ENOC 2008, Saint Petersburg, Russia, June, 30–July, 4 2008

AMPLITUDE CONTROL OF A SELF-VIBRATION MACHINE

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Abstract
In this paper, a spring-mass-damper system with an electrical circuit (3rd-order system) is investigated to establish a self-vibration machine. The design of feedback gain makes the system a self-excited oscillator. The mechanical part (spring-mass-damper system) in the 3rd-order system becomes self-vibration machine which is robust against variation of the mass.

Key words
Self-vibration machine, Control of oscillation, Center manifold, Normal form.

1 Introduction

Self-vibration machine [1]·[2] is a machine vibrating with the natural frequency which is varied depending on the variation of the system parameters of mass and stiffness. Because the energy efficiency of the self-excited machine is kept independent of the variation, there are many applications for realizing high-performance machines, for example, ultrasonic transducer[3] and vibratory drilling[4]. Also, the characteristic that the resonance frequency is traced to the varied natural frequency depending on the variation of the parameters is applicable to the measurement of the natural frequency and such a characteristic is recently utilized to make AFM (atomic force microscope) much higher resolution [5].

In this paper, we consider a spring-mass-damper system which is actuated by a linear motor and establish a self-vibration machine under feedback control. A method of the amplitude control is also discussed by introducing the center manifold theory[6].

2 Realization of self-vibration machine

2.1 Analytical model and equation of motion

We consider an analytical model of spring-mass-damper system as shown in Fig.1 and investigate the method to make the system a self-vibration machine

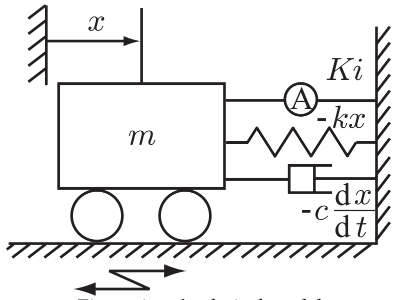


Figure 1. Analytical model.

and to control the magnitude of the amplitude. The thrust force of a linear motor is proportional to the current. The equation of motion of the spring-mass-damper system is expressed as follows:

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+c\frac{\mathrm{d}x}{\mathrm{d}t}+kx=Ki, \tag{1}$$

where K is the thrust constant of the linear motor. To realize a self-vibration machine, we actuate the input voltage of the linear motor. The dynamics of the linear motor is described by an equivalent L-R circuit. We input the voltage to the circuit which is proportional to the velocity of the mass. Under this feedback input, the circuit equation is expressed as follows:

$$L\frac{\mathrm{d}i}{\mathrm{d}t}+Ri+P\frac{\mathrm{d}x}{\mathrm{d}t}=V, \tag{2}$$

where i, L, R, and P are reactance, resistance, and back electromotive force constant of the linear motor. The input voltage is set as

$$V=K_l\frac{\mathrm{d}x}{\mathrm{d}t}, \tag{3}$$

where K_l is the linear feedback gain. Introducing representative length values of $X=KI/k$, I (rating current of the linear motor), $T = \sqrt{m/k}$ and using dimensionless displacement $x^* = x/X$, dimensionless current $i^* = i/I$, and dimensionless time $t^* = t/T$ yield the following equations of the spring-mass-damper system and the circuit for Eqs. (1) and(2):

$$\frac{\mathrm{d}^2x^*}{\mathrm{d}t^{*2}} + \gamma^*\frac{\mathrm{d}x^*}{\mathrm{d}t^*} + x^* = i^*, \tag{4}$$

$$\frac{\mathrm{d}i^*}{\mathrm{d}t^*} + R^*i^* + P^*\frac{\mathrm{d}x^*}{\mathrm{d}t^*} = V^*, \tag{5}$$

where the dimensionless parameters are $\gamma^* = c/\sqrt{mk}$, $R^* = RT/L$, $P^* = KP/kL$. The dimensionless input voltage is

$$V^* = K_l{}^*\frac{\mathrm{d}x^*}{\mathrm{d}t^*}, \tag{6}$$

where $K_l{}^* = K_lK/LI$.

2.2 Analysis of equation of motion and equation of circuit

Introducing the state variables of $x_1 = x*$, $x_2 = \mathrm{d}x^*/\mathrm{d}t^*$, $x_3 = i^*$ yields the state equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -\gamma & 1 \\ 0 & K_l - P & -R \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{7}$$

or

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = A\boldsymbol{x},\ \ \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -\gamma & 1 \\ 0 & K_l - P & -R \end{bmatrix} \tag{8}$$

In the above equations and hereafter, the symbol $*$ to denote dimensionless value is dropped for simplicity.

We clarify that the mechanical part of the spring-mass-damper system in the above 3rd order system including the dynamics of the circuit can accomplish the behavior of the self-vibration machine. In such a system, the 3 eigenvalues of the 3rd-order system have to be one negative real and a pair of complex with positive real part. The real eigenvalue has to be negative for drift-free oscillation of the mechanical part. Figure 2 shows the root locus corresponding to the parameter values of an experimental system. Under the appropriate choice of the feedback gain K_l, the above eigenvalues for the self-vibration machine are obtained. Figure 3 shows time histories of the displacement of the mechanical system and the current in the linear motor. It can be seen that the drift-free self-excited oscillation occurs and the amplitude grows with time.

3 Amplitude control of self-vibration machine

In the preceeding section, a self-vibration machine is realized, but the amplitude grows with time. In this section, we propose a nonlinear feedback control method

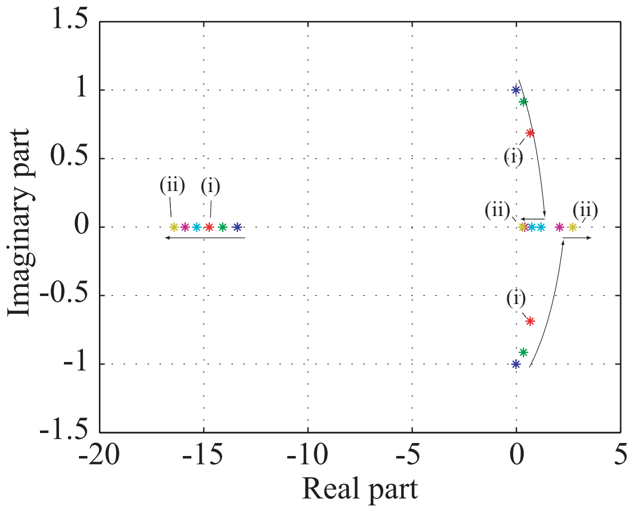


Figure 2. Root locus of A with γ=0.00272, R=13.41, and P=0.3506 under changing K_l from 0 to 50 ((i): K_l=20, (ii): K_l=50).

to keep the resonance amplitude constant. We design the appropriate nonlinear feedback gain on the center manifold. The design is much easier than that for the original system because the system order is reduced to second order.

3.1 Feedback control to make center subspace

We seek the linear feedback control so that the eigensubspace of the dynamics of the 3rd-order system consists of the center and stable subspace. Then, the linear operator A has a pair of pure imaginary eigenvalues and a negative real eigenvalue, and by $\boldsymbol{x} = Q\boldsymbol{y}$ the linear operator A can be transformed into

$$Q^{-1}AQ = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & -\eta \end{bmatrix}. \tag{9}$$

where ω and η are positive. The characteristic equation of A is $\Phi_A(\lambda) = \det(\lambda I - A)$ is expressed as follows:

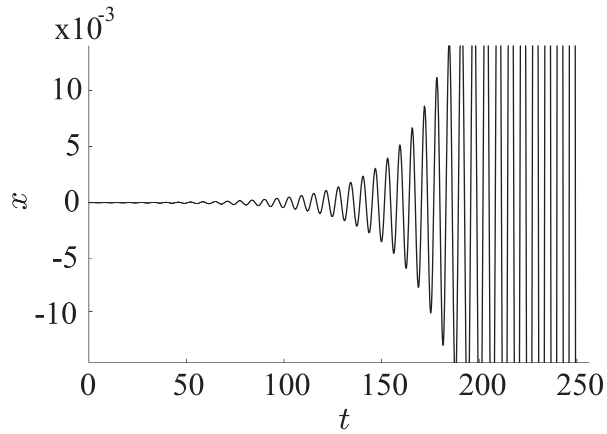
$$\Phi_A(\lambda) = \lambda^3 + (\gamma + R)\lambda^2 + \{\gamma R - K_l + P + 1\}\lambda + R, \tag{10}$$

which is equal that of Eq. (9):

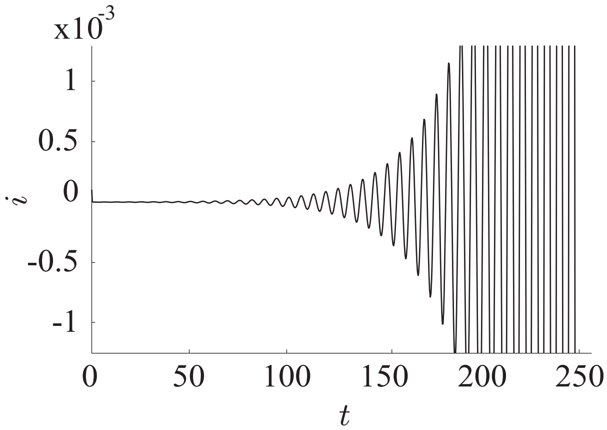
$$\Phi_A(\lambda) = \lambda^3 + \eta\lambda^2 + \omega^2\lambda + \eta\omega^2. \tag{11}$$

From Eqs. (10) and (11) the following equations for the above request for the eigenvalues:

$$\eta = \gamma + R > 0, \tag{12}$$



(a) Dimensionless displacement



(b) Dimensionless current

Figure 3. Time histories of dimensionless displacement and current with γ=0.00272, R=13.41, and P=0.3506, under K_l=1.5.

$$\omega^2 = \gamma(R - K_l) + P + 1 > 0, \qquad (13)$$

$$K_l = \gamma R + P + 1 - \frac{R}{R+\gamma}, \qquad (14)$$

Equation (12) can always be satisfied. We need to choose the feedback gain according to Eq. (14) under the condition Eq. (13) and the feedback gain is expressed K_{cr}.

3.2 Design of nonlinear feedback gain on center manifold

To perform the amplitude control, we apply nonlinear feedback control and design the nonlinear feedback gain on the center manifold. We set the dimensionless input voltage as follows:

$$V = K_{cr}(1+\epsilon)\frac{dx}{dt} + \mu\left(\frac{dx}{dt}\right)^3, \qquad (15)$$

where $\epsilon << 1$

Then, the state equation is expressed as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -\gamma & 1 \\ 0 & K_{cr} - P & -R \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + (K_{cr}\epsilon x_2 + \mu x_2^3)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad (16)$$

The transformation by Q yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & -\eta \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \{K_{cr}(q_{21}y_1 + q_{22}y_2 + q_{23}y_3) + \mu(q_{21}y_1 + q_{22}y_2 + q_{23}y_3)^3\}\begin{bmatrix} \tilde{q_{13}} \\ \tilde{q_{23}} \\ \tilde{q_{33}} \end{bmatrix} \qquad (17)$$

where q_{ij} and $\tilde{q_{ij}}$ are ij components for Q and Q^{-1}, respectively. By suspension-trick we obtain the center manifold as follows: [6]

$$y_3 = \chi_{300}y_1^3 + \chi_{210}y_1^2y_2 + \chi_{120}y_1y_2^2 + \chi_{030}y_2^3 + \chi_{101}\epsilon y_1 + \chi_{011}\epsilon y_2. \qquad (18)$$

The dynamics reduced on the center manifold is governed with

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} K_{cr}\tilde{q_{13}}q_{31}\epsilon & \omega + K_{cr}\tilde{q_{13}}q_{32}\epsilon \\ -\omega + K_{cr}\tilde{q_{23}}q_{31}\epsilon & K_{cr}\tilde{q_{23}}q_{32}\epsilon \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \mu\begin{bmatrix} \alpha_1y_1^3 + \alpha_2y_1^2y_2 + \alpha_3y_1y_2^2 + \alpha_4y_2^3 \\ \alpha_5y_1^3 + \alpha_6y_1^2y_2 + \alpha_7y_1y_2^2 + \alpha_8y_2^3 \end{bmatrix}, \qquad (19)$$

where

$$\alpha_1 = \tilde{q_{13}}q_{21}^3, \alpha_2 = 3\tilde{q_{13}}q_{21}^2q_{22}, \alpha_3 = 3\tilde{q_{13}}q_{21}q_{22}^2, \alpha_4 = \tilde{q_{23}}q_{21}^3, \alpha_5 = \tilde{q_{23}}q_{21}^3, \alpha_6 = 3\tilde{q_{23}}q_{21}^2q_{22}, \alpha_7 = 3\tilde{q_{23}}q_{21}q_{22}^2, \alpha_8 = \tilde{q_{23}}q_{22}^3 \qquad (20)$$

Furthermore, the normal form [7] of (19) under the nonlinear coordinate transformation:

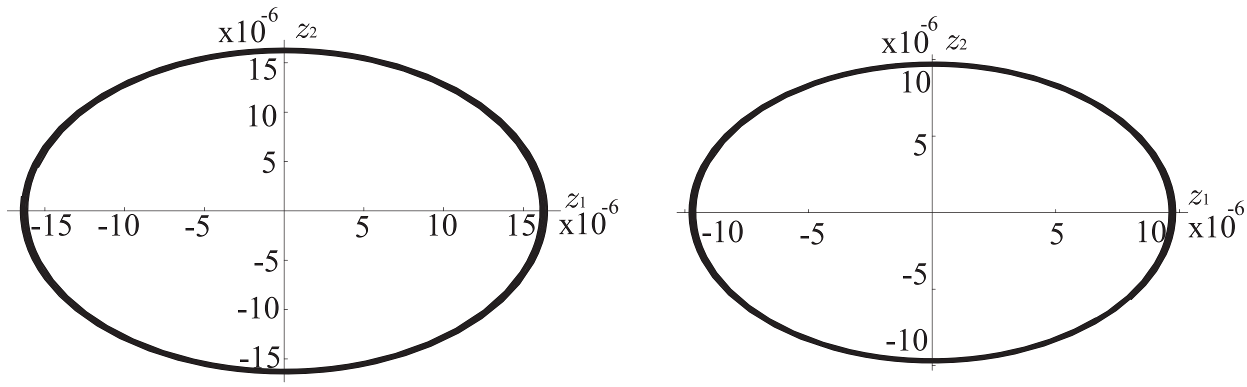
$$\boldsymbol{y} = \boldsymbol{z} + \boldsymbol{h}(\boldsymbol{z}) \qquad (21)$$

is

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} K_{cr}\tilde{q_{13}}q_{31}\epsilon & \omega + K_{cr}\tilde{q_{13}}q_{32}\epsilon \\ -\omega + K_{cr}\tilde{q_{23}}q_{31}\epsilon & K_{cr}\tilde{q_{23}}q_{32}\epsilon \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \mu\begin{bmatrix} (az_1 + bz_2)(z_1^2 + z_2^2) \\ (az_2 - bz_1)(z_1^2 + z_2^2) \end{bmatrix}, \qquad (22)$$

where

$$a = \frac{1}{8}(3\alpha_1 + \alpha_3 + \alpha_6 + 3\alpha_8), \quad b = \frac{1}{8}(\alpha_2 + 3\alpha_4 - 3\alpha_5 - \alpha_7) \qquad (23)$$



(i) $\mu = 10$ (ii) $\mu = 1000$

Figure 4. Phase space with γ=0.00272, R=13.41 P=0.3506, K_l=0.387 and ϵ=0.05. (i): μ=10, (ii): μ=1000.

Figure 4 shows the phase of two kinds of nonlinear feedback gain. From comparison between them, by changing the nonlinear feedback gain, we carry out the amplitude control. Figure 5 are corresponding time histories. The amplitudes of the displacement of the mass and the current in the linear motor are constant in the steady state. Therefore, it can be seen that the setting of the nonlinear feedback gain realizes desired response amplitude of the self-vibration machine.

4 Experimental apparatus and procedure

Figure6 shows the experimental apparatus. The primary coil of the linear motor is fixed to the base. The secondary permanent magnet of the linear motor (Syowa-Densen-Denran Corporation; VCM26-02R; maximum thrust force of 8 N and thrust constant of 2 N/A) is supported by a slide bearing (IKO Corporation, BSU66-100A) and its relative position can be moved with respect to the base. The mass of the secondary permanent magnet corresponds to the mass of the system in Fig.1. To provide the linear restoring force, we attach a spring (Samini Corporation) to linear-motor.

The displacement is measured by using a laser sensor (KEYENCE Corporation, LB-01/LB-60) and the displacement signal is input to a PC through an AD-board (Interface Corporation, PCI-3523A), in which the control voltage V is calculated in real time. The signal corresponding to V of Eq.15 is fed from the PC to the power amplifier (KIKUSUI Corporation, PBX40-10) through a DA-board (Interface Corporation, PCI-3523A). The amplified signal is input to the linear motor.

5 Conclusion

A method is presented to make the mass-spring-damper system a self-vibration system. The system is regarded as the 3rd-order system. By eigenvalue analysis, it is shown that the mass-spring-damper system is self-excited and a self-vibration system is established. For the amplitude control, the application of cubic velocity feedback is proposed and the feedback gain is design on the center manifold.

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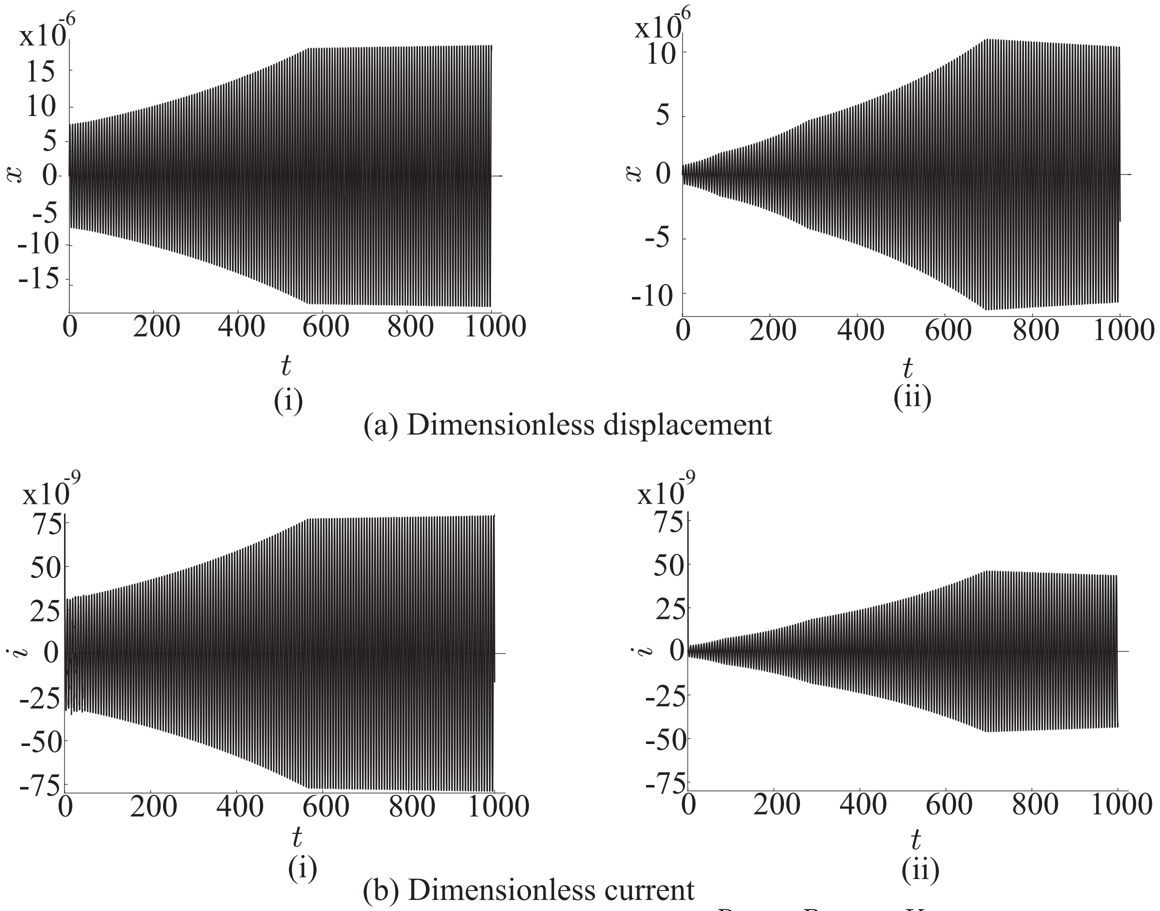

Figure 5. Time histories of dimensionless displacement and current with γ=0.00272, R=13.41 P=0.3506, K_l=0.387 and ϵ=0.05. (i): μ=10, (ii): μ=1000.

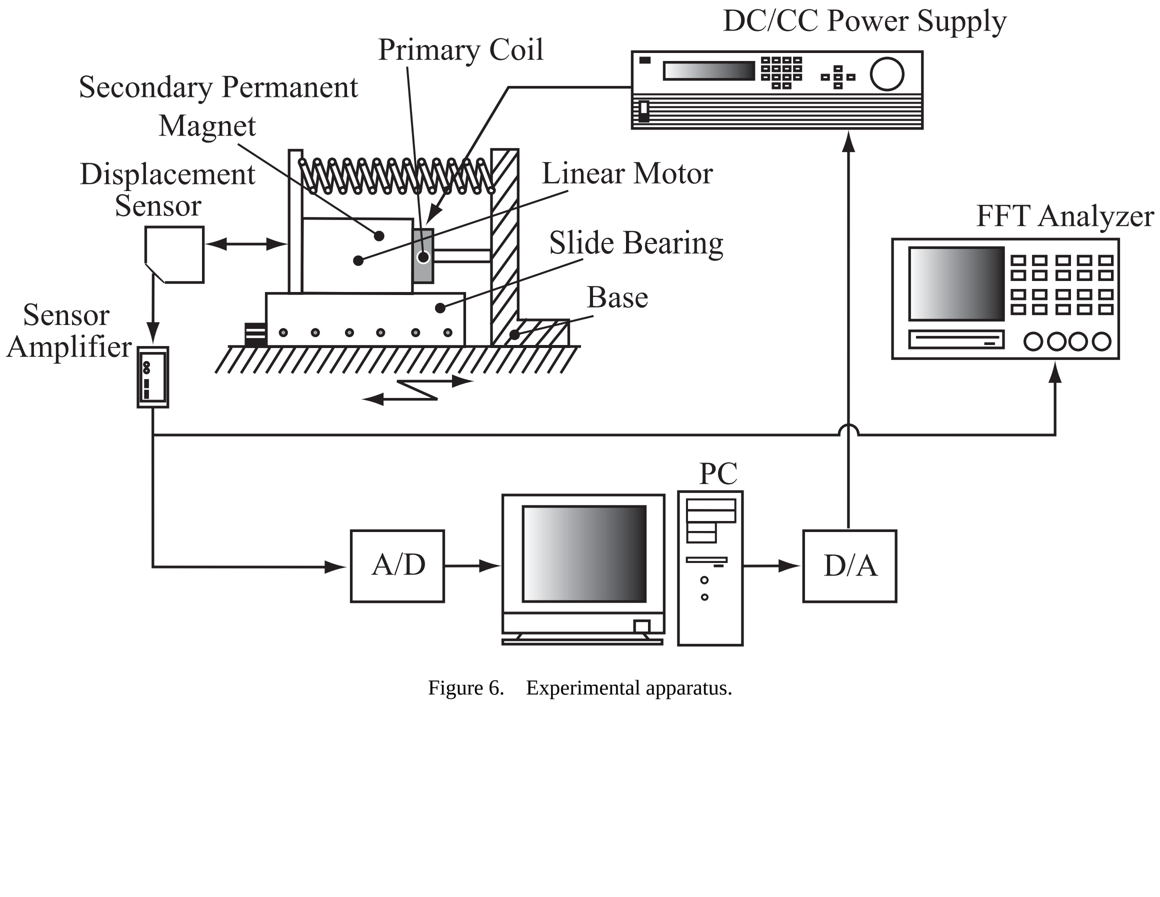

Figure 6. Experimental apparatus.