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FORMATION MANEUVERS VIA ADAPTIVE SUPER TWISTING APPROACH

Rubén Hernández, Oscar Salas, Jesús De León-Morales
Facultad de Ingeniería Mecánica y Eléctrica
Universidad Autónoma de Nuevo León
Cd. Universitaria, 66450 San Nicolás de los Garza N.L., México
sutereo@gmail.com, salvador.sp@gmail.com, drjleon@gmail.com

Abstract

In this paper, a synchronization and formation control for groups of mobile robots of unicycle structure under the behavior-based approach is presented. According to this approach, complex maneuvers are decomposed into a sequence of maneuvers between formation patterns. With the aim of driving the movements of the robots, a decentralized control strategy based on feedback linearization and Adaptive Super Twisting Algorithm (ASTA) are introduced. The proposed control scheme increases robustness against unknown dynamics without overestimating the gain. Simulation results illustrate the effectiveness of the proposed control.

Key words

Coordinated Control, Robust Adaptive Control, Formations, Mobile Robots.

1 INTRODUCTION

Applications of robotics in daily life are being increased enormously, for example service robots for cleaning [Yuan, 2011] or transporting [Tsay, 2003]. In case of cleaning robots, a large area involved can be cleaned faster by a set of robots instead of a single robot.

Formation and coordinated motion control have attracted considerable attention as there task that can be performed more efficiently by a group of robots, such as surveillance, exploration, search and rescue. These cooperative works involve robots moving with a definite formation to maximize detection capabilities and at the same time changing their formation in case of malfunction from an agent. Besides, the robots process the information captured by each one of the agents and can change their formation patterns. Moving large objects represents another example of coordinated tasks, where the robots move maintaining a rigid formation to displace the oversize object.

In the last years, several approaches have been proposed to solve the problem of formation control and coordinated movement control, such as the virtual structure method, which considers the whole system as a single rigid structure or entity and then the desired path is assigned to the structure while maintaining a rigid formation [Ren, 2004]. The leader-follower method consists in designing a robot as the leader responsible for guiding all the other robots involved in the formation, in such a way that they reach their desired positions and keep the composed formation while moving [Brandao, 2009].

The behavior-based methods, where desired behaviors are prescribed for each robot and the final action is derived from a weighting of the relative importance of each behavior [Lawton, 2003]. The basic concept of the potential field methods is to fill the workspace of the robots with an artificial potential field where the robot is attracted to the target position and then is repelled from obstacles [Zheng, 2011]. Sliding modes control approach is used in many applications; enabling high gain accuracy tracking and insensitivity to disturbances and plant parameter variations in nonlinear systems. Super-Twisting Algorithm is a control based on sliding modes technique, which is designed to converge in a finite-time and ensures robustness under uncertainties. However, STA controller needs to know the bounds of uncertainties and perturbations present on the system.

This paper concerns to the formation control of a group of mobile robots, through the coupled dynamics methodology based on a behavior strategy. To deal with the formation problem, a composed control approach based on feedback linearization and adaptive super twisting control algorithm is proposed. This scheme improves robustness as the bounds of uncertainties and perturbations are not necessary to be known.

This paper is organized as follows: Section 2 is devoted to a system description and the analysis of a mathematical model from a mobile robot. In section

3, an Adaptive Super-Twisting Control is derived with the aim of providing robustness under non-modeled dynamics and parametric uncertainties. In section 4, the formation control problem as motion between a sequence of formation patterns and the corresponding coupled dynamics control is addressed. Simulation results are given in section 5, with the aim of illustrating the feasibility and performance of the proposed scheme. Finally, conclusions of this work are drawn.

2 DYNAMICAL MODEL OF A DIFFERENTIAL DRIVE MOBILE ROBOT

Unicycle mobile robots (Fig. 1) have high mobility, combining a high traction by using pneumatic tires while having simple wheel configuration. This paper considers the model proposed by [Zhang,1998], assuming null the uncertainties vector and the moment of inertia combination of the rotor motor, gear and wheel. Besides, the center of mass is considered as located in the center of the line joining the wheels.

Then, the model of the i^{th}-robot of multi robot system is considered as follows

$$\begin{pmatrix} \dot{r}_{xi} \\ \dot{r}_{yi} \\ \dot{\theta}_i \\ \dot{v}_i \\ \dot{\omega}_i \end{pmatrix} = \begin{pmatrix} v_i cos(\theta_i) \\ v_i sin(\theta_i) \\ \omega_i \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_i} & 0 \\ 0 & \frac{1}{I_i} \end{pmatrix} \begin{pmatrix} F_i \\ \tau_i \end{pmatrix} \quad (1)$$

where $r_i = (r_{xi}, r_{yi})^T$ is the inertial position of the i-th robot, θ_i is the orientation, v_i is the linear speed, ω_i is the angular speed, F_i and τ_i are the force and torque applied at the center of the line connecting the wheels. m_i is the mass, and J_i is the moment of inertia. Now, the equations of motion (1) can be written as

$$X_i = f(X_i) + g_i u_i \quad (2)$$

where $X_i = (r_{xi}, r_{yi}, \theta_i, v_i, \omega_i)$, $u_i = (F_i, \tau_i)^T$, $f(\cdot)$ and $g(\cdot)$ can be deduce from (1).

In this work, the robot hand position formation control will be considered. Hand position can be defined as a point located a distance L_i along the line that is perpendicular to the wheel axis and intersects r_i. Thus, the hand position is given by

$$\Pi_i = r_i + L_i \begin{pmatrix} cos(\theta_i) \\ sin(\theta_i) \end{pmatrix} \quad (3)$$

Taking the derivative of (3) with respect to time, it follows that

$$\dot{\Pi}_i = \begin{pmatrix} cos(\theta_i) & -L_i sin(\theta_i) \\ sin(\theta_i) & L_i cos(\theta_i) \end{pmatrix} \begin{pmatrix} v_i \\ \omega_i \end{pmatrix}, \quad (4)$$

From the second derivative of (3) we have

$$\ddot{\Pi}_i = \begin{pmatrix} -u_i\omega_i sin(\theta_i) - L_i\omega_i^2 cos(\theta_i) \\ u_i\omega_i cos(\theta_i) - L_i\omega_i^2 sin(\theta_i) \end{pmatrix} + \begin{pmatrix} \frac{1}{m_i}cos(\theta_i) & -\frac{L_i}{J_i}sin(\theta_i) \\ \frac{1}{m_i}sin(\theta_i) & \frac{L_i}{J_i}cos(\theta_i) \end{pmatrix} \begin{pmatrix} F_i \\ \tau_i \end{pmatrix} \quad (5)$$

Taking into account

$$det \begin{pmatrix} \frac{1}{m_i}cos(\theta_i) & -\frac{L_i}{J_i}sin(\theta_i) \\ \frac{1}{m_i}sin(\theta_i) & \frac{L_i}{J_i}cos(\theta_i) \end{pmatrix} \neq 0$$

the system (2) has constant relative degree equals 2, and can be output feedback linearized about the hand position [Lawton, 2003]. With this aim, define the map $\varphi : \Re^5 \rightarrow \Re^5$ as

$$\zeta_i = \varphi(X_i) = \begin{pmatrix} r_{xi} + L_i cos(\theta_i) \\ r_{yi} + L_i sin(\theta_i) \\ v_i cos(\theta_i) - L_i\omega_i sin(\theta_i) \\ v_i sin(\theta_i) + L_i\omega_i cos(\theta_i) \\ \theta_i \end{pmatrix} \quad (6)$$

where its inverse is given by

$$X_i = \varphi^{-1}(\zeta_i) = \begin{pmatrix} \zeta_{1i} - L_i cos(\zeta_{5i}) \\ \zeta_{2i} - L_i sin(\zeta_{5i}) \\ \zeta_{5i} \\ \frac{1}{2}\zeta_{3i}cos(\zeta_{5i}) + \frac{1}{2}\zeta_{4i}sin(\zeta_{5i}) \\ -\frac{1}{2L_i}\zeta_{3i}sin(\zeta_{5i}) + \frac{1}{2L_i}\zeta_{4i}cos(\zeta_{5i}) \end{pmatrix} \quad (7)$$

Equations (2)-(3) can be written in the transformated coordinates as

$$\begin{aligned} \begin{pmatrix} \dot{\zeta}_{1i} \\ \dot{\zeta}_{2i} \end{pmatrix} &= \begin{pmatrix} \zeta_{3i} \\ \zeta_{4i} \end{pmatrix} \\ \begin{pmatrix} \dot{\zeta}_{3i} \\ \dot{\zeta}_{4i} \end{pmatrix} &= \begin{pmatrix} -u_i\omega_i sin(\theta_i) - L_i\omega_i^2 cos(\theta_i) \\ u_i\omega_i cos(\theta_i) - L_i\omega_i^2 sin(\theta_i) \end{pmatrix} \\ &+ \begin{pmatrix} \frac{1}{m_i}cos(\theta_i) & -\frac{L_i}{J_i}sin(\theta_i) \\ \frac{1}{m_i}sin(\theta_i) & \frac{L_i}{J_i}cos(\theta_i) \end{pmatrix} u_i \\ \dot{\zeta}_{5i} &= \frac{-1}{2L_i}\zeta_{3i}sin(\zeta_{5i}) + \frac{1}{2L_i}\zeta_{4i}cos(\zeta_{5i}). \end{aligned} \quad (8)$$

From the output feedback linearizing control given by

$$u_i = \begin{pmatrix} \frac{1}{m_i}cos(\theta_i) & -\frac{L_i}{J_i}sin(\theta_i) \\ \frac{1}{m_i}sin(\theta_i) & \frac{L_i}{J_i}cos(\theta_i) \end{pmatrix}^{-1} \times \left[V_i - \begin{pmatrix} -v_i\omega_i sen(\theta_i) & -L_i\omega_i^2 cos(\theta_i) \\ v_i\omega_i cos(\theta_i) & -L_i\omega_i^2 sin(\theta_i) \end{pmatrix} \right] \quad (9)$$

it follows that

$$\begin{aligned}
\begin{pmatrix} \dot{\zeta}_{1i} \\ \dot{\zeta}_{2i} \end{pmatrix} &= \begin{pmatrix} \zeta_{3i} \\ \zeta_{4i} \end{pmatrix} \\
\begin{pmatrix} \dot{\zeta}_{3i} \\ \dot{\zeta}_{4i} \end{pmatrix} &= v_i \\
\dot{\zeta}_{5i} &= \frac{-1}{2L_i}\zeta_{3i} sin(\zeta_{5i}) + \frac{1}{2L_i}\zeta_{4i} cos(\zeta_{5i}) \\
\Pi_i &= \begin{pmatrix} \zeta_{1i} \\ \zeta_{2i} \end{pmatrix}
\end{aligned} \quad (10)$$

Last equation denotes the internal dynamics which are rendered non-observable and uncontrollable by the transformation (6). The zero dynamics are found by setting $\zeta_{1i}, \zeta_{2i}, ..., \zeta_{4i} = 0$ and then $\dot{\zeta}_{5i} = 0$, having stable zero dynamics, but not asymptotically stable. As $\zeta_{5i} = \theta_i$ and $(\zeta_{3i}, \zeta_{4i})^T$ represent the velocity of the hand position, this implies that the angle θ_i will stop moving when the hand position stops moving. Hereafter, the input-output dynamics of each robot will be represented by the double integrator system

$$\ddot{\Pi}_i = v_i \quad (11)$$

Owing to parameters variations, uncertainties and non-modeled dynamics, model (10) is an approximation of the behavior from the real system, to deal with this problem it is necessary to consider the design of robust control laws. With this aim, next section address the synthesis of robust controllers for v_i, based on Adaptive Super-Twisting Approach.

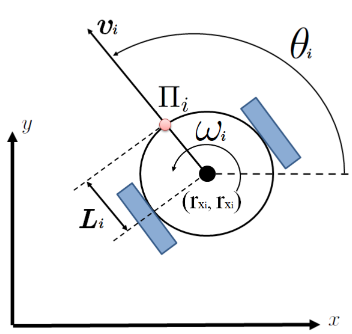

Figure 1: Differential-drive Mobile Robot.

3 ADAPTIVE SUPER-TWISTING CONTROL ALGORITHM

In this section, the synthesis of control law based on a super-twisting adaptive control algorithm, which has been proposed in [Shtessel, 2012], is presented. Furthermore, the bounds of uncertainties and perturbations present on the system do not require to be known. The gains of the controller are adapted in order to attenuate the chattering and do not require to know the bounds of the uncertainties. The main advantage of such algorithm is that it reduce the chattering and increase the robustness of the high order sliding mode approach. The controller designed ensure its convergence in a finite-time and augment the robustness of the system under uncertainties.

Now, consider the super-twisting control algorithm (see [Levant, 2003]), which is given by

$$\begin{aligned} u &= -K_1|s|^{1/2}\text{sign}(s) + \upsilon, \\ \dot{\upsilon} &= -K_2\text{sign}(s), \end{aligned} \quad (12)$$

where u represents the control signal, K_1, K_2 are the control gains and s is a sliding variable.

From the adaptive super-twisting control algorithm (ASTA) approach, the gains K_1 and K_2 are chosen such that they are functions of the sliding surface dynamics as follows

$$K_1 = K_1(t, s, \dot{s}) \ , K_2 = K_2(t, s, \dot{s}). \quad (13)$$

Now, in order to design an adaptive super-twisting control for the uncertain nonlinear system

$$\dot{x} = f(x,t) + g(x,t)u, \quad (14)$$

where $x \in \Re^N$ is the state, $u \in \Re$ the control input, $f(x,t) \in \Re^N$ is a continuous function.

We introduce the following assumptions.

Assumption B1. The sliding variable $s = s(x,t) \in \Re$ is designed so that the desired compensated dynamics of the system (14) are achieved in the sliding mode $s = s(x,t) = 0$.

Assumption B2. The relative degree of the system (14) with the sliding variable $s(x,t)$ with respect to u is equal to 1, and the internal dynamics are stable.

Then, the dynamics of the sliding variable s are given by

$$\dot{s} = a(x,t) + b(x,t)u. \quad (15)$$

where $a(x,t) = \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} f(x,t)$, $b(x,t) = \frac{\partial s}{\partial x} g(x)$.

Assumption B3. The function $b(x,t) \in \Re$ is unknown and different to zero $\forall x$ and $t \in [0, \infty)$. Furthermore, $b(x,t) = b_0(x,t) + \Delta b(x,t)$, where $b_0(x,t)$ is the nominal part of $b(x,t)$ which is known, and there exists δ_1 an unknown positive constant such that $\Delta b(x,t)$ satisfies

$$\left| \frac{\Delta b(x,t)}{b_0(x,t)} \right| \leq \delta_1.$$

Assumption B4. There exists δ_2 an unknown positive constant such that the derivative of function $a(x,t)$ is bounded

$$|\dot{a}(x,t)| \leq \delta_2. \tag{16}$$

The objective of the ASTA approach is to design a continuous control without overestimating the gain, to drive the sliding variable s and its derivative $\dot{s}$ to zero in finite time, under boundary disturbances of type additives and multiplicatives with unknown bounds δ_1 and δ_2.

Then, the closed loop system (15) becomes

$$\begin{aligned} \dot{s} &= a(x,t) - K_1 b(x,t)|s|^{1/2}\mathrm{sign}(s) + b(x,t)\upsilon, \\ \dot{\upsilon} &= -K_2\mathrm{sign}(s), \end{aligned} \tag{17}$$

Furthermore, consider the following change of variable

$$\varsigma = (\varsigma_1, \varsigma_2)^T = (|s|^{1/2}\, sign(s), b(x,t)\upsilon + a(x,t))^T, \tag{18}$$

Then, the system (15) can be written as

$$\dot{\varsigma} = \tilde{A}(\varsigma_1)\varsigma + \tilde{g}(\varsigma_1)\varrho(x,t), \tag{19}$$

where

$$\tilde{A}(\varsigma_1) = \frac{1}{2\,|\varsigma_1|}\begin{pmatrix} -2b(x,t)K_1 & 1 \\ -2b(x,t)K_2 & 0 \end{pmatrix}, \qquad \tilde{g}(\varsigma_1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

where $\bar{\varrho}(x,t) = \dot{b}(x,t)\upsilon + \dot{a}(x,t) = 2\varrho(x,t)\frac{\varsigma_1}{|\varsigma_1|}$. To prove the closed loop stability of the system,

Assumption B5. $\dot{b}(x,t)\upsilon$ is bounded with unknown boundary δ_3 $i.e.$ $|\ \dot{b}(x,t)\upsilon\ | < \delta_3$.

Then, system can be rewritten as follows

$$\dot{\varsigma} = \bar{A}(\varsigma_1)\varsigma, \quad \bar{A}(\varsigma_1) = \frac{1}{|\varsigma_1|}\begin{pmatrix} \frac{-b}{2}(x,t)K_1 & \frac{1}{2} \\ \varrho(x,t) - b(x,t)K_2 & 0 \end{pmatrix} \tag{20}$$

where $|\varsigma_1| = |s|^{1/2}$, it is appealing to consider the quadratic function

$$V_0 = \varsigma^T \tilde{P}\varsigma, \tag{21}$$

where $\tilde{P}$ is a constant, symmetric and positive matrix, as a strict Lyapunov candidate function for (12). Taking its derivative along the trajectories of (12), we have

$$\dot{V}_0 = -\,|s|^{-1/2}\,\varsigma^T\tilde{Q}\varsigma, \tag{22}$$

almost everywhere, where $\tilde{P}$ and $\tilde{Q}$ are related by the Algebraic Lyapunov Equation

$$\bar{A}^T\tilde{P} + \tilde{P}\bar{A} = -\tilde{Q}. \tag{23}$$

Since $\bar{A}$ is Hurwitz for $b(x,t)K_1 > 0$, $2b(x,t)K_2 + 2\varrho(x,t) > 0$, for every $\tilde{Q} = \tilde{Q}^T > 0$, there exist a unique solution $\tilde{P} = \tilde{P}^T > 0$ of the (23), so that V_0 is a strict Lyapunov function.

Remark 1. The stability of the equilibrium $\varsigma = 0$ of (20) is completely determined by the stability of the matrix $\bar{A}$. However, classical versions of Lyapunov 's theorem [Filippov, 1988] cannot be used since they require a continuously differentiable, or at least locally Lipschitz continuous Lyapunov function, though V_0 (21) is continuous but not locally Lipschitz. Nonetheless, as it is explained in Theorem 1 in [Moreno, 2012], it is possible to show the convergence properties by means of Zubov 's theorem [Pozniak, 2008], that requires only continuous Lyapunov functions. This argument is valid in all the proofs of the present paper, so that no further discussion of these issues will be required.

From Assumption B4 and B5, it follows that

$$0 < \varrho(x,t) < \delta_2 + \delta_3 = \delta_4.$$

Notice that, as ς_1 and ς_2 converge to 0 in finite time, it follows that s and $\dot{s}$ converge to 0 in finite time, too.

The control design based on ASTA approach is formulated in the following theorem.

Theorem 3.1. *Considering system (15) satisfying assumptions $B3$, $B4$ and $B5$ for unknown gains $\delta_1, \delta_2 > 0$. Then, for any initial conditions $x(0)$, and $s(0)$, there exists a finite time $0 < t_F$ and a parameter μ, as soon as the condition*

$$K_1 > \frac{\delta_1\left(\lambda + 4\epsilon_*^2\right) + \epsilon_*}{\lambda} + \frac{\left[2\epsilon_*\delta_1 - \lambda - 4\epsilon_*^2\right]^2}{4\epsilon_*\lambda},$$

holds, if $|s(0)| > \mu$, so that a real 2-sliding mode, i.e. $|s| \leq \eta_1$ and $|\dot{s}| \leq \eta_2$, is established $\forall t \geq t_F$, under the action of ASTA control (12) with the adaptive gains

$$\begin{aligned} \dot{K}_1 &= \begin{cases} \omega_1\sqrt{\dfrac{\gamma_1}{2}}\mathrm{sign}(|s| - \mu), & \text{if } K_1 > K_*, \\ K_*, & \text{if } K_1 \leq K_*, \end{cases} \\ K_2 &= 2\epsilon_* K_1, \end{aligned} \tag{24}$$

where $\epsilon_*, \lambda, \gamma_1, \omega_1, \mu$ are arbitrary positive constants, and $\eta_1 \geq \mu$, $\eta_2 > 0$. $\diamond$

Proof of Theorem 3.1. See [Shtessel, 2012].

Now, according to (30), the sliding surface for the control (12)-(13) is defined as

$$s = [\tilde{\Pi}_i + \psi_i(2\tilde{\Pi}_i - \tilde{\Pi}_{i-1} - \tilde{\Pi}_{i+1})] \\ +\lambda_i[\dot{\Pi}_i + \psi_i(2\dot{\Pi}_i - \dot{\Pi}_{i-1} - \dot{\Pi}_{i+1})] \quad (25)$$

where $\psi_i = K_f{}^{-1} K_g = \psi_i^T \geq 0$.

4 FORMATION CONTROL PROBLEM

Let N be the number of mobile robots in the formation. A formation pattern is defined to be a set

$$P = \{\Pi_1^d, ..., \Pi_N^d\} \quad (26)$$

where Π_i^d is the desired location of the hand position of the $i-th$ robot. In this paper will be considered the class of formation control problems where the group of robots is required to commute through a sequence of formation patterns $P_j, j = 1, ..., J$, where is assumed that the sequence of formation patterns are designed in such a way as to avoid robot collisions. Besides, it is desirable to maintain the robots in the shape as the destination pattern.

There are two competing objectives. The first objective it to move the robots to their final destination as specified in the formation pattern. The second objective is to maintain formation during the transition. A diagram of the competing objectives is illustrated in Figure 2.

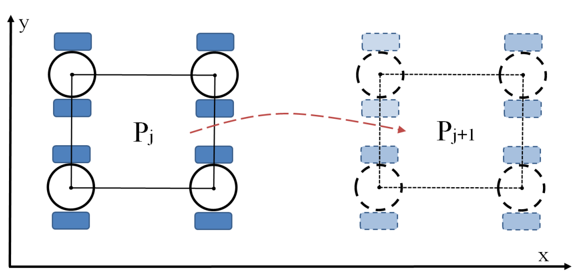

Figure 2: Competing objectives.

In order to incorporate both competing objectives, error functions will be defined. Let ε_g be the total error between the current position of the robots and the desired formation pattern

$$\varepsilon_g = \sum_{i=1}^{N} \tilde{\Pi}_i^T K_g \tilde{\Pi}_i \quad (27)$$

where K_g is a symmetric positive definite matrix, and $\tilde{\Pi}_i = \Pi_i - \Pi_i^d$. Similarly, define ε_f as the formation error

$$\varepsilon_f = \sum_{i=<N>} (\tilde{\Pi}_i - \tilde{\Pi}_{i+1})^T K_f (\tilde{\Pi}_i - \tilde{\Pi}_{i+1}) \quad (28)$$

where $K_f = K_f^T \geq 0$, and where the robot index is defined as modulo N, i.e. $\tilde{\Pi}_{N+1} = \tilde{\Pi}_1$, and $\tilde{\Pi}_0 = \tilde{\Pi}_N$. $i =< N >$ is used to indicate summation around the ring defined by the formation pattern.

The total error for the formation coordination problem is the sum of ε_g and ε_f

$$\varepsilon(t) = \varepsilon_f(t) + \varepsilon_g(t) \\ = \sum_{i=N} \{\tilde{\Pi}_i^T K_g \tilde{\Pi}_i \\ +(\tilde{\Pi}_i - \tilde{\Pi}_{i+1})^T K_f (\tilde{\Pi}_i - \tilde{\Pi}_{i+1})\} \quad (29)$$

where K_f and K_g weight the relative importance of formation keeping versus goal convergence. The formation control objective is to drive $\varepsilon \to 0$ asymptotically.

Now, a control strategy to drive $\varepsilon(t)$ from (29) to zero, taking into account the dynamics (11) is proposed. The coupled dynamics approach couples the dynamics of the robots by incorporating relative position and velocity information between neighbors in the control strategy. This approach requires that each robot knows the relative position and velocity of two other robots, as well as their desired positions in the target formation pattern.

Thus, the control law employed is given by

$$v_i = -K_g \tilde{\Pi}_i - D_g \dot{\Pi}_i \\ -K_f(\tilde{\Pi}_i - \tilde{\Pi}_{i+1}) - D_f(\dot{\Pi}_i - \dot{\Pi}_{i-1}) \\ -K_f(\tilde{\Pi}_i - \tilde{\Pi}_{i-1}) - D_f(\dot{\Pi}_i - \dot{\Pi}_{i+1}) \quad (30)$$

where K_f and D_f are symmetric positive semidefinite matrices, and K_g and D_g are symmetric positive definite. The first two terms in (30) drive the robot to its final position in the formation pattern. The second two terms maintain formation with the $i-1$ robot, and the last two terms maintain formation with the $i+1$ robot.

Theorem 4.1. *If the robot formation (1) is subjected to the control strategy defined in (9)-(8) and the error function (29) converges to zero asymptotically. Furthermore, if the formation is initially at rest, i.e.* $\dot{\Pi}(0) = 0$, *then the formation error is bounded by*

$$\varepsilon(t) \leq \varepsilon(0) - \sum_{i=1}^{N} \dot{\Pi}_i^T \dot{\Pi}_i \quad (31)$$

Now, in order to probe Theorem 4.1, let us introduce the Kronecker product notation and the following lemma.

Lemma 3.1

Let C be the Hankel matrix defined by the row vector

$$(2, -1, 0, ..., 0, -1) \in \Re^N \quad (32)$$

then $C \in \Re^{N\times N}$ is symmetric positive definite. If $\xi = (\xi_1^T, ..., \xi_N^T)^T$ where $\xi_i \in \Re^p$, then

$$\sum_{i=<N>} (\xi_i - \xi_{i+1})^T J(\xi_i - \xi_{i+1}) = \xi^T (C \otimes J)\xi \quad (33)$$

where $\otimes$ denotes the Kronecker product of two matrices. If the terms $J(\xi_i - \xi_{i-1}) + J(\xi_i - \xi_{i+1})$ are stacked in a a column vector, the resulting vector can be written as $(C \otimes J)\xi$. In addition, if $J \in \Re^{p\times p}$ is positive definite, then $(C \otimes J) \in \Re^{N_p \times N_p}$ is positive definite.

Proof of Theorem 4.1. Let us

$$\tilde{\Pi} = (\tilde{\Pi}_1^T, ..., \tilde{\Pi}_N^T)^T \quad (34)$$

$\varepsilon(t)$ can be rewritten as

$$\varepsilon = \frac{1}{2}\tilde{\Pi}^T (I_N \otimes K_g + C \otimes K_f)\tilde{\Pi} \quad (35)$$

Consider the Lyapunov function candidate

$$V = \varepsilon + \frac{1}{2}\sum_{i=1}^{N} \dot{\Pi}_i^T \dot{\Pi}_i \quad (36)$$

which can be written as

$$V = \frac{1}{2}\tilde{\Pi}^T (I_N \otimes K_g + C \otimes K_f)\tilde{\Pi} + \frac{1}{2}\dot{\Pi}^T \dot{\Pi} \quad (37)$$

The time derivative of V is

$$\dot{V} = \dot{\Pi}^T \left[(I_N \otimes K_g + C \otimes K_f)\tilde{\Pi} + v\right] \quad (38)$$

where $v = (v_1^T, ..., v_N^T)^T$. Thus, the control law (30) can be rewritten as

$$\begin{aligned} v = &-(I_N \otimes K_g + C \otimes K_f)\tilde{\Pi} \\ &-(I_N \otimes D_g + C \otimes D_f)\dot{\Pi} \end{aligned} \quad (39)$$

from *Lemma 3.1*, it follows that

$$\dot{V} = -\dot{\Pi}^T (I_N \otimes D_g + C \otimes D_f)\dot{\Pi} \quad (40)$$

which is negative semidefinite. $\diamond$

5 SIMULATION RESULTS

In this section, simulation results are provided to illustrate the effectiveness of the proposed methodology.

The simulation of the systems and the control algorithms were developed in the MATLAB/Simulink environment, with a sampling time of $0.006s$. Controller parameters are displayed on the Tables 1 and 2.

A triangular formation of four robot has been chosen, the robots are commanded to transition through the series of formation patterns given by

$$\begin{aligned}
\mathcal{P}_0 &= \left\{ \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 4.5 \\ 0 \end{pmatrix} \right\} \\
\mathcal{P}_1 &= \left\{ \begin{pmatrix} 3.5 \\ 0.5 \end{pmatrix}, \begin{pmatrix} 2 \\ 0.5 \end{pmatrix}, \begin{pmatrix} 3.5 \\ 1.5 \end{pmatrix}, \begin{pmatrix} 5 \\ 0.5 \end{pmatrix} \right\} \\
\mathcal{P}_2 &= \left\{ \begin{pmatrix} 3.5 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 3.5 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right\} \\
\mathcal{P}_3 &= \left\{ \begin{pmatrix} 4.5 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 4.5 \\ 3 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \end{pmatrix} \right\} \\
\mathcal{P}_4 &= \left\{ \begin{pmatrix} 5.5 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 5.5 \\ 4 \end{pmatrix}, \begin{pmatrix} 7 \\ 3 \end{pmatrix} \right\}
\end{aligned}$$

Table 1: ASTA X-axis Control parameters

i	ω_{xi}	λ_{xi}	μ_{xi}	γ_{xi}	ϵ_{*xi}
1	0.0012	14	0.12	1.1	0.35
2	0.0017	14	0.1	1.1	0.35
3	0.0017	14	0.1	1.1	0.35
4	0.0017	14	0.1	1.1	0.35

Table 2: ASTA Y-axis Control parameters

i	ω_{yi}	λ_{yi}	μ_{yi}	γ_{yi}	ϵ_{*yi}
1	0.0013	14	1.13	1.1	0.5
2	0.0013	14	1.13	1.1	0.5
3	0.0014	14	1.14	1.1	0.5
4	0.0011	14	1.15	1.1	0.5

The corresponding trajectories over the plane XY, are shown in the Figure 3. Response from the different mobile robot can be seen in the Figure 4.

From Figures 3-4 can be seen competing objectives, the robots move to desired positions holding the formation pattern along the transition from point to point.

6 CONCLUSIONS

In this work, the problem of transitioning a group of robots through a sequence of formation patterns has

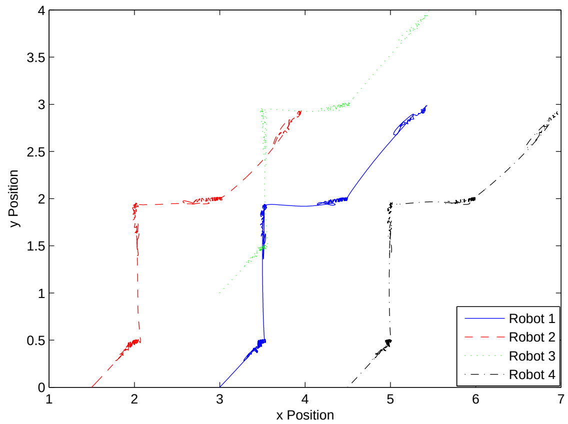

Figure 3: XY plane trajectories.

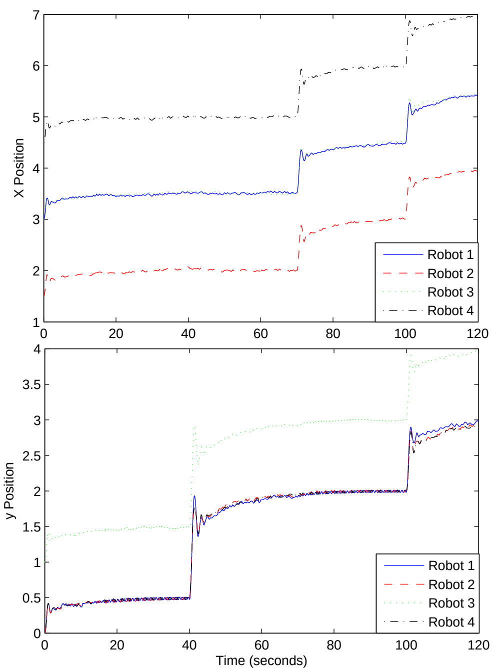

Figure 4: Robots position.

been considered. The group objective is to maintain the desired formation pattern during the transition. With this aim, a control strategy that combines feedback linearisation with an adaptive super-twisting control algorithm has been proposed. Simulation results demonstrated the effectiveness of the proposed scheme.

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