COHERENT RESONANCE IN NEURON ENSEMBLE WITH ELECTRICAL COUPLINGS

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Abstract
We find that the regularity in the spiking behaviour of a neuronal network maximizes at a certain level of environment noise. This effect referred to as coherence resonance is demonstrated in a random complex network of Rulkov neurons. An external stimulus added to some of neurons excites them, and then activates other neurons in the network. The network coherence is also maximized at the certain stimulus amplitude, coupling strength, and the number of stimulated neurons. The coherence enhancement is characterized by the normalized standard deviation from the average inter-spike interval and by the signal-to-noise ratio calculated from power spectra of the excited neurons.

Key words
Neural network, electrical coupling, coherent resonance, Rulkov map.

1 Introduction

Noise can lead to more order in the dynamics. To be mentioned here are the effects of noiseinduced order in chaotic dynamics [Matsumoto and Tsuda, 1983], synchronization by external noise, and stochastic resonance [Benzi, Sutera, and Vulpiani, 1981; Jung, 1993; Moss, Pierson and O'Gorman, 1994; Andreev, Makarov, Runnova, Pisarchik and Hramov, 2013]. Also, noise has been shown to play a stabilizing role in ensembles of coupled oscillators and maps [Hakim and Rappel, 1994]. Especially interesting is the phenomenon of stochastic resonance, which appears when a nonlinear system is simultaneously driven by noise and a periodic signal. At a certain noise amplitude the periodic response is maximal.

The interest in mathematical modeling of neuronal synchronization has significantly increased after neurobiological experiments with two electrically coupled neurons [Elson, Selverston, Huerta, Rulkov, Rabinovich, and Abarbanel, 1998], where various synchronous states have been identified. In order to simulate cooperative neuron dynamics, numerous models based on either iterative maps of differential equations in various coupling configurations have been developed [Elson, Selverston, Huerta, Rulkov, Rabinovich, and Abarbanel, 1998]. Depending on the coupling strength and synaptic delay time, coupled neurons generate spike sequences that are matching in their timings, or bursts either with lag or anticipation [Lang, Lu, and Kurths, 2010]. When three or more oscillators are accounted for, a large number of coupling configurations can be realized. In the theory of graphs or complex networks, these basic configurations are called network motifs.

We explore a simple neural model, the Rulkov map [Rulkov, 2002; Rulkov, Timofeev and Bazhenov, 2004]. Although this model is not explicitly inspired by physiological processes in the membrane, it is capable of generating extraordinary complexity and quite specific neural dynamics (silence, periodic spiking, and chaotic bursting), thus replicating to a great extent most of the experimentally observed regimes [Elson, Selverston, Huerta, Rulkov, Rabinovich, and Abarbanel, 1998], including spike adaptation, routes from silence to bursting mediated by subthreshold oscillations, emergent bursting, phase and antiphase synchronization with chaos regularization [Rulkov, 2002], and complete and burst synchronization.

2 The Investigation Model

Each neuron-like Rulkov element is described by the following system of equations with synaptic coupling [Rulkov, Timofeev and Bazhenov, 2004]:

$$\begin{aligned} x_{n+1} &= f(x_n, x_{n-1}, y_n + \beta_n), \\ y_{n+1} &= y_n - \mu(x_n + 1) + \mu\sigma + \mu\sigma_n + \mu A^{\xi}\xi_n, \end{aligned} \tag{1}$$

where x and y are fast and slow variables associated with membrane potential and gating variables, respectively, α, σ and $\mu \in (0, 1]$ are parameters which regulate the system dynamics, ξ is Gaussian noise with zero mean and unity standard deviation, A^{ξ} is the noise amplitude, and f is a piecewise function defined as

$$f(x_n, x_{n-1}, y_n) = \begin{cases} \alpha/(1- \\ -x_n) + y_n, & \text{if } x_n \leq 0, \\ \alpha + y_n, & \text{if } 0 < x_n < \alpha + \\ & y_n \text{ and } x_{n-1} \leq 0, \\ -1, & \text{if } x_n \geq \alpha + y_n \\ & \text{or } x_{n-1} > 0, \end{cases} \tag{2}$$

It is constructed in a way to reproduce different regimes of neuron-like activity, such as spiking, bursting and silent regimes.

The parameters β_n and σ_n are defined as

$$\begin{aligned} \beta_n &= \beta^e I_n^{ext} + \beta^{syn} I_n^{syn}, \\ \sigma_n &= \sigma^e I_n^{ext} + \sigma^{syn} I_n^{syn}, \end{aligned} \tag{3}$$

where β^e and σ^e are coefficients used to balance the effect of external current I_n^{ext} defined as

$$I_n^{exp} = \begin{cases} 0, & n < t_s, \\ A, & n \geq t_s, \end{cases} \tag{4}$$

β^{syn} and σ^{syn} are coefficients of chemical synaptic coupling, and I_n^{syn} is a synaptic current given as

$$I_{n+1}^{syn} = \gamma I_n^{syn} - g_{syn} * \begin{cases} (x_n^{post} - x_{rp}), & \text{spike}^{\text{pre}}, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

where $g_{syn} \geq 0$ is the strength of synaptic coupling. Indexes pre and $post$ correspond to the presynaptic and postsynaptic variables, $\gamma \in [0, 1]$ is the synaptic relaxation time defining a portion of synaptic current preserved in the next iteration, and x_{rp} is a reversal potential determining the type of synapse, inhibitory or excitatory.

In our modeling we take values of the parameters $\alpha = 3.65$, $\sigma = 0.06$ and $\mu = 0.0005$ so that each neuron being autonomous demonstrates silent regime dynamics. Also we assume $\beta^e = 0.133$, $\sigma^e = 1.0$, $\beta^{syn} = 0.1$, $\sigma^{syn} = 0.5$ and $x_{rp} = 0$. Investigation system is a motif of N neurons coupled to each other with a random coupling strength g_{syn} and relaxation time. The values of them are randomly chosen from 0.0 to 0.1 and from 0.0 to 0.5 respectively. In the investigating system we apply an external stimulus to Na neurons. Stimulus is a current impulse of the following form: from the start it equals to 0, at the moment t_s when we apply it current starts equal to A. The values of variables are chosen so that without the external stimulus each neuron is in a silent regime but with starting the application of stimulus excited neurons start periodically generate spikes.

3 The Analysis

From the system we take signals as time series of fast variable x from all neurons. Additionally we calculate signal averaging over all neurons and analyse them. In figure 1 we can see these signals for systems of 100 neurons for different number of excited neurons. On them we can see phenomenon of grouping. It consists in periodically spiking unexcited neurons so that we can see areas of time on time series (d, e, f) where all unexcited neurons spike and areas where they all are silent and these areas periodically follows one by one. We can notice that for small and big values of Na we dont see grouping.

We analyse influence of amplitudes of external stimulus and internal noise. In figures 2 and 3 we can see dependencies of time series of x from these parameters. Increasing the stimulus amplitude leads to increasing frequency of grouping and grouping durations and decreasing time range between them. Also we can see decreasing oscillation amplitude of average signal. Increasing noise amplitude in its turn leads to decline of grouping effect, signal starts be more noise-like. Also we can see oscillations in time area where external amplitude $A = 0$ so noise starts excite neurons.

For analyse phenomena of periodical grouping we calculate dependencies of signal-to-noise ratio (SNR) from number of neurons in the system N, number of excited neurons Na, amplitude of external stimulus A and amplitude of internal noise A^{ξ}. SNR measured from power spectra of average signal in dB as an excess of main frequency amplitude over background noise [Campos-Meja, Pisarchik, and Arroyo-Almanza,

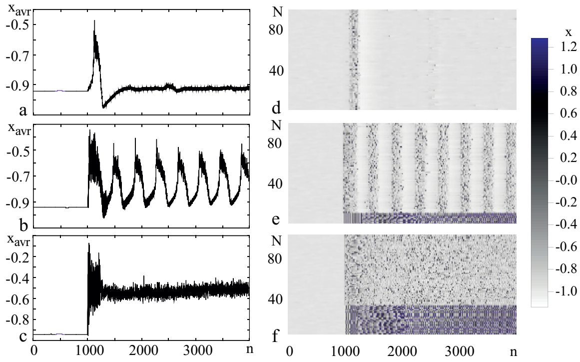

Figure 1. (Left panel) Time series of average membrane potential and (right panel) membrane potential of all neurons in the network of $N = 100$ neurons, when the stimulus with amplitude $A = 1$ is applied to (a) $Na = 1$ neuron, (b) $Na = 10$ neurons, and (c) $Na = 30$ neurons, under noise with amplitude $A^{\xi} = 0.1$. The periodic grouping in ISI is observed for $Na = 10$.

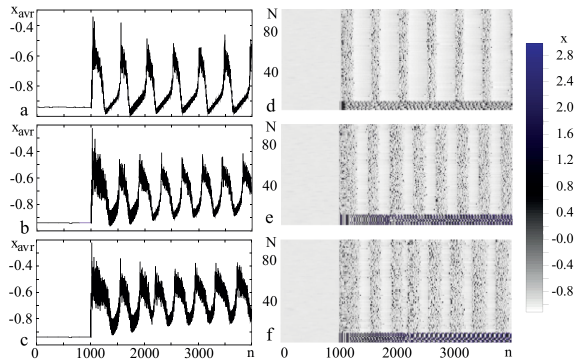

Figure 2. (Left panel) Time series of average membrane potential and (right panel) membrane potential of all neurons in the network of $N = 100$ neurons, when the stimulus with (a) $A = 0.5$, (b) $A = 1.5$, and (c) $A = 2.5$ is applied to $Na = 10$ neurons, under noise with $A^{\xi} = 0.1$.

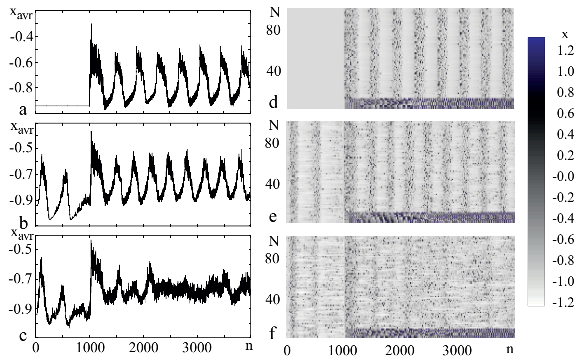

Figure 3. (Left panel) Average membrane potential and (right panel) membrane potential of all $N = 100$ neurons under the influence of noise with (a) $A^{\xi} = 0$, (b) $A^{\xi} = 1$, and (c) $A^{\xi} = 2$ and stimulus with $A = 1$ applied to $Na = 10$ neurons.

2013].

In figure 4, a we can see dependence of SNR from number of neurons in the system when we excite 10 of them. At small values of $N(< 38)$ signal-to-noise ratio is small too but for increasing N from 38 leads to rapid increasing SNR from 5 to 30 and then it stays near of this level until $N = 140$ when SNR starts slowly decrease. So for $Na = 10$ we have optimal values of $N = 38 - 140$ at which SNR takes the highest value. In figure 4, b we can see dependence of SNR from number of neurons being applying by external stimulus for system of 100 neurons. We can say that optimal values of Na are from 4 to 18. For this area of Na SNR takes the highest values. Moving away from it signal-to-noise ratio value decreases to 0.

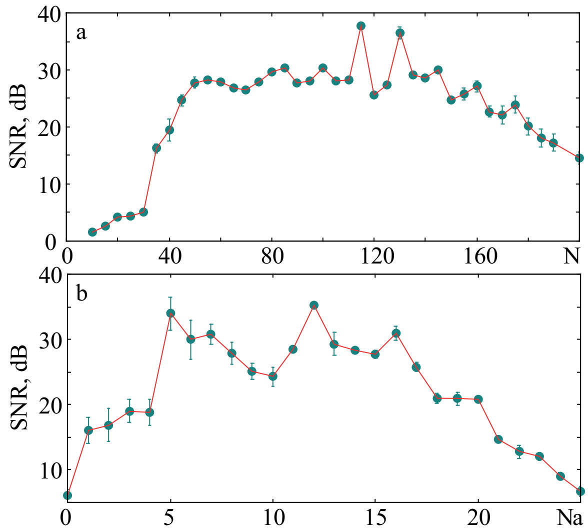

Figure 4. Signal-noise ratio (SNR) versus number of neurons in the system N (a) for $A^{\xi} = 0.1, A = 1.0, Na = 10$ and versus number of neurons being applying by external stimulus Na (b) for $A^{\xi} = 0.1, A = 1.0, N = 100$.

Figure 5, a shows signal-to-noise ratio dependence from external stimulus amplitude, on which we can see the phenomenon of coherent resonance when for a certain values of external stimulus amplitude ($A = 1.3 - 1.6$) SNR takes the maximum value. For $A > 1.6$ signal-to-noise ratio takes the same value. Decreasing external stimulus amplitude from 1.3 to 0 leads to decreasing SNR. In figure 5, b we can see influence of internal noise amplitude to signal-to-noise ratio. For $A^{\xi} = 0.3$ SNR takes the maximum value and decreases to 4 with decreasing A^{ξ}.

To investigate the coherent resonance phenomenon we plotted the 2-dimensional diagram of SNR from amplitudes of external stimulus A and noise A^{ξ} (fig. 6) on which we can see the areas of coherent resonance where SNR values are high. These areas of parameters are colored by black. We can see two white areas ($A < 0.2$, $A^{\xi} < 0.25$ and $0.5 < A^{\xi} < 1.0$) where signal-to-noise ratio is the lowest. There are 3 black areas for $0.8 < A < 1.7$ and $0.0 < A^{\xi} < 1.3$. Also

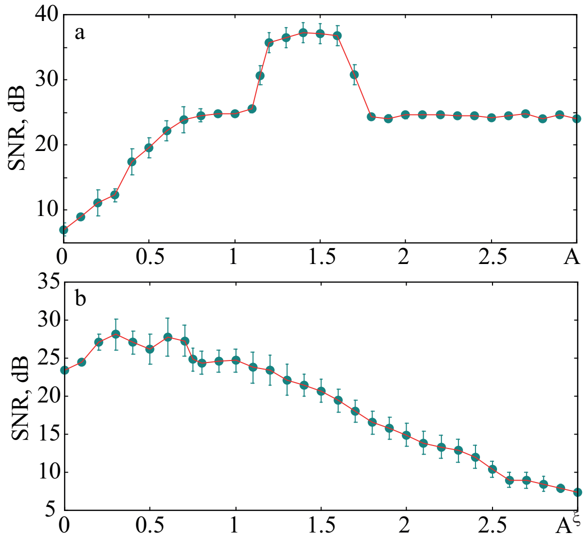

Figure 5. Signal-noise ratio (SNR) versus external stimulus amplitude A (a) for $A^{\xi} = 0.1, Na = 10, N = 100$ and versus noise amplitude A^{ξ} (b) for $A = 1.0, Na = 10, N = 100$.

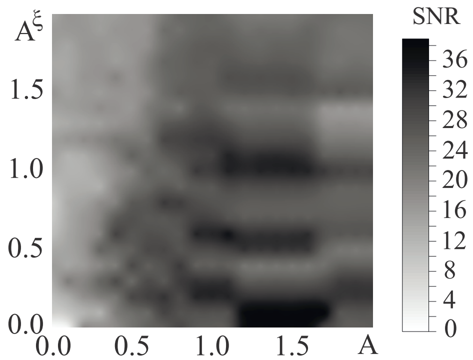

Figure 6. Two-parameter diagram of SNR from amplitudes of external stimulus A and noise A^{ξ}. SNR amplitude is defined by color.

we can see that main area of black color is located for $A > 0.5$ and $A^{\xi} < 2.4$. And for $A > 1.7$ SNR value does not change for the constant noise amplitude.

4 Conclusion

The macroscopic signal from motif of Rulkov elements with random coupling between them and internal noise presence under external stimulus demonstrates phenomenon of grouping when all unexcited neurons start spiking periodically during the time interval. And at the averaging signal from all neurons we see periodically grouping. Changing such parameters as number of neurons in the system, number of excited neurons, amplitudes of external stimulus and internal noise we can see phenomenon of coherent resonance when at the certain values of these parameters signal-to-noise ratio takes the maximal values.

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