x} \in X_{ju}$, $j \in \Im_u$, $t \in [t_0, t_K]$,

where

$$H_{iu}(t, \mathbf{x}, \mathbf{u}, \mathbf{v}^1, \mathbf{v}^2, \mathbf{v}^3) = \frac{\partial W_{jiu}(t, \mathbf{x})}{\partial t} + \left(\frac{\partial W_{jiu}(t, \mathbf{x})}{\partial \mathbf{x}}\right)^T f(t, \mathbf{x}, \mathbf{u}, \mathbf{v}^1, \mathbf{v}^2, \mathbf{v}^3) + f_{ESi}(t, \mathbf{x}, \mathbf{u}, \mathbf{v}^1, \mathbf{v}^2, \mathbf{v}^3), \; i = \overline{1,3}$$

is the Hamilton-Kelley function;

$\Im_u$ is a finite set of subscripts of separation D_u of the set $\mathbf{X}$

2) separation D_ν of the set $\mathbf{X}$, functions $V_{l\nu}(t, \mathbf{x}), l = \overline{1,3}$ of a class C^1 by D_ν and strategy of the MMS-C $\mathbf{u} \in U$ such as

$$H_{l\nu}(t, \mathbf{x}, \mathbf{u}^r, \mathbf{R}\mathbf{u}^r) = \max_{u^i \in U_i} H_{l\nu}(t, \mathbf{x}, \mathbf{u}^r \| \mathbf{u}_l, \mathbf{R}(\mathbf{u}^r \| \mathbf{u}_l)) = 0, \; l = \overline{1,3};$$

for every $\mathbf{x} \in X_{j\nu}$, $j \in \Im_\nu$, $t \in [t_0, t_K]$,

where

$$H_{l\nu}(t, \mathbf{x}, \mathbf{u}^1, \mathbf{u}^2, \mathbf{u}^3, \mathbf{v}) = \frac{\partial W_{jl\nu}(t, \mathbf{x})}{\partial t} + \left(\frac{\partial W_{jl\nu}(t, \mathbf{x})}{\partial \mathbf{x}}\right)^T f(t, \mathbf{x}, \mathbf{u}^1, \mathbf{u}^2, \mathbf{u}^3, \mathbf{v}) + f_{Cl}(t, \mathbf{x}, \mathbf{u}^1, \mathbf{u}^2, \mathbf{u}^3, \mathbf{v}), \; l = \overline{1,3};$$

$\Im_\nu$ is a finite set of subscripts of separation D_ν of the set $\mathbf{X}$.

3) for $\mathbf{x} \in \mathbf{x}(t_K)$

$\overline{V}_{iu}(t_K, \mathbf{x}) = \Phi_{ESi}(t_K, \mathbf{x})$ for every $\mathbf{u} \in U$, $i = \overline{1,3}$,

$V_{l\nu}(t_K, \mathbf{x}) = \Phi_{Cl}(t_K, \mathbf{x})$ for $\mathbf{u} = \mathbf{R}\mathbf{v}$, $l = \overline{1,3}$,

then the set $\{\mathbf{u}^r, \mathbf{R}_1\mathbf{u}, \mathbf{R}_2\mathbf{u}, \mathbf{R}_3\mathbf{u}\} = \{\mathbf{u}^r, \mathbf{R}\mathbf{u}\}$ is hierarchical equilibrium solution (SHEHDG) in two-level control system (in two-step HDG) and payoffs of upper and lower levels respectively are

$$\mathbf{J}_C(\mathbf{u}^r, \mathbf{R}\mathbf{u}^r) = \mathbf{V}_\nu(t_0, \mathbf{x}_0) = (\mathbf{V}_{1\nu}(t_0, \mathbf{x}_0), \mathbf{V}_{2\nu}(t_0, \mathbf{x}_0), \mathbf{V}_{3\nu}(t_0, \mathbf{x}_0));$$

$$\mathbf{J}_{ES}(\mathbf{u}^r, \mathbf{R}\mathbf{u}^r) = \mathbf{V}_u(t_0, \mathbf{x}_0) = (\mathbf{V}_{1u}(t_0, \mathbf{x}_0), \mathbf{V}_{2u}(t_0, \mathbf{x}_0), \mathbf{V}_{3u}(t_0, \mathbf{x}_0)).$$

Analysis of statement 1 indicates that obtained sufficient conditions are hardly applicable. Certain results of control laws-strategies synthesis can be obtained in the class of linear-quadratic problems with generalization of results (Wisebord and Zhukovsky, 1980).

Defined approximation in SHEHDG obtaining methods that lets to solve problem is based on the strategies in the form of approximate PCCL and its parametrization on the program time step. Thus program time step j of PCCL corresponds to the set $t \in [t_{j-1}, t_K]$. For example, on the first program time step at $j = 1$ PCCL set of time is $t \in [t_0, t_K]$.

5. GENERAL STRUCTURAL PROPERTIES OF ITERATION ALGORITHM OF SHEHDG OBTAINING ON THE BASIS OF PCCL

Skipping substantiation of mathematical conditions of unique and existence of the solution and algorithmic features of the method consider general structure of iterations of the algorithm on the program time step of PCCL on the basis of structural properties of SHEHDG.

Every iteration k of algorithm at program time step j of PCCL consists of the following five steps.

Step 1. Solution of the problem (12) – (15) of the equilibrium point $\mathbf{u}_k^r(\mathbf{v}_{k-1}^r)$ obtaining on the MMS-C level.

At the first iteration acceptable $\mathbf{u} \in U$ or initial equilibrium approximation of the PCCL of the MMS-C is specified.

Step 2. Forming of the initial approximation $\tilde{\mathbf{v}}_k^r$ on the basis of the problem (9) – (11).

Step 3. Solution of the problem (9) – (12) with determination of $\mathbf{v}_k^r$ on the level of MMS-ES on the basis of solution $\mathbf{u}_k^r$ obtained at step 1.

Step 4. Comparison of the results of the iteration k and previous iteration ($\mathbf{u}_{k-1}^r \to \mathbf{u}_k^r$, $\mathbf{v}_{k-1}^r \to \mathbf{v}_k^r$).

Step 5. Stop of computation in the case of proximity of the efficiency indexes or program time step of PCCL at the current and previous iterations. “Else” generation of new initial approximations $\tilde{\mathbf{u}}_{k+1}^r$ and then transition to the Step 1 of the $(k+1)$ iteration.

The main elements of approximated equilibrium existence and of algorithm convergence are homogeneity of subsystems in MMS levels, compactness of the sets of control $\mathbf{U}$, $\mathbf{V}$ and property of efficiency indexes $\mathbf{J}_C$, $\mathbf{J}_{ES}$ convexity.

6. CONCLUSIONS

In the article problem definition of Stackelberg’s hierarchical equilibrium of hierarchical differential games. Approximate iteration procedure of obtaining of equilibrium coordination-control of two-level system consisted of the multi-object multi-criteria system of upper level with the “lead right” (MMS-Center) and multi-object multi-criteria system of lower level (MMS-Executive system) is formed. Example of such kind of system is considered in the form of two-stage system of aircraft guidance-stabilization system. Obtained in the article theoretical results can be applied to multilevel system with more than two levels with inter-level coordination provided by upper levels.

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