Control of an active damper based on magneto-sensitive fluid and rubber

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***Abstract*—The damper based on magneto-sensitive fluid and rubber is one of the most promising new devices for structural vibration reduction. It is described by a dynamical system with controlled viscosity and stiffness. The problems of the parametric control of damping and generation of harmonic oscillations for the presented dynamical system are solved.**

I. Introduction

Magneto-sensitive (MS) fluids and rubbers are a class of smart materials whose mechanical properties change instantly by the application of a magnetic field.

Interest in MS materials derives from their ability to provide simple, quiet, rapid-response interfaces between electronic controls and mechanical systems. That MS media have the potential to radically change the way electromechanical devices are designed and operated has long been recognized. A wide range of potential applications is presumably the reason for the intense research in recent years [1-4].

The MS medium is composed of polarizable particles, dispersed in a carrier medium, having a size on the order of a few microns. The MS effect is optimized by choosing a particle material with a high magnetic saturation. In general an alloy of iron is used in MS media. Typical particle volume fractions are between 0,1 and 0,5. Carrier media are chosen based upon their rheological and tribological properties and on their temperature stability. Examples for MS fluids and rubbers are silicone, petroleum based oils, mineral oils, polyesters, water, synthetic hydrocarbon oils and rubber-like elastomers.

During the manufacturing process of MS medium, the isotropy condition inherent of the filler material is maintained in the final composite. Therefore, these materials are considered isotropic and non-conductive. However, MS materials become non-homogeneous due to the presence and distribution of particles in the carrier filler.

In the paper an original dynamical system with controlled viscosity and stiffness is used for modeling of an active dumper based on MS fluid and rubber. The problems of the parametric control of damping and generation of harmonic oscillations for the presented dynamical system are solved by the Lyapunov method [5]. All theoretical results are based on the theorem about asymptotic stability in reference to the part of variables [6] as well as the Barbashin-Krasovski and Chetaev theorems.

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II. Two Parametric Dynamical Model for the Active Dumper

In practice as usual dampers based on MS (or magnetorheological) fluids are used [1,2].

We present an original two parametric model of active damper with MS fluid and rubber elements:

$$\frac{d^2x}{d\tau^2}+\gamma\left(1+u_f\right)\frac{dx}{d\tau}+\left(1+u_r\right)x=f(\tau)\,, \qquad (1)$$

where x is the relative axial displacement of the dampers piston, $\tau=(EL/m)^{1/2}t$ is the dimensionless time, $\gamma=\nu(mEL)^{-1/2}<<1$ is the dimensionless viscosity of the damper without magnetic field, $u_f\approx 1.27\,\delta_f H^2\left(H_f^2-H^2\right)^{-1}$ $(|H|<H_f)$ is the additional viscosity of MS fluid and $u_r=k_rH^2$ $(|H|\le H_r)$ is the additional stiffness of MS rubber in a magnetic field with density H, $k_r\approx\delta_r\mu_r\mu_0$ is the coefficient of magnetic sensitivity of MS rubber, $\mu_0\approx 1.26\cdot 10^{-6}$ is the magnetic permeability of a vacuum, $f(\tau)$ is an external dimensionless force. Here m is the mass of loading damper, ν is the viscosity of MR fluid without magnetic field, L is the length of MS rubber element, μ_f and μ_r is the magnetic permeability of MS fluid and rubber, respectively; δ_f and δ_r is the iron particle volume fraction in MS fluid and rubber, respectively. Here we use approximations for additional viscosity and stiffness from the authors papers [3,4]. It is well known that for real MS rubbers $k_rH_r^2\le 0.4$ [3].

III. Damper Free Oscillations

Consider the problem of damping free oscillations generated by non-zero initial conditions, i.e. for $f(\tau)=0$. In this case the energy of the dynamical system (1) satisfies the following condition:

$$E(\tau)=\frac{1}{2}\left(x^2+\dot{x}^2\right)\to +0\,.$$

Here and what follows the upper point is $d/d\tau$.

Synthesize the controls input u_f and u_r from the condition of decreasing the Lyapunov function $V(\tau)=E(\tau)\ge 0$ on the trajectories of the closed loop system [5], i.e.

$$\dot{V}(\tau) = -\gamma \dot{x}^2 \left(1 + u_f\right) - x\dot{x}\, u_r < 0 .$$

Consider an arbitrary constant $C \geq 0$ and choose controls in the following form:

$$u_f = C , \qquad u_r = F(x\dot{x}) , \tag{2}$$

where F is continuous, strictly increasing function such what $F(0) = 0$ and $0 \leq F(p) \leq k_r H_r^2$ for every $p \in R$.

In this case for the energy of dynamical system (1) the differential equation is fulfilled

$$\dot{E} = -\alpha \dot{x}^2 - x\dot{x}\, F(x\dot{x}) , \tag{3}$$

where $\alpha = \gamma(1 + C) > 0$.

Proposition 1: The extended dynamical system (1)-(3) is asymptotically stable in reference to the variable E . The closed loop system (1), (2) has the asymptotically stable equilibrium $x_1 = x = 0$, $x_2 = \dot{x} = 0$.

Proof: The asymptotic stability follows from the application of the theorem on asymptotic stability in reference to the part of variables [6] to the function V .

Example 1: In Figure 1 the phase portraits of the closed loop system (1), (2) are presented. The function $F(p) = 0.4\left(1 - \exp(-p)\right)h(p)$ was used, where $h(p)$ is the Heaviside function. The phase portrait (a) and (b) corresponds to the parameter $\alpha = 1$ and $\alpha = 5$ in the relation (3), respectively.

VI. Generation of Harmonic Oscillations

Consider the problem of parametric generation of harmonic oscillations with the desired frequency ω and amplitude A in the system (1). For this reason, introduce the functions of energy and desired energy of the dynamical system (1)

$$E(\tau) = \frac{1}{2}\left(\omega^2 x^2 + \dot{x}^2\right) , \qquad E_* = \omega^2 A^2 .$$

Define a new control for the reduced stiffness by the formula $u_2 = 1 - \omega^2 + u_r$, where $1 \leq \omega^2 \leq 1 + k_r H_r^2$. Synthesize the controls input u_f and u_2 from the condition of decreasing the Lyapunov function $V(\tau) = \frac{1}{2}\left(E(\tau) - E_*\right)^2 \geq 0$ on the trajectories of the closed loop system [5], i.e.

$$\dot{V}(\tau) = -\left(E - E_*\right)\left(\gamma \dot{x}^2 \left(1 + u_f\right) + x\dot{x}\, u_2\right) < 0 .$$

Choose controls in the following form:

$$u_f = F_1(E - E_*)h(E - E_*) , \qquad u_2 = F_2\left(x\dot{x}(E - E_*)\right) , \tag{4}$$

where $h(p)$ is the Heaviside function, F_1 and F_2 are continuous, strictly increasing functions such what $F_1(0) = 0$, $F_2(0) = 0$ and $1 - \omega^2 \leq F_2(p) \leq 1 - \omega^2 + k_r H_r^2$ for every $p \in R$.

As a result, we have the following additional differential equation:

$$\frac{d}{d\tau}\left(E - E_*\right) = -\gamma \dot{x}^2 \left[1 + F_1(E - E_*)h(E - E_*)\right] - \\ - x\dot{x}\, F_2\left(x\dot{x}(E - E_*)\right) , \tag{5}$$

Proposition 2: The extended dynamical system (1), (4), (5) is asymptotically stable in reference to the variable $E - E_*$. The closed loop system (1), (4) has the desired harmonic oscillation if $E > 0$ and $\omega^2 > 1 + \gamma$. It has also the unstable equilibrium point (0,0).

Proof: The asymptotic stability follows from the application of the theorem on asymptotic stability in reference to the part of variables [6] to the function V . Because $\dot{E} \geq 0$ in small environ of the point $(0,0)$ then the equilibrium is unstable in accordance to the Chetaev theorem.

Example 2: In Figure 2 the phase portraits of the closed loop system (1), (4) are presented for the harmonic oscillation with the desired frequency $\omega = 1.1$ and amplitude $A = 0.7$. Here the following functions were used: $F_1(p) \equiv 0$ and $F_2(p) = 0.2\left(1 - \exp(-\lambda p)\right)h(p)$ with a parameter $\lambda > 0$. The phase portrait (a) and (b) corresponds to the parameter $\lambda = 1$ and $\lambda = 5$, respectively.

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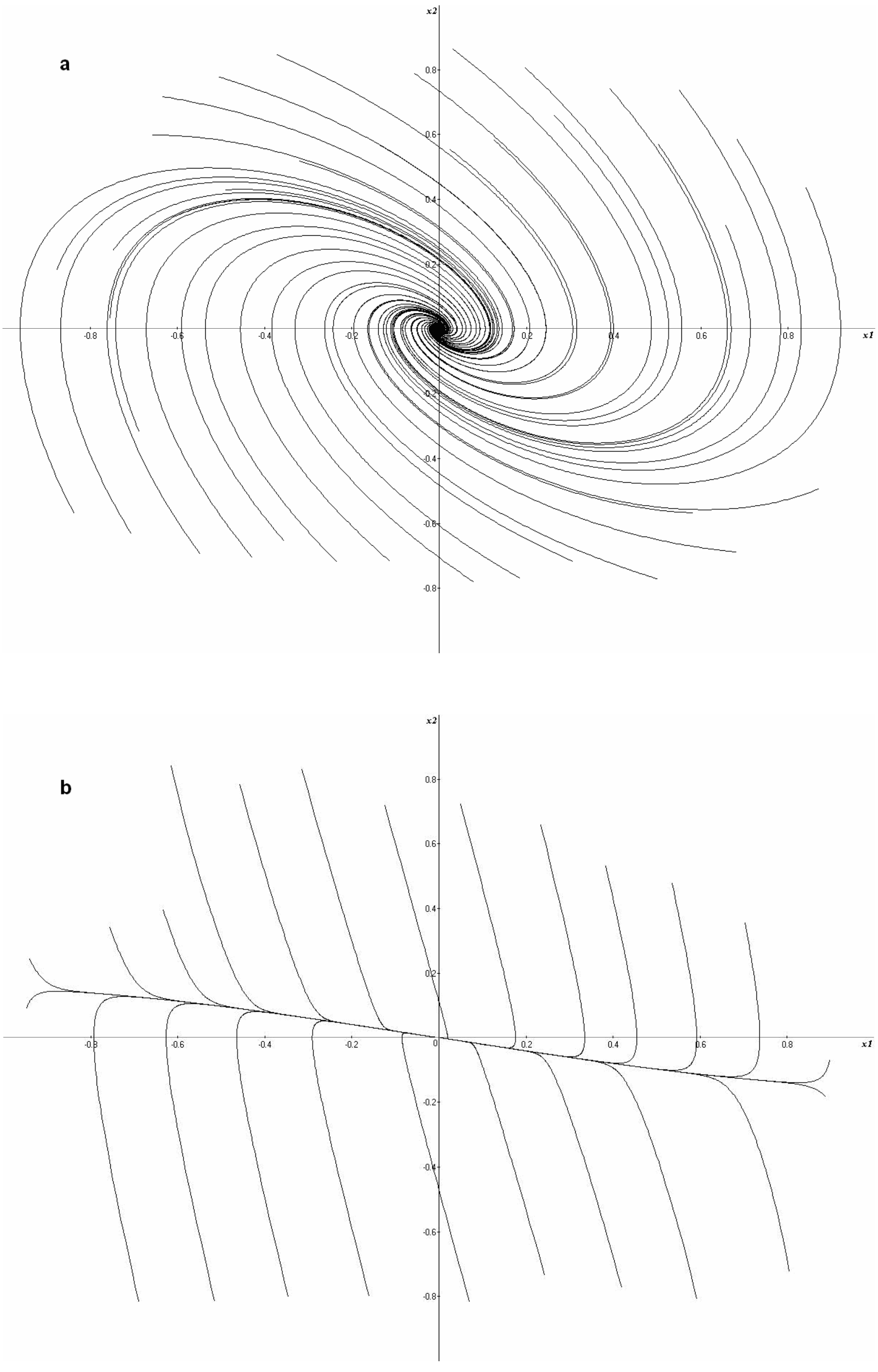

Fig. 1. The phase portraits of the closed loop system (1), (2) described in the Example 1.

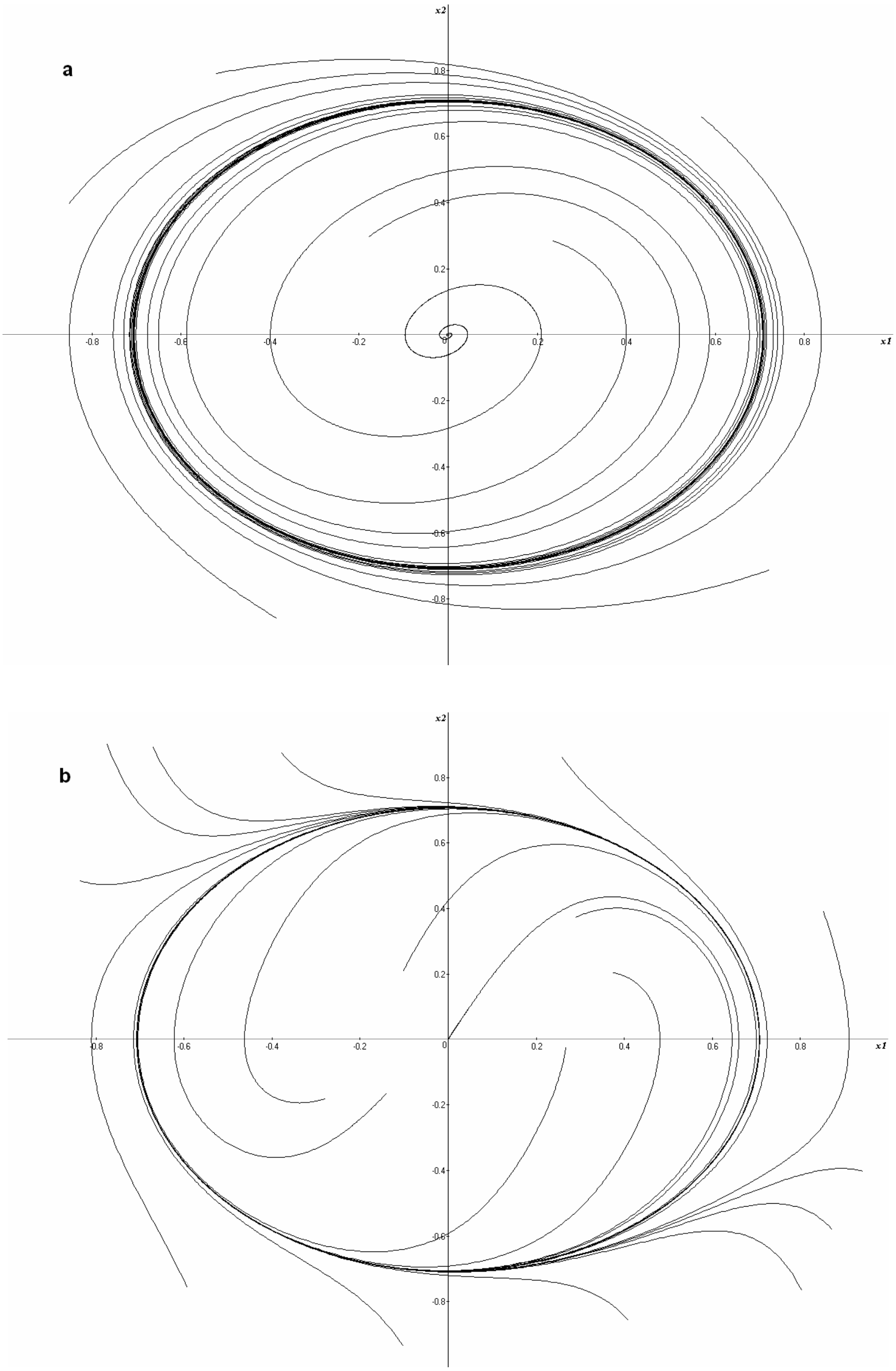

Fig. 2. The phase portraits of the closed loop system (1), (4) described in the Example 2.