ENOC-2008, Saint Petersburg, Russia, June, 30–July, 4 2008

SLIDING MODE CONTROL OF A VEHICLE WITH NON-LINEARITIES

Nurkan Yagiz*
Dept. of Mechanical Eng.
Istanbul University
Turkey
nurkany@istanbul.edu.tr

Yuksel Hacioglu
Dept. of Mechanical Eng.
Istanbul University
Turkey
yukselh@istanbul.edu.tr

Yener Taskin
Dept. of Mechanical Eng.
Istanbul University
Turkey
ytaskin@istanbul.edu.tr

Abstract
A robust sliding mode controller for active suspensions of a non-linear half vehicle model is introduced in this study. After designing a non-chattering sliding mode controller, a four degrees of freedom non-linear half car model, which allows wheel hop and includes suspension system with non-linear spring, piecewise linear damper with dry friction, is presented. Then, designed controller is applied to this model in order to evaluate its performance. The results have shown that, the designed controller does not cause any problem in suspension working limits, while increasing the ride comfort of passengers.

Key words
Active suspensions, non-linear half car model, sliding mode.

1 Introduction

The main functions of the vehicle suspension systems are to provide road holding and to suppress the vehicle body vibrations coming from the road surface unevenness. In fact, these design criteria conflict each other, that road holding needs hard suspension springs whereas ride comfort needs soft suspension springs. Since appropriate improvements of semi-active and active suspension systems have a potential to improve the ride comfort and vehicle maneuverability, this research area has remained attractive for many years. Hrovat presented a survey paper which deals with optimal control of quarter-car, half-car and full-car models [Hrovat, 1997]. Also, the inherent trade-off in active suspension design is discussed and some semi-active and non-linear control design techniques are addressed. Williams investigated the damping ratio for passive suspensions and then sky-hook and LQR based active suspensions are designed and applied to a quarter-car model [Williams, 1997]. Ahmadian and Pare studied the performance of the three semi-active control policies, including the well known skyhook control and the two others referred to as groundhook and hybrid control on a quarter car test rig [Ahmadian and Pare, 2000]. They used a magnetorheological damper in their study. Verros *et al.* examined quarter-car models involving suspensions with bilinear damping and trilinear stiffness properties [Verros, Natsiavas and Papadimitriou, 2005]. Semi-active suspensions have a limited performance on improving ride comfort if they are compared with their active counterparts. Huisman *et al.* presented a continuous time control strategy for an active quarter car suspension system, based on optimal control theory [Huisman, Veldpaus, Voets and Kok, 1993]. Teja and Srinivasa proposed a stochastically optimized PID controller for a linear quarter car model [Teja and Srinivasa, 1996].

As a variable structure controller, sliding mode control has become widespread after a paper by Utkin [Utkin, 1977]. This method is insensitive to parameter variations and external disturbances during the sliding phase. Sam *et al.* presented a new robust strategy utilizing the proportional-integral sliding mode control scheme in controlling the active suspension system [Sam, Osman and Ghani, M. R. A. 2004]. Yagiz *et al.* applied the non-chattering sliding mode control to automotive suspension system [Yagiz, Yuksek and Sivrioglu, 2000]. The superiorities of this method are its simplicity, high performance and robust behaviour. It is shown that the ride comfort is improved. Yagiz examined the dynamic behaviour of a non-linear full vehicle model having active suspensions and a controlled passenger seat [Yagiz, 2004]. The dry friction on dampers causes the non-linearity. Using a controller under the passenger seat, the ride comfort is improved greatly.

The aim of this study is to improve the ride comfort of passengers using active suspensions. Therefore a four degrees of freedom half car model with nonlinear suspension spring and piecewise linear damper with dry friction is used in numerical analysis. The performance improvement of the

* Author for correspondence (e-mail: nurkany@istanbul.edu.tr, tel: +90 212 4737048, fax: +90 212 4737180)

suspension system is disclosed by comparing the non-chattering sliding mode controller with the uncontrolled case. The new contributions of the present study come from its high performance in control of vehicle body displacements and accelerations while preserving the suspension working space.

2 Controller Design

In sliding mode controlled systems, n[th] order tracking problem is transformed into first order stability problem, which makes the problem easy to cope with. The control action has the state errors progress on stable and unstable trajectories and reach the sliding surfaces. Then, state errors quickly reach the zero value.

The state space form of a non-linear dynamic system can be written as,

$$\dot{\mathbf{x}} = \mathbf{f}\left(x,t\right) + \mathbf{B}\,\mathbf{u} \tag{1}$$

where $\mathbf{x} = \left[x_1, \ldots, x_n, x_{n+1}, \ldots, x_{2n}\right]^T$. The second half of the states are the time derivatives of the first half for mechanical systems, respectively. $2n$ is the number of the states. For a control system, the sliding surface can be selected as,

$$\boldsymbol{\sigma} = \mathbf{G}\,\Delta\mathbf{x} \tag{2}$$

Here $\Delta\mathbf{x}$ is the difference between the reference value and system response. $\mathbf{G}$ includes the sliding surface slopes and has positive elements. For stability, the following Lyapunov function candidate, which is proposed for a non-chattering action, has to be positive definite and its derivative has to be negative semi-definite.

$$\mathbf{v}\left(\boldsymbol{\sigma}\right) = \frac{\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\sigma}}{2} > 0 \tag{3}$$

$$\frac{d\,\mathbf{v}\left(\boldsymbol{\sigma}\right)}{dt} = \frac{\dot{\boldsymbol{\sigma}}^{\mathrm{T}}\boldsymbol{\sigma}}{2} + \frac{\boldsymbol{\sigma}^{\mathrm{T}}\dot{\boldsymbol{\sigma}}}{2} \leq 0 \tag{4}$$

The equation (2) is separated as follows:

$$\boldsymbol{\sigma} = \boldsymbol{\Phi}\left(t\right) - \boldsymbol{\sigma}_a(x) \tag{5}$$

where

$$\boldsymbol{\Phi}\left(t\right) = \mathbf{G}\boldsymbol{x}_r \tag{6}$$

$$\boldsymbol{\sigma}_a(x) = \mathbf{G}\boldsymbol{x} \tag{7}$$

If the limit condition is applied to equation (4), and from equation (1) and equation (2) the controller force for the limit case is obtained:

$$\mathbf{u}_{eq} = (\mathbf{GB})^{-1}\left(\frac{d\boldsymbol{\Phi}\left(t\right)}{dt} - \mathbf{G}\,\mathbf{f}\left(x,t\right)\right) \tag{8}$$

Equivalent control is valid only on the sliding surface. So an additional term should be defined to pull the system to the surface. For this purpose derivative of the Lyapunov function can be selected as follows.

$$\dot{\mathbf{v}} = -\boldsymbol{\sigma}^{\mathrm{T}}\,\boldsymbol{\Gamma}\,\boldsymbol{\sigma} \leq 0 \tag{9}$$

Equating (9) to (4) and by carrying out necessary calculations, total control input is found as,

$$\mathbf{u} = \mathbf{u}_{eq} + (\mathrm{GB})^{-1}\,\boldsymbol{\Gamma}\,\boldsymbol{\sigma} \tag{10}$$

$(\mathbf{GB})^{-1}$ is always invertible and equal to mass matrix for mechanical systems. $\boldsymbol{\Gamma}$ is a positive definite matrix, and value of terms are decided by trial at the design stage. However, if the knowledge of $\mathbf{f}\left(x\right)$ and $\mathbf{B}$ are not well known, the calculated equivalent control inputs will be completely different from the needed equivalent control inputs. Thus, in this study, it is assumed that the equivalent control is the average of the total control. For estimation of the equivalent control, an averaging filter, here a low pass filter, can be designed as follows.

$$\hat{\mathbf{u}}_{eq} = \frac{1}{\tau\,s + 1}\mathbf{u}(t - \delta\,t) \tag{11}$$

The main idea, which is used in the design of the low pass filter, is based on that low frequencies determine the characteristics of the signal and high frequencies come from unmodeled dynamics. Finally the non-chattering control input is defined as

$$\mathbf{u}(t) = \hat{\mathbf{u}}_{eq} + (\mathrm{GB})^{-1}\,\boldsymbol{\Gamma}\,\boldsymbol{\sigma} \tag{12}$$

3 Vehicle Model

Figure 1 presents the non-linear vehicle model used in this study. This model has four degrees of freedom which are the body bounce y_2, pitch motion of the vehicle body θ and displacements of the front and rear wheels y_{1f} and y_{1r}. In this model M, I, m_f and m_r represent the sprung mass, mass moment inertia of the vehicle body for pitch motion, front unsprung mass and rear unsprung mass, respectively. Road inputs to the front and rear wheels are y_{0f} and y_{0r}. Also u_f and u_r represents the control forces applied to the front and rear suspensions. In Figure 1, a and b are the distances of the front and rear suspensions to the centre of gravity of the vehicle body and V is the velocity of the vehicle.

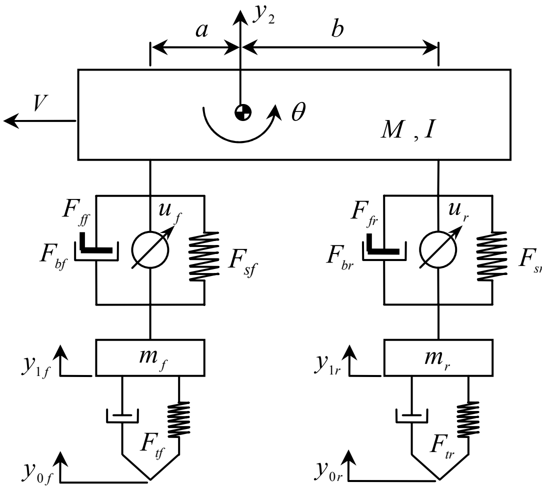

Figure 1 Half car model.

The suspension springs are non-linear stiffening ones as seen in Figure 2.a and the force produced between ends obeys the relation below,

$$F_{sj}(\Delta y_j) = k_{s1j}\Delta y_j + k_{s2j}\left(\Delta y_j\right)^3 \qquad (13)$$

Here j=f stands for the front suspension and j=r stands for the rear suspension. k_{s1j} and k_{s2j} denotes the spring coefficients for the linear and non-linear part of the spring force expression. Front and rear suspension deflections are defined as below, respectively:

$$\Delta y_f = y_2 - a\sin\theta - y_{1f} \qquad (14)$$

$$\Delta y_r = y_2 + b\sin\theta - y_{1r} \qquad (15)$$

The suspension damper is piecewise linear and has different coefficients for compression and extension. Thus, different forces are obtained for compression and extension as seen in Figure 2.b. The force produced through the damper obeys the relation below:

$$F_{bj}(\Delta\dot{y}_j) = \begin{cases} b_{ej}\,\Delta\dot{y}_j & ; \quad \Delta\dot{y}_j > 0 \\ b_{cj}\,\Delta\dot{y}_j & ; \quad \Delta\dot{y}_j \le 0 \end{cases} \qquad (16)$$

Here $\Delta\dot{y}_j$ denotes the derivative of the related suspension deflection. b_{ej} and b_{cj} are the damping coefficients for extension and compression cases, respectively. Also it is proposed that there exist dry friction on the front and rear dampers. The dry friction model has a viscous band [Yagiz, 2004] as seen Figure 2.c. The dry friction force depends on the relative speed between related damper ends and obeys the following relation:

$$F_{fj}(\Delta\dot{y}_j) = \begin{cases} R & ; \quad \Delta\dot{y}_j > \varepsilon \\ \dfrac{R}{\varepsilon}\Delta\dot{y}_j & ; \quad -\varepsilon \le \Delta\dot{y}_j \le \varepsilon \\ -R & ; \quad \Delta\dot{y}_j > -\varepsilon \end{cases} \qquad (17)$$

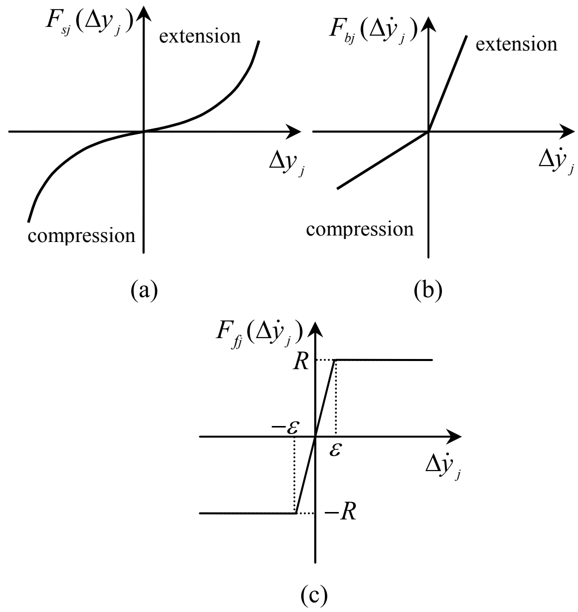

Figure 2 a) Nonlinear suspension spring model b) Piecewise linear damper model
c) Dry friction model

The tyre forces are defined below which includes wheel hop phenomena.

$$F_{tj} = \left\{k_{tj}\left(y_{1j} - y_{0j}\right) + b_{tj}\left(\dot{y}_{1j} - \dot{y}_{0j}\right)\right\}\delta(y_{1j}, y_{0j}) \qquad (18)$$

$$\delta(y_{1j}, y_{0j}) = \begin{cases} 1 & ; \quad y_{1j} - y_{0j} \le 0 \\ 0 & ; \quad y_{1j} - y_{0j} > 0 \quad (\textit{wheel hop}) \end{cases} \qquad (19)$$

Here, again, j=f stand for the front suspension and j=r stand for the rear suspension. k_{tj} is the tyre stiffness and $y_{1j} - y_{0j}$ is the tyre deflection and wheel hop takes place when tyre deflection is greater than zero. Thus in this case the tyre force terms in the equations of the motion vanish.

Equations of motion for the half car model are given below. They include non-linear suspension springs, piecewise linear dampers with dry friction and wheel hops.

$$M\ddot{y}_2 + F_{bf} + F_{br} + F_{sf} + F_{sr} + F_{ff} + F_{fr} = u_f + u_r - Mg \quad (20)$$

$$I\ddot{\theta} - a\cos\theta\left(F_{bf} + F_{sf} + F_{ff}\right) + b\cos\theta\left(F_{br} + F_{sr} + F_{fr}\right) = bu_r - au_f \quad (21)$$

$$m_f\ddot{y}_{1f} - F_{bf} - F_{sf} + F_{tf} - F_{ff} = -u_f - m_f g \quad (22)$$

$$m_r\ddot{y}_{1r} - F_{br} - F_{sr} + F_{tr} - F_{fr} = -u_r - m_r g \quad (23)$$

Also, in order to prevent the suspension gap loss during control action, which causes the controllers not to function properly, it is proposed that the reference value for the vehicle body displacement y_{2ref} is the interpolation of the displacements of the front and rear unsprung masses y_{1f} and y_{1r} under the body mass centre. Then,

$$y_{2ref} = \frac{1}{a+b}\left(y_{1f}b + y_{1r}a\right) \quad (24)$$

4 Numerical Results

In this section the performance of the half car model with the sliding mode controller is evaluated. Numerical parameters of the vehicle model and the parameters for the controller designed are given in Appendix. The road input to the vehicle model is given in Figure 3. The road input to the rear wheel is the same but delayed by $\Delta t = (a+b)/V$.

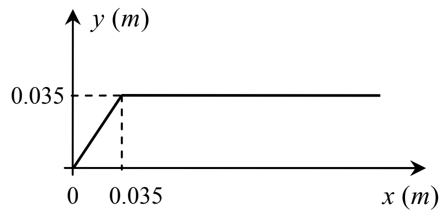

Figure 3 Road disturbance

For the road input given, comparison of the time responses of the vehicle body bounce and pitch motions and related accelerations are presented in Figure 4. It is seen that by using the sliding mode controller, magnitudes of the body bounce and pitch motion are greatly reduced with respect to the uncontrolled ones. Also it should be noted that there is an improvement in the vertical and angular accelerations of the vehicle body, which indicate that the ride comfort of passengers is increased.

Suspension deflections for the uncontrolled and sliding mode controlled cases are given in Figure 5. It is clear from this figure that the maximum suspension deflections for the sliding mode controlled case do not exceed the uncontrolled ones and there is no permanent deflection, which indicates that the ride comfort is improved without degenerating the suspension working limits.

Control forces acting on the front and rear suspensions are given in Figure 6.

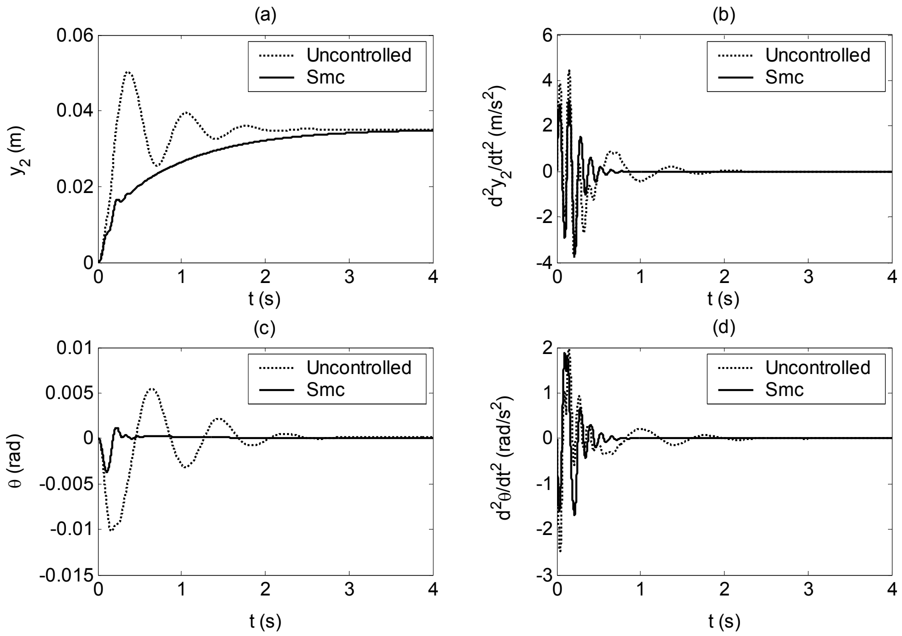

Figure 4 The uncontrolled and sliding mode controlled vehicle responses
a) Body bounce b) Vertical acceleration of the vehicle body c) Pitch motion
d) Angular acceleration of the vehicle body

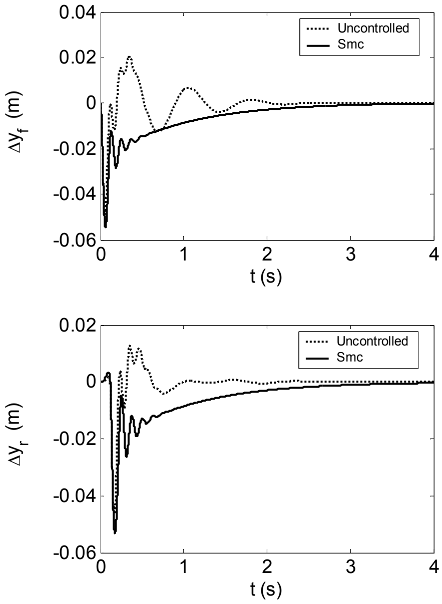

Figure 5 Suspension deflections for the uncontrolled and sliding mode controlled cases

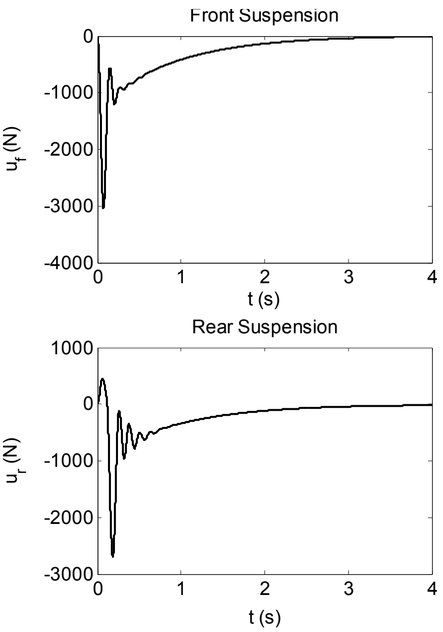

Figure 6 Control forces

5 Conclusion

Improving the ride comfort in vehicles is an active research area due to developing technology. Thus, in order to improve the ride comfort in vehicles, a robust sliding mode controller was preferred in this paper. The used model, which is four degrees of freedom half car, has strongly non-linear characteristics as a result of its non-linear suspension components and wheel hop besides the geometrical non-linearity of motion equations. In order to validate its performance, time domain analysis of the half car model was carried out. The results, have demonstrated that the magnitudes of the body displacement, pitch motion and related accelerations were decreased which result in increased ride comfort. Also, it has been verified that the designed controller has not caused any problem in suspension working limits.

References

Ahmadian, M. and Pare, C.A. (2000) A quarter-car experimental analysis of alternative semiactive control methods. *Journal of Intelligent Material Systems and Structures*, 11, 604-612.

Hrovat, D. (1997) Survey of advanced suspension developments and related optimal control applications. *Automatica*, 33(10), 1781-1817.

Huisman, R.G.M., Veldpaus, F.E., Voets, H.J.M. and Kok, J.J. (1993) An optimal continuous time control strategy for active suspensions with preview. *Vehicle System Dynamics*, 22, 43-55.

Sam, Y. M., Osman, J. H. S. and Ghani, M. R. A. (2004) A class of proportional-integral sliding mode control with application to active suspension system. *Systems & Control Letters,* 51, 217-223.

Teja, S.R. and Srinivasa, Y.G. (1996) Investigations on the stochastically optimized PID controller for a linear quarter-car road vehicle. *Vehicle System Dynamics*, 26, 103-116.

Utkin, V.I. (1977) Variable structure systems with sliding modes. *IEEE Transactions on Automatic Control*, 22(2), 212-222.

Verros, G., Natsiavas, S. and Papadimitriou, C. (2005) Design optimization of quarter-car models with passive and semi-active suspensions under random road excitation. *Journal of Vibration and Control*, 11, 581–606.

Williams, R.A. (1997) Automotive active suspensions Part 1: basic principles. *Proceedings of the Institution of Mechanical Engineers Part D*, 211(6), 415-426.

Yagiz, N. (2004) Comparison and Evaluation of different control strategies on a full vehicle model with passenger seat using sliding modes. *Int. Journal of Vehicle Design*, 34(2), 178-192.

Yagiz, N., Yuksek, I. and Sivrioglu, S. (2000) Robust control of active suspensions for a full vehicle model using sliding mode control. *JSME International Journal, Series C*, 43(2), 253-258.

Appendix

Numerical parameters of the half car model

M = 1000 kg	k_{tf} = k_{tr} = 400000 N/m
I = 1600 kg.m^2	$b_{tf} = b_{tr}$ = 30 N/m/s
m_f = m_r = 150 kg	$b_{ef} = b_{er}$ =2500 N/m/s (ext.)
k_{s1f} = 20000 N/m	$b_{cf} = b_{cr}$ =1500 N/m/s (compr.)
k_{s2f} = 320000 N/m	V = 72 km/h
k_{s1r} = 18000 N/m	a = 1 m , b = 1.2 m
k_{s2r} = 280000 N/m	R = 22 N , ε = 0.012 m/s

Controller Parameters*

τ = 0.1	Γ = 25	α = 1.5

* Controller parameters are the same for the front and rear suspensions.