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ON THE EFFECT OF CORRECTING THE ELECTRO-HYDRAULIC SERVO DRIVE ON THE STABILITY AND CONTROLLABILITY OF AN AIRCRAFT

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Abstract
Response of the simplest servo system to an input perturbation is considered for two variants of correction of the servo drive to provide solutions for its stability and controllability. Numerical computations are employed to study the effect of the system's main parameters on the dynamic characteristics of an aircraft.

Key words
mathematical model, electro-hydraulic controlling systems, stability, rolling systems.

1 Introduction

At present, electro-hydraulic controlling systems (EHCS) for various objects, including undercarriage struts of aircrafts, are becoming more and more widely used [Krapivko and Lar'kin, 1975; Lar'kin, 1975; Popov, 1977]. In investigating the dynamics of EHCS the main attention is known to be paid to the issues of stability and quality of the regulation processes. Numerous works tackle these issues. For example, [Krapivko and Lar'kin, 1975] studied the effect of Q-factor, attachment rigidity, response times, additional feedbacks on the dynamic characteristics of the servo drive, as well as the relation between the dynamic stiffness of a servo drive and its nonlinear characteristics, and a conclusion was made about a possibility of developing a linear mathematical model of a servo drive that could be used in analyzing stability problems. At the same time, providing the stability may require correction of the servo drive, involving the introduction of leakages between the cavities of the slave cylinder or connecting its cavities to a "resistance-capacitance" unit, called "damper", which is a case housing a spring-loaded piston. These issues remain unstudied in the available literature. To this end, using a newly introduced model, the present work investigates the stability and controllability of the servo drive-mass system for the above mentioned variants of correction, presents analytical correlations of components of the dynamic stiffness of the servo drive with the parameters of the system analyzed, as well as computational results of the behavior of the main characteristics of the system analyzed (displacement of the damper piston and the turn angle of the controlled object) when changing the servo drive parameters. The computations are performed using the MAPLE package for analytical computations.

In doing so, the controllability of the system is assessed judging by its response to the input perturbation in the form of a unit function, and its stability is assessed, judging from the presence or absence of fluctuations of the controlled object during the transient process and after it. The response characteristics used are time and value of the displacement of the controlled object for which the latter one tends to be constant.

2 Equations of Motion

The physical model in question can be schematically represented as shown in Fig.1. It includes an electro-mechanical transducer with a hydraulic amplifier (EMT + HP), a slave cylinder (SC) and a damper (DAM). HP is connected to pumping and drain lines. The SC piston has an orifice (OR), and the channels connecting SC and DAM cavities have orifices OR1 and OR2. The system control unit consists of a comparing element, an amplifier with coefficient k_a, and a feedback with coefficient k_{bf}.

It is assumed that relations between valve displacement, gate current I, angle of rotation of the SC rod and input voltage can be written as follows [Krapivko and Lar'kin 1975]:

$$T\dot{x}_v + x_v = k_{eh} I = A_0(u_{in}/k_{bf} - \chi) \quad (1)$$

where $A_0 = k_{eh}k_a k_{bf}$, k_{eh}, and T are gain factor and time constant of EMT + HP, respectively.

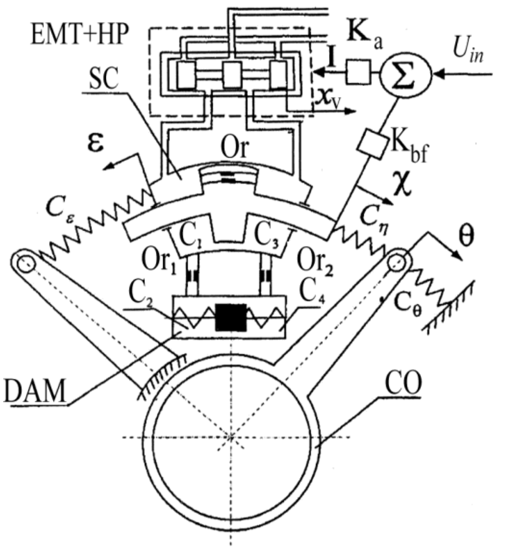

Figure 1. Scheme of the physical model.

The springs with stiffness values C_η and C_ε in the above arrangement depict twisting elasticities of the controlled object (CO) and of the SC supporting element. CO is supported by a spring with stiffness C_θ. Such a description of the arrangement in question can correspond to a rolling system with high velocities, when resistance to wheel slip of the aircraft is accounted for by coefficient C_θ.

Here is how the system works: after applying input voltage u_{in} the valve shifts, for instance, to the right, and the pumping line is connected to cavity C_1 of the SC, and cavity C_2 is connected to the drain line. SC piston rotates to angle χ, causing the CO to turn to angle θ.

It is also assumed that:

- the HP valve and the SC piston are symmetric,
- overlapping of the valve ports is zero,
- valve leakage is absent,
- friction of the valve and the piston is low,
- the effect of the loading on the valve flow is small,
- the dependence of throttle OR and OR1 (OR2) flows on the pressure bump on them is linear.

Then the equation of motion of the servo drive-mass system can be written as follows:

$$\left.\begin{aligned} &\dot{\theta} = y \\ &J\dot{y} = -(h_\theta y + C_\eta(\theta - \chi) + C_\theta \theta) \\ &\left(1 + \frac{C_\eta}{C_\varepsilon}\right)\dot{\chi} = Dx + \frac{C_\eta}{C_\varepsilon} y - (k_{or0} + k_{or10}) C_\eta(\chi - \theta) + k_2 G(\chi_d) \\ &k_1\dot{\chi}_d = k_{or10} C_\eta(\chi - \theta) - k_2 G(\chi_d) \\ &T\dot{x} = -x - \chi + \theta_{in}, \end{aligned}\right\} \quad (2)$$

where

$x = \dfrac{x_v}{A_0}$,

$k_{or0} = \dfrac{k_{or}}{(F_c r_c)^2}, k_{or10} = \dfrac{k_{or1}}{2(F_c r_c)^2}, k_2 = \dfrac{k_{or1}}{2F_c F_d}$, k_{or}, k_{or1} - coefficients of linear relationship of flow through orifices OR and OR1 and pressure bump on them, $\chi_d = z_d / r_c$ - displacement of DAM piston reduced to rotation, z_d - displacement of DAM piston, r_c - radius of action of a force on SC, $D = \dfrac{B_l A_0}{F_c r_c}$ - Q-factor of the servo drive, B_l - coefficient correlating valve leakage with its displacement,

F_c, F_d - areas of SC and DAM pistons;

$$G(\chi_d) = \left.\begin{aligned} &C_{d2}\chi_d + (C_{d2} - C_{d1})\Delta, \chi_d \le -\Delta \\ &C_{d1}\chi_d, -\Delta \le \chi_d \le \Delta \\ &C_{d2}\chi_d - (C_{d2} - C_{d1})\Delta, \chi_d \ge \Delta \end{aligned}\right\} \quad (3)$$

Δ - value of angle χ_d, when DAM piston stops on the support,

C_{d1}, C_{d2} - variable stiffness coefficients of the DAM spring,

$k_1 = \dfrac{F_d}{F_c}$ - ratio of piston areas, J - moment of inertia of CO relative to the rotation axis, h_θ - damping coefficient of the turn of CO, $\theta_{in} = u_{in} / k_{bf}$.

Consider a special case $T = 0, k_{or1} = 0$ (the damper is not connected). The transfer function from θ_{in} to angle θ, is, apparently, of the following form:

$$\theta = \frac{C_\eta D}{a_0 p^3 + a_1 p^2 + a_2 p + a_3}\theta_{in} \qquad (4)$$

where

$$a_0 = J(1+\frac{C_\eta}{C_\varepsilon}), a_1 = J(D + k_{or0}C_\eta) + h_\theta(1+\frac{C_\eta}{C_\varepsilon}),$$

$$a_2 = (D + k_{or0}C_\eta)h_\theta + C_\eta + C_\theta + \frac{C_\eta C_\theta}{C_\varepsilon},$$

$$a_3 = D(C_\eta + C_\theta) + k_{or0}C_\eta C_\theta$$

For the input perturbation in the form of a unit function $\theta_{in} = \theta_{in0}1(\tau)$, the steady state value of variable θ for large enough values of τ is equal to

$$\theta^* = \frac{C_\eta D \theta_{in0}}{D(C_\eta + C_\theta) + k_{or0}C_\eta C_\theta} \qquad (5)$$

Hence, for C_θ=0, $\theta^* = \theta_{in}$. The presence of a spring with stiffness C_θ and account of leakage $k_{or0} \neq 0$ reduces the response of CO to input perturbation. In the absence of leakage in SC ($k_{or0} = 0$), stability can be provided by taking account of friction coefficient h_θ determined, using Hurwitz criterion, from relation

$$h_\theta^2 D(1+\frac{C_\eta}{C_\varepsilon}) + h_\theta[JD^2 + (1+\frac{C_\eta}{C_\varepsilon})(C_\eta + C_\theta + \frac{C_\eta C_\theta}{C_\varepsilon})] - JD\frac{C_\eta^2}{C_\varepsilon} > 0$$

Hence, for h_θ=0, it is impossible to provide stability of the system. Using the initial version (without correction) of the servo drive and a spring of stiffness C_η connected in parallel with a spring of effective stiffness $C_{eff}(\omega)$ and a damper with effective resistance coefficient $h_{eff}(\omega)$, in the regime of steady state vibration with frequency ω, the following expressions can be obtained for them:

$$C_{eff}(\omega) = C_\eta \frac{D^2 + b\omega^2}{D^2 + b^2\omega^2}, b = 1 + \frac{C_\eta}{C_\varepsilon}, \qquad (6)$$
$$h_{eff}(\omega) = C_\eta \frac{D(1-b)}{D^2 + b^2\omega^2}$$

It is evident that the effective resistance coefficient is negative. It means that the servo drive does not absorb energy, which explains the instability for $h_\theta < h_{eff}$.

For $k_{or0} \neq 0, T = 0$ (servo drive without account of spring C_η) relations (6) will take the following form:

$$C_{eff} = \frac{Dk_{or0} + \frac{\omega^2}{C_\varepsilon}}{s}, s = k_{or0}^2 + \frac{\omega^2}{C_\varepsilon^2} \qquad (7)$$
$$h_{eff} = \frac{k_{or0} - \frac{D}{C_\varepsilon}}{s}$$

3 Numerical computations

Introducing dimensionless parameters (with superscript $^-$):

$$D = \bar{D}\sqrt{\frac{a}{m}}, C_\eta = \bar{C}_\eta 2ar^2, C_\varepsilon = \bar{C}_\varepsilon 2ar^2, k_{or0} = \frac{\bar{k}_{or0}}{2\sqrt{amr^2}}, C_\theta = \bar{C}_\theta 2ar^2$$

$$k_{or10} = \bar{k}_{or10}\frac{1}{2\sqrt{amr^2}}, k_2 = \bar{k}_2/\sqrt{am}, T = \bar{T}\sqrt{\frac{m}{a}}, \tau = \bar{\tau}\sqrt{\frac{m}{a}}, \omega = \bar{\omega}\sqrt{\frac{a}{m}}.$$

$$J = \bar{J}2mr^2, h_\theta = \bar{h}_\theta 2\sqrt{amr^2}, C_{d1} = \bar{C}_{d1}a, C_{d2} = \bar{C}_{d2}a,$$

(a, m, r are scale coefficients with dimensions $\frac{н}{м}, \frac{нс^2}{м}, м$, respectively) and using the following re-designation of variables $\chi \to z_1, \chi_\partial \to z_2, \theta \to z_3, \dot{\theta} \to z_4, x \to z_5$. Here, $^-$ over the dimensionless variables and parameters are omitted.

In what follows, the following parameters will be considered constant in numerical computations

$$\bar{C}_\eta = 1{,}217; \bar{C}_\varepsilon = 6{,}086; \bar{C}_\theta = 0{,}24; k_1 = 0{,}5; \bar{J} = 1;$$

$$F_c = 50*10^{-4}m^2; F_d = 25*10^{-4}m^2; r_c = 0{,}1m;$$

The rest of the parameters are considered variable, varying in the following limits:

$$k_{or} \in [0{,}5;2]*10^{-10}m^5/(\text{N}*\text{c}), k_{or1} \in [1;4]*10^{-10}m^5/(\text{N}*\text{c})$$

$$\theta_{in} \in [0{,}05;0{,}15]\text{rad}, T \in [0;0{,}02]\text{c}, D = 4;7;10c^{-1}$$

From Fig. 2a it is evident that variation of z_2 in time takes place with increasing vibration. The system is unstable. The transient process corresponding to the highest of the values of D depicted in the figure, achieves θ^* in the shortest time. Introduction of leakage $(k_{or} \neq 0)$ in the SC piston stops vibration, at the same time reducing value of z_2^* and τ^*, thus degrading controllability (Fig.2b). The values of the parameters used in the computations in the above figure were: $\theta_{in} = 0{,}05rad, D = 10c^{-1}, h_\theta = 0$. The highest z_2^* and τ^* correspond to $k_{or} = 0{,}5*10^{-10}\frac{m^5}{\text{N}*\text{c}}$, the lowest ones – to $k_{or} = 0{,}2*10^{-10}\frac{m^5}{\text{N}*\text{c}}$.

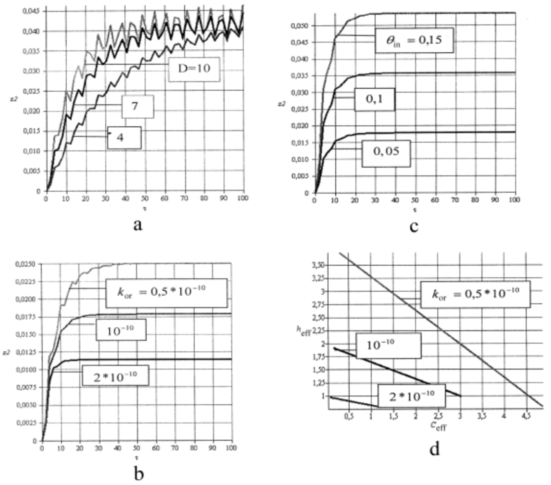

Figure 2.Transient process for angle z_2 for the initial version of the servo drive (without correction) for $\theta_{in}=0{,}05r, k_{or}=0, h_\theta=0$ and various values of Q factor of D (10; 7; 4; c^{-1}).

Increase of θ_{in} in the interval from 0.05 to 0.15 rad ($D=10c^{-1}$, $k_{or}=10^{-10}\frac{m^5}{\text{N*c}}$) leads to increase of values z_2^* and τ^* (Fig.2c). Variation of h_{eff} as a function of C_{eff} for various k_{or} for frequencies within the range of $\bar{\omega}\in[0;3]$ is presented in Fig. 2d. Within the limits of the above frequencies, effective values increase with increasing k_{or}.

Consider the case of a system with a connected damper. Variation of angle z_2 in time for the version with connected DAM for $D=10c^{-1}$ and various k_{or1} is presented in Fig. 3a. Here, coefficient $h_\theta=0$, stiffness $C_{d1}=3*10^6$ N/m, time constant $T=0$, θ_{in} =0,05, k_{or} =0. It evident from the figure that the value of k_{or1} only weakly affects the transient process. Response z_3 to the input signal shows that the transient process weakly depends on k_{or1} (the curves practically merge, Fig. 3b). This is due to a small displacement of angle χ_d resulting from substantial resistance of the spring. In the absence of spring ($C_{d1}=0$) variation of z_2 is shown in Fig.3c, and z_3 - in Fig.3d. It is evident from the figures, that for $k_{or1}=4*10^{-10}\frac{m^5}{\text{N*c}}$ DAM piston comes to the stop earlier than for $k_{or1}=10^{-10}\frac{m^5}{\text{N*c}}$, and z_3 more quickly reaches the value close to the maximum. After the piston comes to the stop, vibration develops.

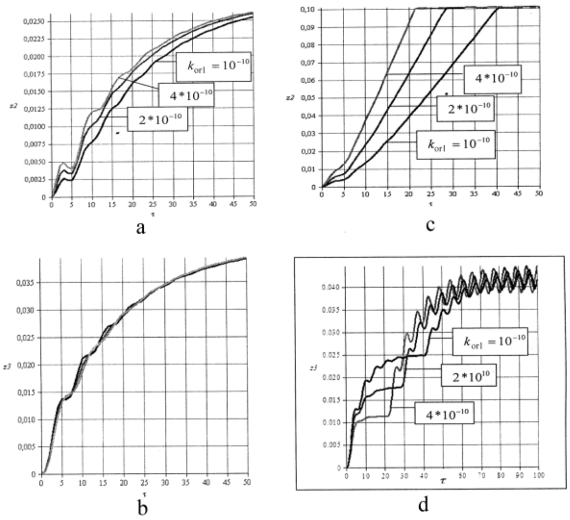

Figure 3.Transients with connected damper.

The computations using complete model (8), accounting for small values of T, show that time variation of coordinates z_3 and z_2 differ very little from those shown in Fig. 3 for the same parameters. If small leakage through SC piston $k_{or}=0{,}2*10^{-10}\frac{m^5}{\text{N*c}}$ is taken into account, the maximum values of output z_3^* decrease (Fig. 4a) as compared with Fig. 3a. At the same time, for C_{d1}=0 there is no vibration developing after DAM piston comes to the stop.

Components of the dynamic stiffness of the servo drive were determined for complete model (2) as:

$$C_{eff}=\frac{CF+GE\omega^2}{F^2+G^2\omega^2}, h_{eff}=\frac{EF-GC}{F^2+G^2\omega^2},$$
$$C=\frac{D(K_{d3}+K_1T\omega^2)}{1+T^2\omega^2}-K_1\omega^2,$$
$$F=K_{or0}K_{d3}-\frac{K_1\omega^2}{C_\varepsilon}, G=(K_{or0}+K_{or10})K_1+\frac{K_{d3}}{C_\varepsilon},$$
$$E=\frac{D(K_1-TK_{d3})}{1+T^2\omega^2}+K_{d3}, K_1=\frac{F_d}{F_c}, K_{d3}=K_2C_{d1}$$

Relations $h_{eff}(C_{eff})$ are given in Figs.4c and 4г for C_{d1}=3000000$\frac{\text{N*c}}{m}$ and C_{d1}=0, respectively. The highest values of h_{eff} and C_{eff} for $k_{or}=0{,}2*10^{-10}\frac{m^5}{\text{N*c}}$ correspond to $k_{or1}=10^{-10}\frac{m^5}{\text{N*c}}$, and the lowest one to $k_{or1}=4*10^{-10}\frac{m^5}{\text{N*c}}$. For $C_{d1}=3000000\frac{\text{N*c}}{m}$, the values of h_{eff} and C_{eff}

are a little lower as compared with the case of C_{d1}=0.

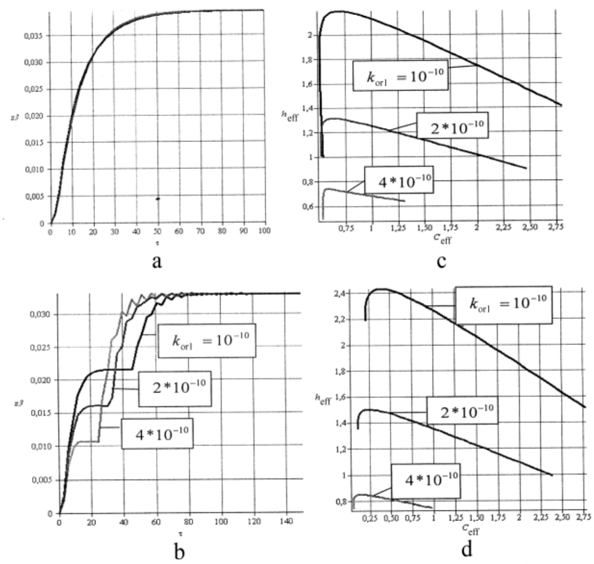

Figure 4. Transients with the leakages through the piston (4a, b), the components of the dynamic stiffness (4c, d).

It is noteworthy that when choosing parameters of a particular system, say, a system with rolling, the available values of stiffness and damping must pertain to the stability region.

4 Conclusions

This paper presents an analysis which is based on a new non-linear mathematical model. The main conclusions about the stability and controllability of an aircraft, depending on the parameters of the electro-hydraulic drive, can be summarized as follows:

1. The initial version of the servo drive (without orifice OR and DAM) sensors the input perturbation, however, stability is not provided.
2. The higher the value of k_{or}, the more the introduction of leakage in the SC piston degrades the controllability characteristics.
3. When DAM with a stiff spring is connected, the effect of coefficient k_{or1} on the transient process is minor. The DAM piston does not reach the stop.
4. To suppress vibration due to z_3 after the DAM piston comes to the stop, only small value of k_{or} or of coefficient h_θ is sufficient.
5. A small value of T produces minor effect on the controllability and stability as compared with the case of T=0.
6. Values of h_{eff} and C_{eff} of the servo drive are determined by the presence or absence of the spring and by the value of k_{or1}.
7. The absence of a spring significantly simplifies the design of DAM, decreasing its overall dimensions.

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