SUPERSTABILITY CONDITIONS BASED ROBUST STABILIZATION OF A CLASS OF UNCERTAIN HYPERCHAOTIC SYSTEMS

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Abstract
In the work we present a superstability conditions based method of the analysis and control of hyperchaotic systems with parametric uncertainty. The constructive ways of checking the system achievability of the superstable dynamics are described. A class of superstabilizable hyperchaotic systems is defined. For this class we show the ways of using superstability conditions for the robust analysis and design of the superstabilizable controller, which provides the given characteristics of the transient response. The efficiency of the presented approach is proved by the numeric simulation result cited in the work.

Key words
Hyperchaotic systems, superstability, robust stabilization.

1 Introduction

For over two decades the attention of researchers in the field of modeling and control of complex system dynamics has been drawn to hyperchaotic systems [Lü and Chen, 2006]. They are described by the system of nonlinear autonomous differential equations with phase space dimensionality equal to four (4D), and their main feature is the dynamic behavior characterized by two or more positive Lyapunov exponents. Consequently, hyperchaotic systems demonstrate more complex behavior which is hard to predict and control compared to 3D chaotic systems. The specific features of hyperchaotic systems are attractive for use in practical applications (secure communication, synchronization, etc.), and in demand for the analysis of the high-dimensional social and economical systems [Yu, Cai and Li, 2012].

The total amount of works in the field of hyperchaos analysis and control can be divided into two groups. In the first group there are the investigations devoted to the construction of new hyperchaotic systems [Li, Tang and Chen, 2005; Chen, Yang, Qi and Yuana, 2007; Qi, van Wyk and Chen, 2008; Hu, 2009; Correia and Rech, 2010; Li and Sprott, 2014], as well as design and circuit implementation of this systems [Takahashi, Nakano and Saito, 2004; Yu, Lü and Chen, 2007; Liu, Feng and Tse, 2010; Yujun, Xingyuan, Mingjun and Huaguang, 2010; Yu, Lu, Yu and Chen, 2012; Li, Hu, Tang and Zeng, 2014; El-Sayed et al., 2014]. The second group consists of the works offering various hypechaotic dynamics control methods [Yang, Liu and Mao, 2000; Yan, 2004; Li, Chen and Tang, 2005; Yan, 2005; Jia, 2007a; Yang, Zhang and Chen, 2009; Dou, Sun, Duan and Lü, 2009; Wang, Cai, Miao and Tian, 2010; Wang and Zhao, 2010; Zhu, 2010; Njah, 2010; Pang and Liu, 2011; Effati, Saberi Nik, and Jajarmi, 2013; Toopchi and Wang, 2014]. As a rule the aim of the control of hyperchaotic systems is the stabilization of unstable state of equilibrium, i.e. chaos suppression. The stability analysis and the design of stable controllers are conducted based on the Lyapunov direct method that provides sufficient stability condition.

The study of the existing results shows that system stabilization often leads to the appearance of the undesirable peak effect – the dramatic increase in the solution norm at the initial phase of the time response. However in practice it is required to find the controller stabilizing the system with the given characteristics of the transient response. Moreover the offered stabilization methods assume that the exact values of system parameters are known. However in practical implementation the system parameters are often subject to uncontrollable perturbations. Therefore the control should be designed in view of possible presence of parametric uncertainty.

In this work we offer a way of analysis and synthesis allowing overcoming the mentioned difficulties and broadening our understanding of the properties of hyperchaotic systems. The method is based on the use of superstability conditions [Polyak and Shcherbakov, 2002a,b], that just like the quadratic stability approach, provide the sufficient stability conditions.

The superstability conditions are convenient as they are formulated in the terms of the system matrix elements, not their eigenvalues. The arising linear restrictions on the system parameters are strict, and can not be performed for any controlled system. However, if superstability is achievable, it will be possible to find effective solutions for a number of complex control problems.

The application of superstability conditions to chaotic systems begins in [Talagaev and Tarakanov, 2012], where the conditions of achieving superstability were studied for a class of the 3D chaotic systems. In [Talagaev, 2014] it is shown that the problems of robust stabilization and restricted perturbation suppression can be solved by applying the superstability conditions to 3D chaotic systems. The aim of this work is the extension of the results achieved earlier on hyperchaotic systems. We single out a class of hyperchaotic systems for which we study the superstability achievement conditions and demonstrate the efficiency of superstability conditions for the robust analysis and control under parametric uncertainty. The offered approach allows revealing superstabilizable hyperchaotic systems and finding the superstabilizing control that provides the given characteristics of transient processes.

2 Superstability of chaotic and hyperchaotic systems

In this part we give the basic properties of the superstable systems and analyze the conditions that make the system superstabilization possible.

2.1 Preliminaries

Consider the linear controlled system

$$\dot{x}(t) = Ax(t) + Bu(t)\,, \quad (1)$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $A = (a_{ij}) \in R^{n\times n}$ is a matrix of system parameters, $B = (b_{ij}) \in R^{n\times m}$ is input matrix.

The matrix A and the system (1) corresponding to it are called superstable, if

$$\sigma(A) = \min_i(-a_{ii} - \sum_{j\neq i} | a_{ij} |) > 0\,,\ i = 1,...,n\,, \quad (2)$$

where $\sigma(A)$ – is the superstability degree of the matrix A. Superstability conditions are formulated in terms of the elements of system matrix. Superstability is a sufficient stability condition. If a system is superstable, it will always be stable (all eigenvalues of A have negative real parts). The inverse proposition is not always true. Thus, superstable systems are the subclass of the stable ones.

Superstable systems possess practically useful properties. With zero input ($u(t) \equiv 0$) the solution norm for any initial condition decreases monotonously and exponentially, that is $\| x(t) \| \le \| x(0) \| e^{-\sigma(A)t}$. Due to this stability with respect to initial conditions there is no drastic increase of a solution norm (peak effect) at the initial stage of the transient response. Unlike superstable systems, the stable ones do not possess this property. If control exists and is restricted ($u(t) \le 1$), then at $\| x(0) \| \le \eta = \| B \|_1 / \sigma(A)$ it will be $\| x(t) \| \le \eta$. It means that the trajectories beginning in the set $\{x \in R^n : \| x \| \le \eta\}$ remain in it at all admissible u.

An important step that expands the theory of superstable systems is the generalization of superstability conditions to nonlinear dynamic systems capable of exhibiting chaotic behavior. It is possible, as superstability (unlike stability) remains under nonlinear perturbations.

2.2 Superstability achievement conditions

Consider a class of controlled nonlinear systems described by the following equation:

$$\dot{x} = Ax + g(x) + Bu\,, \quad (3)$$

where $g : R^n \to R^n$ is a nonlinear part of the system, $g(x)|_{x=0} = 0$. Let at $u(t) \equiv 0$ the parameter values be such that the system (3) will demonstrate chaotic (or hyperchaotic) dynamics. Assume that 1) the dimensionality of the phase space $n \ge 3$; 2) the system is dissipative, 3) the trajectories of the systems evolve in some bounded region $S = \{x : \| x(t) \| \le L\}$; 4) the system has the zero equilibrium state and the Jacobi matrix $J = A$ at $x = 0$.

The control of chaotic system (3) is in designing control laws that stabilize the systems at the zero equilibrium (hence chaos suppression). We replace the requirements for stability provision with the ones for superstability provision. If the system (3) possesses chaotic dynamics, then the equilibrium state $x = 0$ is unstable and the superstability conditions (2) are not performed. Then the superstabilization problem will be as follows. It is necessary to find the state feedback

$$u = Kx\,, \quad K = (k_{ij}) \in R^{m\times n}\,, \quad (4)$$

which provides superstability of the system (3). Substituting of the control law (4) into (3) the closed-loop system will be written as

$$\dot{x}(t) = A_c x(t) + g(x)\,, \quad A_c = A + BK \quad (5)$$

and for the given matrices A, B the superstabilization problem will be reduced to finding the superstabilizing matrix K, that provides the performance of superstability condition $\sigma(A_c) > 0$ for the matrix A_c.

The application of the superstability conditions (2) to the system (5) leads to acute constraints on the elements of the matrix $A_c = A + BK$. They can not be performed for every system. The possibility of system superstabilization (i.e. the existence of the superstabilizing regulator) can be checked in two ways.

Approach 1. The superstability of the matrix $A_c = A + H$, $H = BK$ will provide the performance of conditions

$$-(a_{ii} + h_{ii}(K)) > \sum_{j \neq i} | a_{ij} + h_{ij}(K) | , i = 1,...,n , \quad (6)$$

where $h_{ij}(K) = (BK)_{ij} = b_i K$, b_i is the i th line of the matrix. If we introduce additional variables σ, p_{ij}, the superstability condition (6) for the matrix A_c can be written in the equivalent form. For diagonal elements of the matrix A_c the condition will be written as

$$-(a_{ii} + h_{ii}(K)) - \sum_{j \neq i} p_{ij} \geq \sigma > 0 , \; i = 1,...,n , \quad (7)$$

and for all others it will be

$$-p_{ij} \leq a_{ij} + h_{ij}(K_l) \leq p_{ij} , \; i, j = 1,...,n , \; i \neq j . \; (8)$$

Now to check the superstability of the matrix A_c we need k_{ij} and p_{ij}, $i, j = 1,...,n$ that satisfy the mentioned inequalities to exist at some $\sigma > 0$. The check can be performed by solving the linear programming (LP) problem:

max σ *subject to constraints* (7) *and* (8).

The variables in the LP problem are the matrices K, $P = (p_{ij})$ and the scalar σ. If the LP problem has the solution K^*, σ^*, and for all that $\sigma^* > 0$, then the controller $u = K^* x$ provides the superstability of the closed-loop system (5). The value $\sigma^* = \sigma(A + BK^*)$ is the best estimation, at which its state satisfies the condition $\| x(t) \| \leq \| x(0) \| e^{-\sigma^* t}$. If while solving the LP problem it turns out that $\sigma^* \leq 0$, superstabilization is impossible.

Approach 2. Consider the chaotic system $\dot{x} = Ax + g(x)$. If superstability is achievable, then for the given unstable matrix A there exists a nonempty set of matrices $S = \{W\}$, $W = (w_{ij}) \in R^{n \times n}$, that provide for the matrices $\widetilde{A} = A + W$ the performance of superstability conditions

$$-(a_{ii} + w_{ii}) > \sum_{j \neq i} | (a_{ij} + w_{ij}) | , \; i = 1,...,n . \quad (9)$$

The matrix A of the system is often rarefied, that is some a_{ij} are equal to zero. We restrict the choice of matrices W, that provide the transformation of the elements of the matrix A according to the rule $a_{ij} \to a_{ij} + w_{ij}$. Let in addition to the conditions (9) the elements of the matrices satisfy the condition

$$w_{ij} = 0 \Leftrightarrow a_{ij} = 0 . \quad (10)$$

The restriction (10) allows designing the matrix W structurally equivalent to A. Define the distance between the unstable matrix A and the superstable matrix $\widetilde{A} = A + W$ as $dist(A, \widetilde{A}) = \| A - \widetilde{A} \| = \| W \|$, where $\| \cdot \|$ – is a matrix Euclidean norm. Then the superstabilizability is confirmed via solving the quadratic programming (QP) problem:

min $\| W \|$ *subject to constraints* (9) *and* (10).

If the solution W^* of the QP problem exists, then for the given unstable matrix A we will find the closest superstable matrix $\widetilde{A}^* = A + W^*$. Simultaneously for the system (3) we find the superstabilizing regulator $u = Kx$, which provides the superstability of the matrix $A_c = A + BK$ at $B = I$ and $K = W^*$.

Both approaches allow finding the superstabilizing control, if there exists one. The first approach allows checking whether the given controller (the matrix B is given) is able to superstabilize the system. The second approach is more generic and constructive at the same time. It allows learning whether the superstabilization is possible in principle.

2.3 A class of superstabilizable systems

Having sorted out various hyperchaotic systems, we can see that superstability is a rare property. It is shown in [Talagaev and Tarakanov, 2012] that the structure of the matrix A determines the possibility of accessing superstability by the system (3). For the solution of the QP problem to exist it is enough to perform the condition $a_{ii} \neq 0$ for the diagonal elements of the matrix A. Indeed, if $a_{ii} \neq 0$, it is always possible to find such w_{ii} that the conditions (9) will be performed. This conclusion can serve as a selection criterion for superstabilizable chaotic and hyperchaotic systems. In a general case a class of superstabilizable systems can be written as

$$\dot{x} = Ax + g(x) = Ax + x_1 G_1 x + x_2 G_2 x + ... + x_n F_n x ,$$

where $x \in R^n$, $A = (a_{ij})$, $a_{ii} \neq 0$, $G_l = (f_{ij}^l) \in R^{n \times n}$, $l = \overline{1, n}$.

3D *case*. An example of superstabilizable system among the chaotic systems with the phase space dimensionality $n = 3$ is

$$\dot{x} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} x + x_1 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} x .$$

where $x = (x_1, x_2, x_3)^T$. At $a_{12} a_{21} > 0$, $a_{12} a_{21} < 0$ and $a_{12} a_{21} = 0$ the system becomes Lorenz, Chen and Lu chaotic systems, correspondingly [Lu, Chen and Cheng, 2004]. The results of the analysis and superstabilization are shown in [Talagaev, 2014].

4D *case*. The search of the superstabilizable systems can also be conducted among the systems with the phase space dimensionality equal to four ($n = 4$). Some examples of the superstabilizable hyperchaotic systems are given below: the Li system [Li, Tang and Chen, 2005]

$$\dot{x}_1 = a(x_2 - x_1) + x_4,$$
$$\dot{x}_2 = b_1 x_1 - x_1 x_3 + b_2 x_2,$$
$$\dot{x}_3 = -c x_3 + x_1 x_2,$$
$$\dot{x}_4 = x_2 x_3 + d x_4,$$

the Jia system [Jia, 2007b]

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4, \\ \dot{x}_2 &= b x_1 - x_1 x_3 - x_2, \\ \dot{x}_3 &= -c x_3 + x_1 x_2, \\ \dot{x}_4 &= -x_1 x_3 + d x_4. \end{aligned} \qquad (11)$$

This list is not complete and can be expanded. An example of the system (11) stabilization will be given further.

3 Robust analysis and controller design

The efficiency of the superstability conditions is not limited to the presented analysis and solution of the superstabilization problem for a class of chaotic systems. The field of application of superstability is much wider. In this part we demonstrate the efficiency of superstability conditions for solving the problem of the robust synthesis and analysis.

3.1 Robust analysis

Let the presence of the uncertainty be caused by the inaccuracies in the measurements of parameter values of the linear part of the system

$$\dot{x} = (A_0 + \gamma\Delta)x + g(x).$$

Here $A_0 = (a_{ij}^0)$ is a real nominal matrix, $\Delta = (\delta_{ij})$ is $(n \times n)$-matrix of the values of uncertain factors with the elements $|\delta_{ij}| \le l_{ij}$, $l_{ij} \ge 0$ are given values that form the matrix $L = (l_{ij})$, $\gamma \ge 0$ is the uncertainty range. The matrix L specifies the scale of element measurements $a_{ij} = a_{ij} + \gamma\delta_{ij}$ of the matrix A.

Let the nominal system $\dot{x} = A_0 x + g(x)$ be superstable. Then, for the interval matrix family

$$A_0 + \gamma\Delta$$

there should be found a superstability radius γ^*, for which the superstability is preserved for all $\gamma < \gamma^*$.

Retaining of superstability means performing the conditions

$$-a_{ii}^0 - \gamma\delta_{ii} - \sum_{j \ne i} |a_{ij}^0 + \gamma\delta_{ij}| > 0, \; i = 1,...,n.$$

They will be performed, if

$$-a_{ii}^0 - \gamma l_{ii} - \sum_{j \ne i} (|a_{ij}^0| + \gamma l_{ij}) > 0, \; i = 1,...,n.$$

From this we get a lower estimation of the superstability radius

$$\gamma < \gamma^* = \min_i \frac{-a_{ii}^0 - \sum_{j \ne i} |a_{ij}^0|}{\sum_j l_{ij}}.$$

Having calculated the value of γ^* we get the necessary information, which allows making a judgment on the preservation of superstability for this state of equilibrium in the presence of some uncertainty at the precise measurement of the system parameters. Note that by changing the superstability preservation demand on the stability we complicate the problem substantially.

3.2 Robust superstabilization

Study the problem of the superstabilizing regulator synthesis at parametric uncertainty.

Consider the uncertain system

$$\dot{x} = (A_0 + \gamma\Delta)x + g(x) + Bu. \qquad (12)$$

Let nominal system ($\gamma \equiv 0$) be superstabilizable via the feedback $u = Kx$ and the matrix of the closed-loop system $A_c^0(K) = A_0 + BK$, $A_c^0(K) = (a_{ij}^0(K))$ superstable. Then the uncertain system (12) is robustly superstabilizable via the given feedback, if superstability is retained for the family of matrices

$$A_c = A_c^0(K) + \gamma\Delta.$$

This means that at all the admissible Δ for the matrix A_c there should be performed the conditions

$$-a_{ii}^0(K) - \gamma\delta_{ii} - \sum_{j \ne i} |a_{ij}^0(K) + \gamma\delta_{ij}| > 0, \; i = 1,...,n.$$

Then the robust superstabilization of the system (12) is possible, if (3)

$$\gamma < \gamma_K^* = \min_i \frac{-a_{ii}^0(K) - \sum_{j \ne i} |a_{ij}^0(K)|}{\sum_j l_{ij}}.$$

In particular, if $l_{ij} \equiv 1$, then $\gamma_K^* = \sigma(A_c^0(K))/n$.

3.3 The example

Let's design the superstabilizing regulator for the system (11), written in the form

$$\dot{x} = A_0 x + x_1 G_1 x,$$

where

$$A_0 = \begin{pmatrix} -a & a & 0 & 1 \\ b & -1 & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}, \; G_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$

The system exhibits hyperchaotic behavior when the parameter values are taken as $a = 10$, $b = 28$, $c = 8/3$, $d = 1.3$. Fig. 1 shows the attractor corresponding to the situation when the system has two positive Lyapunov exponents.

In the hyperchaotic regime the superstability conditions for the matrix A are not performed. The characteristic values of the Jacbian matrix at the zero equilibrium are -22.8277, 11.8277, -2.6667 and 1.3000. By adding the controller to the system, we get the following model of the controlled system

$$\dot{x} = A_0 x + x_1 G_1 x + Bu.$$

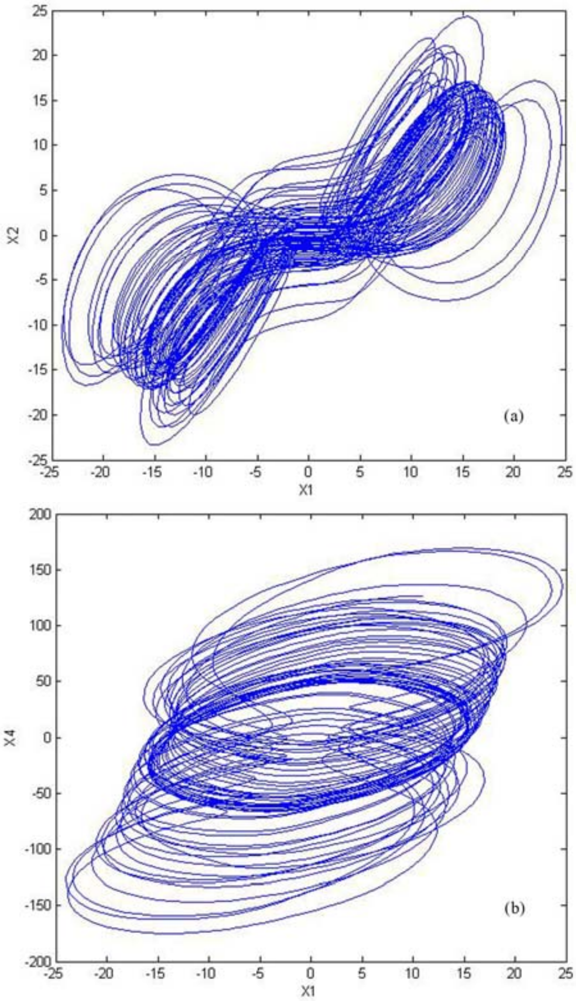

Figure 1. The attractor of the hyperchaotic system (12): (a) x1–x2 plane; (b) x1–x4 plane.

To stabilize the system, we define the feedback controller $u = Kx$. Let's find out if there exists the gain matrix $K \in R^{4\times 4}$ able to provide the performance of superstability conditions for the matrix of the closed-loop system $A_c = A_0 + BK$. While considering the matrix A_0 we see that $a_{ii} \neq 0$. Then if we choose $B = I$ the controller with the gain matrices

$$K_s = diag(k_{11}, k_{22}, k_{33}, k_{44})$$

will be superstabilizable. Actually, from the superstability conditions for the matrix A_c

$$\sigma_1 = -(-a + k_{11}) - a - 1 > 0, \; \sigma_2 = -(-1 + k_{22}) - b > 0,$$

$$\sigma_3 = -(c + k_{33}) > 0, \; \sigma_4 = -(d + k_{44}) > 0$$

it becomes clear that the superstabilization will be provided by

$$k_{11} = -1 - \sigma_1, \; k_{22} = 1 - b - \sigma_2,$$

$$k_{33} = -c - \sigma_3, \; k_{44} = -d - \sigma_4.$$

At that the values $\sigma_1, \sigma_2, \sigma_3, \sigma_4 > 0$ can be chosen in such a way as to provide the desirable stability reserve of the system.

Fig. 2 shows the results of the numerical simulation of the controlled hyperchaotic system with the initial condition $x(0) = (5,5,5,5)$. As can be seen the controller with the chosen superstability degree $\sigma_1 = ... = \sigma_4 = 0.5$ stabilizes the system in such a way that at the initial stage of the time response the peak effect does not occur.

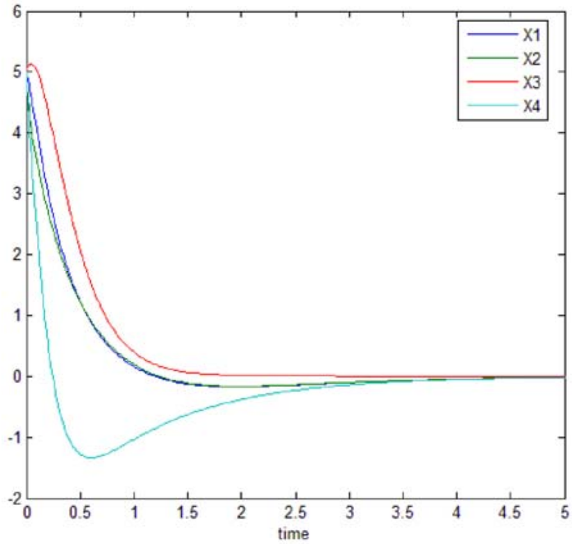

Figure 2. Time responses of the controlled system.

Now assume that nonzero system parameters in the matrix $A = A_0 + \gamma\Delta$ possess the uncertainty $a_{ij} = a_{ij0} + \gamma\delta_{ij}$, $|\delta_{ij}| \leq l_{ij}$, where a_{ij0} are the nominal parameter values for which we designed the superstabilizing regulator earlier. Let $l_{ij} \equiv 1$. Then the uncertain system

$$\dot{x} = (A_0 + \gamma\Delta)x + x_1 G_1 x + Bu$$

is superstabilizable by the feedback controller $u = K_s x$, if $\gamma < \sigma(A_c^0)/n$. As $\sigma(A_c^0) = 0.5$, we get $\gamma_l < 0.125$.

4 Conclusion

The method of the analysis and hyperchaotic system control aimed at providing superstability is presented. It is shown that the implementation of superstability conditions is an effective way of solving the problems of robust analysis and synthesis of hyperchaotic systems with parameter uncertainties. An advantage of the offered approach is the possibility to provide the given transient response characteristics.

References

Chen, Z., Yang, Y., Qi, G. and Yuana, Z. (2007). A novel hyperchaos system only with one equilibrium. Phys. Lett. A, 360(6), pp. 696–701.

Correia M. J. and Rech, P. C. (2010). Hyperchaos in a new four-dimensional autonomous system. Int. J. Bifurc. Chaos, 20(10), pp. 3295–3301.

Dou, F., Sun, H., Duan, W. and Lü, K. (2009). Controlling hyperchaos in the new hyperchaotic system. Com. Nonlin. Sci. Num. Simul., 14(2), pp. 552–559.

Effati, S., Saberi Nik, H. and Jajarmi, A. (2013). Hyperchaos control of the hyperchaotic Chen system by optimal control design. Nonlinear Dyn., 73(1-2), pp. 499–508.

El-Sayed, A. M. A., Nour, H. M., Elsaid, A., Matouk, A.E. and Elsonbaty, A. (2014). Circuit realization, bifurcations, chaos and hyperchaos in a new 4D system. Appl. Math. Comput., 239, pp. 333–345.

Hu, G. (2009). Generating hyperchaotic attractors with three positive lyapunov exponents via state feedback control. Int. J. Bifurc. Chaos, 19(2), pp. 651–660.

Jia, Q. (2007a). Adaptive control and synchronization of a new hyperchaotic system with unknown parameters. Phys. Lett A, 362 (5), pp. 424–429.

Jia, Q. (2007b). Hyperchaos generated from the Lorenz chaotic system and its control. Phys. Lett. A, 366(3), pp. 217–222.

Li Y., Tang W. K. S. and Chen G. (2005) Generating hyperchaos via state feedback control. Int. J. of Bifurc. Chaos, 15(10), pp. 3367–3375.

Li, C. and Sprott, J. C. (2014). Coexisting Hidden Attractors in a 4-D Simplified Lorenz System. Int. J. of Bifurc. Chaos, 24(3), 1450034.

Li, Q., Hu, S., Tang, S. and Zeng, G. (2014). Hyperchaos and horseshoe in a 4D memristive system with a line of equilibria and its implementation. Int. J. Circ. Theory Appl., 42(11), pp. 1172–1188.

Li, Y.X., Chen, G.R. and Tang, W.K.S. (2005). Controlling a unified chaotic system to hyperchaotic. IEEE Trans. Circ. Syst. II-Expr. Briefs, 52(4), pp. 204–207.

Liu, M., Feng, J. and Tse, C. K. (2010). A new hyperchaotic system and its circuit implementation Int. J. of Bifurc. Chaos, 20(4), pp. 1201–1208.

Lü, J. and Chen, G. (2006). Generating multiscroll chaotic attractors: Theories, methods and applications. Int. J. Bifurc. Chaos, 16(4), pp. 775–858.

Lu, J., Chen, G., and Cheng, D. (2004). A new chaotic system and beyond: the generalized Lorenz-like system. Int. J. Bifurc. Chaos, 14(5), pp. 1507–1537.

Njah, A. N. (2010). Tracking control and synchronization of the new hyperchaotic Liu system via backstepping techniques. Nonlinear Dyn., 61(1-2), pp. 1–9.

Pang, S. and Liu, Y. (2011). A new hyperchaotic system from the Lü system and its control. J. Comput. Appl. Math., 235, 2775–2789.

Polyak, B.T. and Shcherbakov, P.S. (2002a). Superstable Linear Control Systems. I. Analysis. Autom. Rem. Cont., 63(8), pp. 1239–1254.

Polyak, B.T. and Shcherbakov, P.S. (2002b). Superstable linear control systems. II. Design. Autom. Rem. Cont., 63(11), pp. 1745–1763.

Qi, G., van Wyk, M. A., van Wyk, B. J. and Chen, G. (2008). On a new hyperchaotic system. Phys. Lett. A, 372(2), pp. 124–136.

Takahashi, Y., Nakano, H. and Saito, T. (2004). A simple hyperchaos generator based on impulsive switching. IEEE Trans. Circ. Syst. II-Expr. Briefs, 51(9), pp. 468–472.

Talagaev, Y.V. (2014). Robust analysis and superstabilization of chaotic systems. In Proc. 2014 IEEE Int. Conf. Control Applications. Antibes, France, Oct. 8-10, pp. 1431–1436.

Talagaev, Y.V. and Tarakanov, A.F. (2012). Superstability and optimal multiparametrical suppression of chaotic dynamics in a class of autonomous systems with quadratic nonlinearities. Differential Equations, 48(1), pp. 153–157.

Toopchi, Y. and Wang, J. (2014). Chaos Control and Synchronization of a Hyperchaotic Zhou System by Integral Sliding Mode control. Entropy, 16(12), pp. 6539–6552.

Wang, H.-X, Cai, G.-L. , Miao, S. and Tian, L.-X. (2010). Nonlinear feedback control of a novel hyperchaotic system and its circuit implementation. Chin. Phys. B, 19(3) pp. 030509.

Wang, X. and Zhao, G. (2010). Hyperchaos generated from the unified chaotic system and its control Int. J. Mod. Phys. B, 24(23), pp. 4619–4637.

Yan, J.J. (2004). H-infinity controlling hyperchaos of the Rossler system with input nonlinearity. Chaos Solitons Fractals, 21(2), pp. 283–293.

Yan, Z.Y. (2005). Controlling hyperchaos in the new hyperchaotic Chen system. Appl. Math. Comput. 168(2), pp. 1239–1250.

Yang, L., Liu, Z.R. and Mao, J.M. (2000). Controlling hyperchaos. Phys. Rev. Lett., 84(1), pp. 67–70.

Yang, Q., Zhang, K. and Chen, G. (2009). Hyperchaotic attractors from a linearly controlled Lorenz system. Nonlin. Anal. Real World Appl., 10(3), pp. 1601–1617.

Yu, H.J., Cai, G.L. and Li, Y.X. (2012). Dynamic analysis and control of a new hyperchaotic finance system. Nonlinear Dyn., 67(3), pp. 2171–2182.

Yu, S., Lu, J., Yu, X. and Chen, G. (2012). Design and implementation of grid multiwing hyperchaotic Lorenz system family via switching control and constructing super-heteroclinic loops. IEEE Trans. Circuits Syst. I, Regul. Pap. 59(5), pp. 1015–1028.

Yu, S.M., Lü, J.H. and Chen, G.R. (2007). Theoretical design and circuit implementation of multidirectional multi-torus chaotic attractors. IEEE Trans. Circuits Syst. I, 54, 2087–2098

Yujun, N., Xingyuan, W., Mingjun, W. and Huaguang, Z. (2010). A new hyperchaotic system and its circuit implementation. Com. Nonlin. Sci. Num. Simul., 15(11), pp. 3518–3524.

Zhu, C. (2010). Controlling hyperchaos in hyperchaotic Lorenz system using feedback controllers. App. Math. Comput., 216(10), pp. 3126–3132.