PHASE CONTROL: COMPARISON BETWEEN PULSED AND SINUSOIDAL PERTURBATIONS

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Abstract
We consider a comparison of the phase control technique when applied as a sinusoidal or square pulsed periodic perturbation. We explore the effect of such perturbations to the different terms of the Duffing oscillator. In both cases, the effects are specially effective when they modulates the cubic and the linear term and ineffective when applied to the driving term although their role is exchanged.
Our results highlight the highly nontrivial role of the phase when applying a second periodic perturbation to a chaotic system.

Key words
Phase Control of Chaos, Optimal Control.

1 Introduction

Chaos is characterized by the sensitive dependence on the initial conditions, which implies also that small but accurately chosen perturbations of a chaotic system can lead to substantial changes in the dynamics, and even to its stabilization in different periodic orbits.
This is the rationale behind different methods of chaos control that have been proposed since the pioneering work by Ott et al. [Ott, 1990]. In that work, a carefully chosen perturbation is applied on one of the system's parameters leading to the stabilization of the system on one of the attractors' unstable periodic orbits.
Such perturbation is computed each time depending on the system's state, and for this reason the OGY method falls in the category of the so-called feedback methods of chaos control. Another important family of chaos control is given by nonfeedback controls. These are applied to periodically driven nonlinear dynamical systems. In these methods, a second small harmonic perturbation is applied to the system [Lima, 1990] and can lead to the stabilization over different periodic orbits. It was soon appreciated that the phase difference between this second harmonic perturbation and the main driving can lead to different behaviours. This was first observed in the control of chaos in a CO2 laser [Meucci, 1994] and in numerical simulations with the Duffing oscillator [Qu, 1995], a model of paramount importance in science and engineering. The concept of *phase control of chaos* was then precisely formulated and applied to the control of chaos in a two-well Duffing system and in its implementation in an electronic circuit [Zambrano, 2006]. Interestingly, we also found that such method can be applied to control other types of complex dynamics, such as crisis induced intermittency [Zambrano, Marino et al., 2006], escapes in an open system [Seoane, 2008] and pulses in an excitable system [Zambrano, 2008], both numerically and in experimental implementations.
This suggest the versatility and the robustness of the phase control method. However, we are far from a complete understanding on the mechanism of this control scheme and on how it can be further optimized. In a recent work, we used the Duffing oscillator to understand how different ways to apply a pure sinusoidal controlling perturbation have different effectiveness in stabilizing the system's trajectories [Meucci, 2016].
A second possibility that we have recently explored is using square perturbations [Meucci, 2017], showing that it is possible to stabilize the trajectories for an adequate value of the phase difference.

2 The system

Our numerical and experimental investigations refer to a double-well Duffing oscillator:

$$\dot{x} = y$$
$$\dot{y} = -\gamma y + x - x^3 + A\sin(2\pi f_d t) \quad (1)$$

where $\gamma = 0.25$ is the damping constant, $A = 0.41$ is the amplitude of the sinusoidal driving signal with frequency $f_d = 1$.

The phase control strategy can be applied indifferently to the linear, the cubic term or the driving term of the Duffing oscillator. In this work, we consider two different periodic perturbations, that is, a pulsed and a sinusoidal one. These two different perturbations are shown in Fig. 1 together with the driving term of the Duffing oscillator whose amplitude A=0.41 is selected to have a chaotic condition.

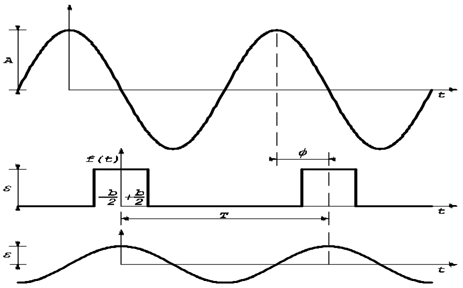

Figure1. Controlling perturbations $f(t)$ of period $T=1/fc$ representing a rectangular pulse of width b and height ε (a) and a pure sinusoidal signal with the same amplitude ε (b). In the upper part, the reference driving signal with amplitude A.

The pulsed control perturbation $f(t)$, consists of a square wave of period $T=1/f_c$ and amplitude epsilon, so that $f(t)= \varepsilon$ for t = [$-b/2$, $b/2$] and 0 during the rest of the period as shown in Fig. 1. In our case we consider the resonant case where the control frequency f_c is equal to the driving frequency f_d. The parameter b is related to the duty cycle D of the square wave through the relation $D = b/T$. A relative phase φ [0, 2π] of the pulsed perturbation $f(t+\varphi)$ with respect to the driving has been introduced considering that it is the key parameter of our control strategy.

In the case of the sinusoidal perturbation, optimal control is obtained by perturbing the linear term of the Duffing oscillator while in the case of pulsed perturbation optimal control is achieved on the cubic term. In both cases, the perturbations are scarcely effective when applied to the main forcing amplitude. The harmonic content in the square wave perturbation provides a simple explanation for this phenomenon.

3 Optimal Control with pulsed perturbations

Optimal control using a pulsed perturbation is obtained by applying the controlling perturbation to the cubic term of the Duffing oscillator:

$$\dot{x} = y$$
$$\dot{y} = -\gamma y + x - (1 + \epsilon f(t))x^3 + A\sin(2\pi f_d t) \quad (2)$$

To characterize the effect of the controlling perturbation we compute the Largest Lyapunov Exponents (LLEs) in the parameter plane ε - φ using the Wolf algorithm [Wolf, 1985] for different values of the duty cycle D. The trajectories were obtained using a fourth-order Runge-Kutta algorithm with a time step $h = 0.001$. After running the integrator for 1000 cycles, to make sure the convergence towards the attractor, the LLE was calculated using another 1000 forcing cycles of the system. Numerical results for six different values of D from $D = 0.02$ to $D = 0.2$ are reported in Fig.2.

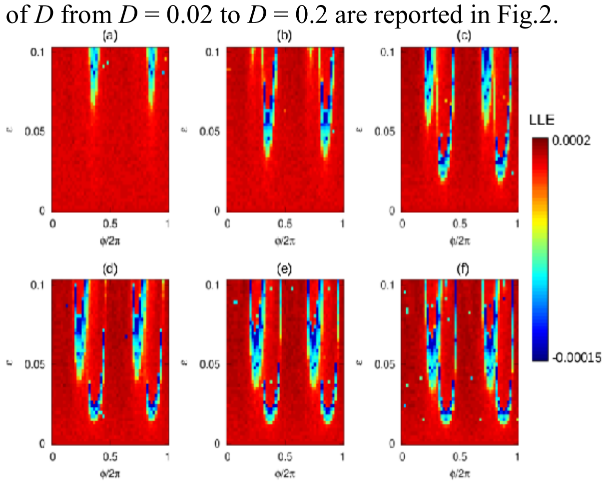

Figure 2. Largest Lyapunov Exponent computed for a pulsed control on the cubic term, (a) $D = 0.02$, (b) $D = 0.04$, (c) $D = 0.08$, (d) $D = 0.12$, (e) $D = 0.16$, (f) $D = 0.2$.

The minimum value of ε required for the control of the system decreases when the duty cycle increases, while the phase remains a crucial parameter to control the system. It is important to observe that the two stability domains separated by a phase difference of π show the appearance of secondary domains as the parameter D increases. For the sake of completeness, in Fig.3 we report a comparison among the three different places where the pulsed perturbation can be applied, that is, the linear term x, the cubic term x^3 and the driving term A. From the comparison, it appears that optimal control is achieved on the cubic term of the oscillator.

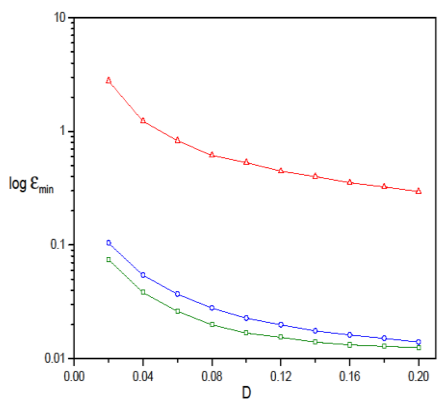

Figure 3. Minimum ε required to control the system for different values of the duty cycle D for control applied on the forcing amplitude A (red, triangles), to the linear term (blue, circles) and to the cubic term (green, squares).

4 Optimal Control with sinusoidal perturbations

The above result is to be compared with the case where pure sinusoidal phase control was considered [Meucci, 2016]. In such a case, phase control is more efficient when applied to the linear term. In Fig. 4 we report both LLE (left panels) and isospike plots (right panels) regarding the detection of different periodic orbits from chaos (in black). From top to bottom we refer to sinusoidal perturbations applied to the linear, cubic and driving term respectively.

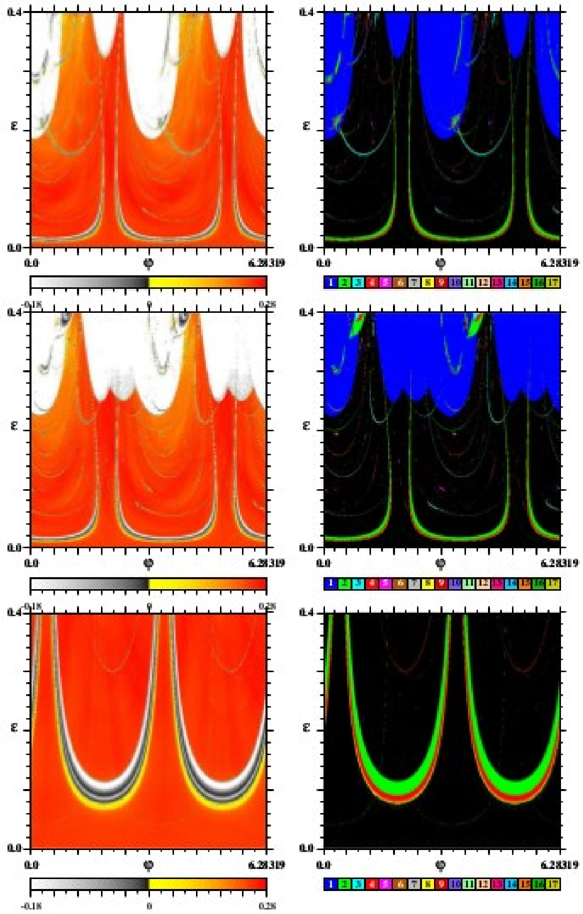

Figure 4. Stability phases predicted by Eqs. (2). Lyapunov exponents (left panels) and isospike diagrams (right panels) obtained for control applied on the linear term, on the cubic term, and on the driving term (top to bottom panels, respectively).

5 Conclusion

In this paper we compare the phase control technique when applied as a sinusoidal or square-pulsed periodic perturbation to a Duffing oscillator.

The harmonic content of the square wave plays a crucial role in optimal control. For a pulsed periodic control, optimal control is achieved on the cubic term, the opposite for a sinusoidal perturbation.

The advantage of using pulsed perturbations is also related to the fact the region of optimal control can be easily detected due to the presence of a saturated regime as the duty cycle is increased. In this regime, if the pulse duration is reduced, we have to compensate with an increase of its amplitude and vice versa. Short-pulsed perturbations will be effective only for a narrow range of the phase difference; long perturbations will lose effectiveness as they imply the effect of several sinusoidal harmonic components with opposite phases, which occurs soon as the duty cycle becomes large. Our investigations confirm the versatility of the phase control technique and how the constraints imposed by the energy play a key role when considering potential applications in physics, engineering and biomedical applications.

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