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NONLINEAR OSCILLATIONS OF FLEXIBLE VERTICAL GYROSCOPIC ROTORS

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Abstract

Highspeed vertical rotors can have a rotating mass above or beneath the shaft support point [Kushul, 1968; Kushul, Zeitman, 1968; Zeitman, 1991]. Those of stipulate some constructional peculiarities: the highly flexible shaft and the massive rotor with the strong gyroscopic properties and vertical rotation axis at the availability resilient supports. It is proposed a new untraditional dynamic model which enables the adequate analytical description of the considered systems. Those of can be excited by the rotor unbalance and also by the support point vibrations. It is investigated the nonlinear oscillations for the system with rotor above the support point as well as a problem of the vertical rotation stability. It is also noted a variation of the vertical spinning stability threshold for the rotor system under investigation.

Key words

vertical rotor, flexible shaft, rotation stability

Introduction

Highspeed vertical rotor systems belong because of their particularities to such structures which we consider to call as the elastic gyroscopic system. Those of vibrations can be not always described within the traditional assumptions. It is therefore proposed a dynamic model where the rotor accomplishes the motion as a flexible normal or inverted gyropendulum [Zeitman, 1991]. Such system is undergone in this case besides general loads due to shaft and bearing deflections the action of inertial forces and couples resulting from new rotor motion form. It is taken into account a shaft buckling by the axial loads with the variable direction depending on the mode of a flexural curve [Zeitman, Kushul, 1968]. Nonlinear differential equations of the rotor motion are attained. Versatiles vibrations in particular from an unbalance of the rotor under discussion are studied. The considered rotors can be with the different constructional composition when the rotor mass centres are respectively placed above or beneath a support point. We shall here the former case study. Therefore arises then a vertical rotation stability problem [Kushul, 1968; Zeitman, Kushul, 1968; Zeitman, 1991; Volokhovskaya, Zeitman, 1992]. It is shown a decrease of the vertical spinning stability threshold by taking into account the shaft flexibility for the assumed dynamic model.

On the Fig.1 is presented the investigated dynamic model of the rotor system under study.

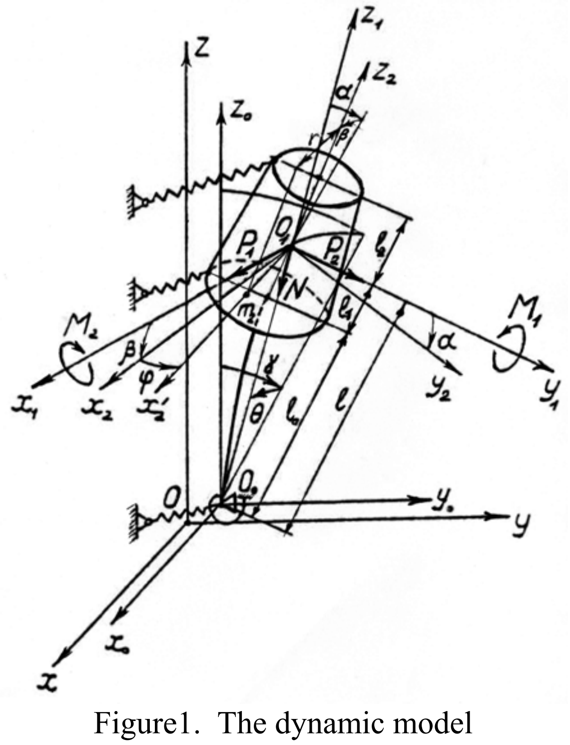

Figure1. The dynamic model

A solid symmetric rotor is fastened on a flexible massless thin shaft. A rotor symmetry axis coincides with a rotation one. The shaft support point O_0 is resiliently strengthened in the direction of the fixed axes coordinates system $Oxyz$. The coordinate axes x_0, y_0, z_0 origin is set in the support point O_0, those of translationaly moving relatively to the unmovable axes x, y, z. The rotor mass centre O_1 is disposed above the shaft support point O_0. It is supposed that one moves on the sphere with the centre in O_0 and its position is defined by spherical coordinates γ, θ concerning the movable axes x_0, y_0, z_0. The same angles determine a space position of the spherical coordinates system $O_1x_1y_1z_1$ characterizing a rotor orientation with an undeformed shaft. A shaft bending brings about an additional turn of the rotor symmetry axis O_1z_2 coinciding with the tangent to a flexural shaft curve in the fixation point of the rotor and shaft (s=l_0). The transverse shaft deflections are read off the straight line O_0O_1 (axis O_1z_1) in the spherical coordinates system $O_1x_1y_1z_1$. The Rezale's axes system $x_2y_2z_2$ is invariably bound up with the rotor symmetry axis. A position of the latter is defined by the Rezale's angles α, β in relation to the spherical system $O_1x_1y_1z_1$. The axes $x'_2\, y'_2\, z'_2$ rotation connected rigidly with the rotor in regard to system $O_1x_2y_2z_2$ is determined by a proper rotation angle φ.

The flexible shaft is acted by restoring couple in O_0 simulating the elastic properties of the support device – a thrustbearing. That of affects of the same point by the viscous damping reaction forces depending linearly or nonlinearly on the respective velocity components.

After these preliminary remarks as well as the short description of the dynamic model in the frames of reference can we now record the rotor motion differential equations. Applying the kinetostatic method in the main theorems of the relative motion dynamics it can be derived the following equations

$$\left.\begin{aligned}
&a_{11}\ddot{\xi}_0+a_{12}\dot{\xi}_0+a_{13}\xi_0+a_{14}\ddot{\theta}+a_{15}\theta+a_{16}\beta=\\
&=\varepsilon\dot{\varphi}^2\cos\varphi+\varepsilon\ddot{\varphi}\sin\varphi+F_1\\
&a_{21}\ddot{\eta}_0+a_{22}\dot{\eta}_0+a_{23}\eta_0+a_{24}\ddot{\gamma}+a_{25}\gamma+a_{26}\alpha=\\
&=\varepsilon\dot{\varphi}^2\sin\varphi-\varepsilon\ddot{\varphi}\cos\varphi+F_2\\
&a_{31}\ddot{\zeta}_0+a_{32}\dot{\zeta}_0+a_{33}\zeta_0=-1+F_3\\
&a_{41}\ddot{\beta}+a_{42}\ddot{\theta}+a_{43}\dot{\varphi}(\dot{\alpha}+\dot{\gamma})+a_{44}\theta+a_{45}\beta+a_{46}\ddot{\xi}_0+a_{47}\xi_0=\\
&=a_{46}\varepsilon(\dot{\varphi}^2\cos\varphi+\ddot{\varphi}\sin\varphi)+a_{41}\varepsilon(1+\ddot{\xi}_0)\cos\varphi+F_4\\
&a_{51}\ddot{\alpha}+a_{52}\ddot{\gamma}+a_{53}\dot{\varphi}(\dot{\beta}+\dot{\theta})+a_{54}\gamma+a_{55}\alpha+a_{56}\ddot{\eta}_0+a_{57}\eta=\\
&=a_{56}\varepsilon(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi)+a_{51}\varepsilon(1+\ddot{\xi}_0)\sin\varphi+F_5\\
&a_{61}\ddot{\beta}+a_{62}\ddot{\theta}+a_{63}\dot{\varphi}(\dot{\alpha}+\dot{\gamma})+a_{64}\theta+a_{65}\beta+a_{66}\ddot{\xi}_0+a_{67}\xi_0=\\
&=a_{66}\varepsilon(\dot{\varphi}^2\cos\varphi+\ddot{\varphi}\sin\varphi)+a_{61}\varepsilon(1+\ddot{\xi}_0)\cos\varphi+F_6\\
&a_{71}\ddot{\alpha}+a_{72}\ddot{\gamma}+a_{73}\dot{\varphi}(\dot{\beta}+\dot{\theta})+a_{74}\gamma+a_{75}\alpha+a_{76}\ddot{\eta}_0+a_{77}\eta_0=\\
&=a_{76}\varepsilon(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi)+a_{71}\varepsilon(1+\ddot{\xi}_0)\sin\varphi+F_7\\
&a_{81}\ddot{\varphi}=a_{82}(\ddot{\gamma}\theta+\dot{\gamma}\dot{\theta})+a_{83}(\eta_0\theta-\xi_0\gamma)+\varepsilon[(1+\ddot{\xi}_0)(\alpha+\gamma)-\\
&-(\ddot{\gamma}+\ddot{\eta}_0)]\cos\varphi-\varepsilon[(1+\ddot{\xi}_0)(\beta+\theta)-(\ddot{\theta}+\ddot{\xi}_0)]\sin\varphi
\end{aligned}\right\}(1)$$

where ξ_0, η_0, ζ_0 - dimensionless displacement coordinates of the support point relative to unmovable system $Oxyz$; ε - dimensionless rotor unbalance; a_{ij}(i=1,…8; j=1,…7) – dimensionless constant equations coefficients depending on the elastic, inertial, geometric and damping rotor system properties; F_i(i=1,…7) – dimensionless complicated nonlinear functions of the coordinates and their derivatives with constant and periodical coefficients. Differentiation with respect to dimensionless time $\tau = t\sqrt{g/l}$ is here designated by points above the letters.

Nonlinear equation (1) describes completely the assumed rotor dynamic model motion as the inverted gyropendulum with the flexible shaft, elastic connections and movable support point. Let us consider the more simple variety of the dynamic model under study – stationarily spinning rotor with the fixed support point and free from external resilient connections except an elastic restoring momentum in a flexible shaft [Kushul, 1968]. It is at first examined the rotor oscillations by the linearized system (1), when nonlinear functions F_i=0(i=1,…7). This step gives the generating one. Introducing the complexes functions of the angle coordinates

$$x=\beta+i\alpha,\ y=\theta+i\gamma,\ i=\sqrt{-1}$$

from the system (1) it can be obtained the motion differential equations for the case under discussion in complexes variables

$$\left.\begin{aligned}&a_{41}\ddot{x}+a_{42}\ddot{y}-a_{45}\omega_* i(\dot{x}+\dot{y})+a_{45}y=\\&=a_{46}\varepsilon\omega_*^2\exp(i\omega_*\tau)\\&a_{61}\ddot{x}+a_{62}\ddot{y}-a_{63}\omega_* i(\dot{x}+\dot{y})+a_{64}x+a_{65}y=\\&=a_{66}\varepsilon\omega_*^2(i\omega_*\tau)\end{aligned}\right\}\qquad(2)$$

where dimensionless coefficients a_{ij} are taken from (1) for the considered occasion and $\omega_*=\omega\sqrt{l/g}$. Those of are the transcendental and algebraic functions of the elastic and inertial rotor parameters. Assuming in the right part of (2) $\varepsilon=0$ it is obtained the model with the ideally balanced rotor and the equations system (2) becomes uniform. It can be got from that of a frequency equation for the calculation the whirling angle velocities at the rotor transverse oscillations

$$a_0\nu_*^4+a_1\nu_*^3+a_2\nu_*^2+a_3\nu_*+a_4=0\qquad(3)$$

Here coefficients $a_i(i=0,\ldots 4)$ are dimensionless transcendental functions of the elastic and inertial rotor system parameters; ν_* - are dimensionless whirling angular velocities which it can be calculated as the roots of equation (3). The general solution of the equations system (2) can be found for complex form as

$$\left.\begin{aligned}&y=\sum_{k=1}^{4}R_k\exp[i(\nu_{*k}\tau+\psi_k)],\\&x=\sum_{k=1}^{4}R_k q_k\exp[i(\nu_{*k}\tau+\psi_k)]\end{aligned}\right\}\qquad(4)$$

where $\nu_{*k}=\nu_k\sqrt{l/g}\,(k=1,...4)$ are the roots of (3), q_k are the coefficients of the vibration forms defining from (2). The amplitudes R_k and phases ψ_k are expressed from (2) through the initial angles and angular rapidities values.

The equalities (4) enable construct the trajectories of the arbitrary rotor point in the projection on the horizontal plane. Those of permit to estimate the rotor displacement quantities at the free vibrations with the different initial conditions. It should be noticed that these trajectories are visibly differed from ones constructed on the base of traditional dynamic model.

The considered gyroscopic rotor vibrations from the unbalance are described from nonuniform equations system (2). The amplitudes stationary values can be obtained applying in (2) a substitution

$$x=X_0\exp(i\omega_*\tau),\ y=Y_0\exp(i\omega_*\tau),\ \omega_*=\omega\sqrt{l/g}$$

with

$$\left.\begin{aligned}&X_0=(\varepsilon\omega_*^2/\Delta_1)(a_{0*}\omega_*^2-a_{65}),\\&Y_0=-(\varepsilon\omega_*^2/\Delta_1)(a_{0*}\omega_*^2+a_{45})\end{aligned}\right\}\qquad(5)$$

Here $\Delta_1=a_{0*}\omega_*^4+a_{2*}\omega_*^2+a_{4*}$ in doing so a_{0*}, a_{2*}, a_{4*} are known functions of the elastic and inertial rotor system parameters but $a_{i*}(i=0,2,4)$ are drawn from (2).

Let us discuss the simplest rotor system version when $l=l_0$ and $\chi=0$ (χ – nondimensional angular stiffness of the restoring momentum in the thrust bearing). The polynomial $\Delta_1(\omega_*)$ in expression (5) is under these conditions equal

$$\begin{aligned}&\Delta_1(\omega_*)=(\sigma^2-\sigma_0^2)(1-f)\omega_*^4-\\&-[(\sigma_0^2-\sigma^2)f-1]\omega_*^2-1\end{aligned}\qquad(6)$$

where σ_0^2, σ^2 - nondimensional polar and equatorial inertial momenta respectively; $f=\vartheta\,\mathrm{ctg}\,\vartheta$, $\vartheta=\lambda_0 l$ – nondimensional flexibility at the shaft buckling; $\lambda_0=mg/EI$, mg – the rotor weight; EI – the flexural shaft stiffness.

The bequadratic equation $\Delta_1(\omega_*)=0$ derived from (6) has no real roots, i.e. critical speeds, if its discriminant is positive, that is

$$D(\Delta_1)=4(\sigma_0^2-\sigma^2)-[(\sigma_0^2-\sigma^2)f+1]^2>0\qquad(7)$$

The modern vertical gyroscopic rotors parameters are such that the difference $\sigma_0^2-\sigma^2>0$ for all practically important cases. The indicated difference numerical value lies in the range 0,01–0,07, i.e. $\sigma_0^2-\sigma^2\ll 1$. The condition (7) performance is merely by virtue of possible in the highly confined domain in so far in the most of the parameter ϑ real value interval exists an unquality $D(\Delta_1)<0$. This means that the rotor is not free from critical speeds. There is nevertheless for the suggested dynamic model an area sufficiently narrow where $D(\Delta_1)>0$ and consequently the rotor system under consideration has no resonance states. The surprising thing is that this occures when it had regard to the springiness of the rotor system suspension every element. The mentioned phenomenon takes place when

$f<(\sigma^2-\sigma_0^2)^{-1}$ or with allowance for the sited numerical data 0,95π<ϑ<0,995π. It should be noted that the values ϑ for the many vertical rotors under operation fall into this band. If $\sigma_0^2<\sigma^2$ this area vanishs. A familiar fact occures in the resonance existence domain when at $\sigma_0^2>\sigma^2$ is there one critical speed and if $\sigma_0^2<\sigma^2$ are there two those of.

The absolutely rigid shaft has ϑ=0, f=1 and from (6) the equation for the critical speed calculation

$$(\sigma_0^2-\sigma^2-1)\omega_*^2-1=0 \quad (8)$$

The new property of the pattern under examination with the rigid shaft follows from (8): it has no critical speed if $\sigma_0^2-\sigma^2<1$ or $C_1<A_1+ml^2$ where C_1, A_1 – polar and equatorial rotor inertia momentum. This ratio is performed as the cited data demonstrate in the majority practical cases.

An application of the vertical gyroscopic rotors with the mass centre above the support point contributes to an appearance opportunity of a stability loss for a vertical rotation rotor regime. The investigated rotor system is under effect of the different force factors. Some ones facilitate to an instability. The main instability factor is in the discussed case the rotor gravity momentum, unbalance forces in the under resonance zone and often electromagnetic reactions forces having a negative stiffness. The joint action of all indicated forces at the rotor system results in the threshold angular speeds appearance dividing the all velocities areas into the stable and unstable rotation [Volokhovskaya, Zeitman, 1992].

The symmetry axis position of the unbalanced rotor close to vertical is characterized by the stationary periodical motion equations (2) solution in the form (5). The necessary stability conditions of one can be derived from the variations equations for it. Those of coincide thoroughly with (2) without right parts. The integration of ones results in to the characteristic equation (3). The sought stability conditions are got from the roots ones of the polynomial n-th power.

Applying the Sturm procedure are obtained the rotor vertical rotation conditions

$$\left.\begin{aligned}
&3a_1^2-8a_0a_2>0\\
&U_1=a_1^2a_2^2-3a_1^3a_3-18a_0^2a_3^2+14a_0a_1a_2a_3-\\
&-6a_0a_1^2a_4-4a_0a_2^3+16a_0^2a_2a_4>0\\
&U_2=4U_1^2(16a_0a_4-a_1a_3)+4U_1U_3(a_1a_2-\\
&-6a_0a_3)-U_3^2(3a_1^2-8a_0a_2)>0\\
&U_3=a_1^2a_2a_3-4a_0a_2^2a_3+3a_0a_1a_3^2-9a_1^3a_4+\\
&+32a_0a_1a_2a_4-48a_0^2a_3a_4
\end{aligned}\right\} \quad (9)$$

where $a_0 \ldots a_4$ are coefficients from (3).

It is of use to consider some practical cases for the primary analysis is examined the simplest instance employed above in (6) when χ=0, $l=l_0$. The coefficients a_k(k=0,…4) are then

$$a_0=\sigma^2(1-f),\ a_1=-\sigma_0^2\omega_*(1-f),\ a_2=-(1+\sigma^2 f),\ a_3=\sigma_0^2\omega_* f,\ a_4=-1 \quad (10)$$

where f, σ_0^2, σ^2 are taken from (6).

The values ϑ are confined for all practically important cases by $0<\vartheta\le 5$. The function f is ruptured in this range and varies doubly its sign. The first unequality (9) has after the substitution of the coefficient values from (10) the quadratic of the species

$$3\sigma_0^4(1-f)\omega_*^2+8\sigma^2(1+\sigma^2 f)=0 \quad (11)$$

There are in occasion under review by the ϑ variation data the following change intervals for function f

1. $0\le f\le 1$ and $-1/\sigma^2<f<0$. The unequality is satisfied for any value ω_*.

2. $1<f<\infty$. Nevertheless is here appeared a limitation

$$\omega_*^2>8\sigma^2(\sigma^2 f+1)/3\sigma_0^4(f-1) \quad (12)$$

3. $-\infty<f<-1/\sigma^2$. Here is that of analogous (12)

$$\omega_*^2>8\sigma^2(\sigma^2 f+1)/3\sigma_0^4(1-f) \quad (13)$$

The unequality (11) is satisfied if the rotor parametres depending on the magnitude and sign of f are obeyed to the conditions (12), (13).

The function U_1 expressing the second unequality (9) turns into biquadratic polynomial relative to ω_*

$$\begin{aligned}
U_1=&3\sigma_0^8 f(1-f)^3\omega_*^4+\sigma_0^4(1-f)^2(1+\sigma^2+\\
&+10\sigma^2 f-3\sigma^4 f^2)\omega_*^2+4\sigma^2(1-f)(1+\sigma^2 f)(1+4\sigma^2-\\
&-2\sigma^2 f+\sigma^4 f^2)
\end{aligned} \quad (14)$$

The behaviour investigation of U_1 in the given intervals of f variation has shown the different cases the stability boundary formation depending on the coefficients change from f. The unequality

U_1>0 is however satisfied at all values ω_* as (11) in the range 0 $\leq f(\vartheta)$<1. There is thus here a single stability condition, namely that of U_2>0 from unequalities (9).

The function U_2 is the polynomial of fifth power in regard to parameter $z = \sigma_0^4 \omega_*^2$. Its coefficients are the complicated functions of f and σ^2. An equation U_2=0 enables to calculate the threshold angular speeds defining the stability boundaries. The equation has one real positive root z_1 in the examined range for f and by 0,05 $\leq \sigma^2 \leq$ 1. The relationship curves of nondimensional angular speeds which are the threshold speeds from ϑ at the fixed values σ^2 are presented on Fig.2. The instability domains U_2<0 lies under curves but above the ones are placed the stable rotation fields (U_2>0). These demonstrate the extension of the instability areas because of the shaft flexibility.

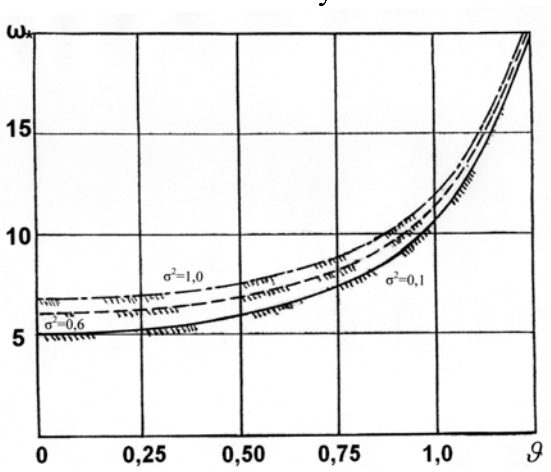

Figure2. The stability/instability speed curves

Conclusion

It is here proposed a new dynamic model for the vibrations investigation of the elastic gyroscopic system. The vertical rotors with definite constructional particularities are related to such class those of. The oscillations of the suggested model are described by nonlinear motion equations. It has in the general occasion at the unstationary vibrations eight a freedom degree. Their number is essentially reduced at the limitations in practically important cases.

The vertical rotor system investigation with the centre mass above the support point and with the applied elastic constraints at the resiliently fastened suspension shows the influence of shaft buckling to the discussed scheme oscillations characteristic. A number of its parameters particularly the buckling shaft stiffness affects to the stability/instability fields variation of the rotor vertical rotation. The derived results enable to affirm that the rotor suspension point excitation even by the periodical forces in virtue of motion equations structure brings about to the essential vibration regimes extension of the vertical rotor system model.

References

Kushul M.Ya. (1968) Motion of a gyroscope with a vlexible axis acted on by gravity and elastic constraints for small angles of nutation. The stability of its vertical rotation. Applied mathematics and Mechanics, N.4, Vol. 32, P.575-588.

Volokhovskaya O.A., Zeitman M.F. (1992) On rotor system rotation stability with flexible vertical shaft in magnetoelastic suspension. *Problemy mashinostroeniya i nadyoznosti mashin, №1,* pp.10-17 (in Russsian).

Zeitman M.F., Kushul M.Ya. (1968). Vertical rotors bending oscillations in gravity field. *Mashinovedeniye*, №3, pp.23-34 (in Russian).

Zeitman M.F. (1991). Flexural oscillations of highspeed ultracentrifuges rotors at their movement as gyropendulum. *Problemy mashinostroeniya i nadyoznosti mashin*, №4, pp.6-15 (in Russian).