SIMPLIFIED ADAPTIVE CONTROL WITH GUARANTEED H_∞ PERFORMANCE

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Abstract: Robust simplified adaptive controllers for continuous-time systems with uncertainties and disturbances are considered. Sufficient conditions for closed-loop stability and prescribed H_∞ disturbance attenuation level of the proposed simplified adaptive control scheme, are introduced, under an almost-strictly-positive- realness requirement on the plant. A numerical example is given, which demonstrates the proposed method.

Keywords: H_∞ control, adaptive control, Robust SPR, SAC, strictly positive real, simplified adaptive control systems.

1. INTRODUCTION

Adaptive control methods cope with unknown changing plant parameters by adjusting the control law to the varying plant on-line. They may be divided into explicit (indirect) control, which separately applies plant-parameters identification and control schemes, and implicit (direct) control, where the control gains are directly computed (without identifying the plant parameters). Simplified Adaptive Control (SAC) is a class of direct adaptive controller schemes which has received considerable attention in the literature for continuous-time systems (Sobel, Kaufman and Mabius, 1982; Sobel, 1989; Kaufman, Barkana and Sobel, 1998). Robustness of SAC controllers facing polytopic uncertainties has already been established (Kaufman et al., 1998; Yaesh and Shaked, 2006; Ben Yamin et al., 2006) allowing application to real engineering problems (see e.g. reference Yossef et al., 2004). The stability of continuous-time SAC is related to the Strictly Positive Real (SPR) property of the controlled plant. Bar-Kana (Bar-Kana, 1986) has recently provided a proof of the fact that any proper minimum-phase linear system with positive definite input-output feed-through matrix D is Almost Strictly Positive Real (ASPR). In addition, any strictly minimum-phase transfer function with minimal realization A, B, C where $CB > 0$ is ASPR(Kaufman et al., 1998).

In the present paper, the relationship between optimal H_∞ control and SAC will be discussed. The objective is to use SAC while satisfying some H_∞ -norm bound γ. Note that SAC can stabilize an uncertain system without knowing the explicit system dynamics. Sufficient conditions are derived for the stability of the closed-loop dynamics of the SAC scheme with disturbance attenuation level γ. These sufficient conditions are expressed in terms of Bilinear Matrix Inequalities (BMI), which can be solved using local iterations. When the ASPR requirement is replaced by the more restrictive SPR requirement, Linear Matrix

Inequalities (LMIs) are obtained. A numerical example is given which illustrates the method.

Throughout the paper the superscript 'T' stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, $\mathcal{R}^{n\times m}$ is the set of all $n\times m$ real matrices, and the notation $P>0$, for $P\in\mathcal{R}^{n\times n}$ means that P is symmetric and positive definite. The trace of a matrix Z is denoted by $tr\{Z\}$. The convex hull defined by the polytope vertices $\Omega_j, j=1,...N$ is denoted by $\mathcal{C}o\{\Omega_j,\ j=1,...N\}$ and $col\{a,\ b\}$ for vectors a and b denotes the augmented vector $[a^T\ b^T]^T$. In symmetric block matrices we use $*$ as an ellipsis for terms that are induced by symmetry.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following continues-time linear system:

$$\begin{aligned}\dot{x}(t) &= A\,x(t)+B_1\,w(t)+B_2u(t), \quad x(0)=x_0\\ z(t) &= C_1\,x(t)+D_{11}\,w(t)+D_{12}\,u(t)\\ y(t) &= C_2\,x(t)+D_{21}\,w(t)+D_{22}\,u(t)\end{aligned} \tag{1}$$

where $x(t)\in\mathcal{R}^n$ is the system state, $z(t)\in\mathcal{R}^r$ is the objective vector ,$y(t)\in\mathcal{R}^m$ is the plant output, $w(t)\in\mathcal{R}^m$ is the exogenous disturbance which is energy bounded and $w(t)\in\mathcal{L}_2$ and $u(t)\in\mathcal{R}^m$ is the control input. A, B, C_1 , C_2, D_{11}, D_{12}, D_{21} and D_{22} are constant matrices of appropriate dimensions. We assume that $D_{22}>0$.

Remark 1. In the general case, for any proper but not strictly proper system, when D_{22} is not positive definite but satisfies $D_{22}+D_{22}^T>0$, we define $u(t)=D_{22}^T\widehat{u}(t)$ and obtain the following representation for (1)

$$\begin{aligned}\dot{x}(t) &= A\,x(t)+B_1\,w(t)+\widehat{B_2}\,\widehat{u}(t)\\ z(t) &= C_1\,x(t)+D_{11}\,w(t)+\widehat{D_{12}}\,\widehat{u}(t)\\ y(t) &= C_2\,x(t)+D_{21}\,w(t)+\widehat{D_{22}}\,\widehat{u}(t)\end{aligned} \tag{2}$$

where $\widehat{B_2}=B_2D_{22}^T$, $\widehat{D_{12}}=D_{12}D_{22}^T$ and $\widehat{D_{22}}=D_{22}D_{22}^T>0$. We therefore, assume in the sequel, without loss of generality, that $D_{22}>0$.

It is required to achieve a stable closed-loop system so that the standard H_∞ cost function J satisfies

$$J\triangleq ||z||_2^2-\gamma^2||w||_2^2<0 \tag{3}$$

for any $w(t)\neq 0$ and $w(t)\in\mathcal{L}_2$ by employing a SAC controller $u(t)$ that is obtained by the measurement feedback scheme

$$u(t)=-K(y(t))y(t), \tag{4}$$

where $K(y(t))$ does not explicitly depend on the system parameters (A, B_1 etc.). Note that the closed-loop system (1) is stable with a disturbance attenuation level γ when (3) is satisfied.

To this end, we assume that there exists a feedback gain matrix K_e which achieves (3). We will show in the sequel that using SAC (Kaufman et al., 1998) we can calculate the gain K_e. It is emphasized here that SAC implementation requires neither explicit knowledge of K_e nor the exact knowledge of the system dynamics.

Consider the control law

$$\begin{aligned}u(t) &= -K_e\,y(t)+\widetilde{u}(t)\\ &= -K_e\,(C_2\,x(t)\ +\\ &\qquad D_{21}\,w(t)+D_{22}\,u(t))+\widetilde{u}(t)\end{aligned} \tag{5}$$

where $\tilde{u}(t)$ is an auxiliary input signal to be determined in the sequel. We define $\widehat{K}_e=(I+K_eD_{22})^{-1}K_e$ and note that $(I+K_eD_{22})^{-1}=(I-\widehat{K}_eD_{22})$. The algebraic loop for $u(t)$ in (5) thus results in

$$\begin{aligned}u(t) = -\widehat{K}_e(C_2x(t)\ &+D_{21}\,w(t))+\\ &(I-\widehat{K}_eD_{22})\widetilde{u}_k.\end{aligned} \tag{6}$$

Substituting (6) in (1) and defining $\widetilde{A}\equiv(A-B_2\widehat{K}_eC_2)$, $\widetilde{C_1}\equiv(C_1-D_{12}\widehat{K}_eC_2)$, $\widetilde{B_1}\equiv(B_1+B_2\,(I-\widehat{K}_eD_{22})D_{21})$, $\widetilde{B_2}\equiv B_2(I-\widehat{K}_eD_{22})$, $\widetilde{C_2}\equiv(I-D_{22}\widehat{K}_e)C_2$, $\widetilde{D_{11}}\equiv(D_{11}+D_{12}\,(I-\widehat{K}_eD_{22})D_{21})$, $\widetilde{D_{12}}\equiv D_{12}\,(I-\widehat{K}_eD_{22})$, $\widetilde{D_{21}}\equiv(D_{21}+D_{22}\,(I-\widehat{K}_eD_{22})D_{21})$ and , $\widetilde{D_{22}}\equiv D_{22}\,(I-\widehat{K}_eD_{22})$, we obtain the closed-loop system

$$\begin{aligned}\dot{x}(t) &= \widetilde{A}\,x(t)+\widetilde{B_1}\,w(t)+\widetilde{B_2}\widetilde{u}(t)\\ z(t) &= \widetilde{C_1}\,x(t)+\widetilde{D_{11}}\,w(t)+\widetilde{D_{12}}\widetilde{u}(t)\\ y(t) &= \widetilde{C_2}\,x(t)+\widetilde{D_{21}}\,w(t)+\widetilde{D_{22}}\widetilde{u}(t).\end{aligned} \tag{7}$$

It is required to assure that the closed-loop (7) is stable for any $w(t)\in\mathcal{L}_2$ and has a disturbance attenuation level γ.

3. MIXED H_∞ AND SAC FOR CONTINUOUS-TIME LINEAR SYSTEMS

Consider the following direct adaptive control scheme known as SAC (Kaufman et al., 1998):

$$u(t)=-K(t)\,y(t)=-\widehat{K}(t)(C_2x(t)+D_{21}\,w(t)) \tag{8}$$

where, after solving the algebraic loop in $u(t)$, it is found that

$$\widehat{K}(t)=(I+K(t)D_{22})^{-1}K(t). \tag{9}$$

The SAC gain adaptation formula is

$$\dot{K}(t)=y(t)y^T(t)-\beta K(t), \quad K(0)=\epsilon I \tag{10}$$

where $\beta \geq 0$ is a scalar and ϵ is a positive scalar. Next, an upper-bound on $\widehat{K}(t)$ is calculated. Since it was assumed that $D_{22} > 0$, we have

$$\begin{aligned}\widehat{K}(t) &= (I+K(t)D_{22})^{-1}K(t)D_{22}D_{22}^{-1} \\ &= D_{22}^{-1} - (I+K(t)D_{22})^{-1}D_{22}^{-1} \\ &= D_{22}^{-1} - F(t)\end{aligned}$$

where $F(t) = (D_{22} + D_{22}K(t)D_{22})^{-1}$ so that $F(t) > 0$ and $F(t) \leq D_{22}^{-1}$. Namely:

$$0 < \widehat{K}(t) \leq D_{22}^{-1}. \tag{11}$$

Remark 2. The equality in (11) is achieved when $K(t) \to \infty$. Without the β-term in (10), $K(t)$ may steadily increase when $y(t) \neq 0$. With the β-term, $K(t)$ is obtained from a first-order filtering of $y(t)y^T(t)$ and thus cannot diverge, unless $y(t)$ diverges (Kaufman et al., 1998).

Define $\delta(t) = K_e - K(t)$. The control law (8) is obtained by substituting $\widetilde{u}(t) \triangleq \delta(t)y(t)$ in (5). Thus, we obtain

$$\dot{\delta}(t) = -\dot{K}(t) = -y(t)y^T(t) + \beta K(t) \tag{12}$$

and the following holds:

$$\begin{aligned}\delta(t)\dot{\delta}(t) &= \delta(t)(-y(t)y^T(t) + \beta K(t)) \\ &= -\widetilde{u}(t)y(t)^T + \beta\delta(t)K(t).\end{aligned} \tag{13}$$

We are now in a position to state the main result of this section.

Theorem 1. For an ASPR plant, the adaptive scheme consisting of the plant (1), the control law (8) and the gain adaptation formula (10) has bounded gains and states and a disturbance attenuation level γ, for any $\beta \geq 0$ and any $w(t) \in \mathcal{L}_2$ if the following BMI holds:

$$\Gamma \leq 0, \tag{14}$$

where

$$\Gamma \triangleq \begin{bmatrix} \widetilde{A}^T P + P\widetilde{A} & P\widetilde{B_2} - \widetilde{C_2}^T & P\widetilde{B_1} & \widetilde{C}_1^T \\ * & -\widetilde{D_{22}} - \widetilde{D_{22}}^T & \widetilde{D}_{21} & \widetilde{D}_{12}^T \\ * & * & -\gamma^2 I & \widetilde{D}_{11}^T \\ * & * & * & -I \end{bmatrix} \tag{15}$$

Proof : In order to establish the results we consider the radially-unbounded Lyapunov function candidate

$$\begin{aligned}V(x(t), K(t)) = x^T(t)Px(t) + \\ Tr\{\delta(t)\delta(t)^T\} > 0.\end{aligned} \tag{16}$$

Note that $V(0, K_e) = 0$ and $V(x(t), K(t)) > 0$ for all $\{x(t), K(t)\} \neq \{0, K_e\}$. Note also that $V_k(x(t), K(t)) \to \infty$ if $\|x(t)\| \to \infty$ or $\|K(t)\| \to \infty$. To obtain (3) we must have

$$\dot{V}(t) \leq \gamma^2 w^T(t)w(t) - z^T(t)z(t) \tag{17}$$

The derivative of (16) is given by

$$\begin{aligned}\dot{V}(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ -2Tr\{\delta(t)\dot{\delta(t)}^T\}.\end{aligned} \tag{18}$$

Define $\mathcal{S} = \dot{V}(t) - \gamma^2 w(t)^T w(t) + z^T(t)z(t)$. Then,

$$\begin{aligned}\mathcal{S} = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) - 2Tr\{\delta(t)\dot{\delta(t)}^T\} \\ -\gamma^2 w(t)^T w(t) + z^T(t)z(t).\end{aligned} \tag{19}$$

Substituting (13) in (19), we have

$$\begin{aligned}\mathcal{S} = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) - 2Tr\{\widetilde{u}(t)y(t)^T\} \\ +2\beta Tr\{\delta(t)K^T(t)\} - \gamma^2 w(t)^T w(t) + z^T(t)z(t).\end{aligned}$$

Using (7) we obtain

$$\begin{aligned}\mathcal{S} = &(\widetilde{A}\,x(t) + \widetilde{B}_1\,w(t) + \widetilde{B}_2\widetilde{u}(t))^T(t)Px(t) + \\ &x^T(t)P(\widetilde{A}\,x(t) + \widetilde{B}_1\,w(t) + \widetilde{B}_2\widetilde{u}(t)) \\ &-\widetilde{u}(t)^T(\widetilde{C}_2\,x(t) + \widetilde{D}_{21}\,w(t) + \widetilde{D}_{22}\,\widetilde{u}(t)) \\ &-(\widetilde{C}_2\,x(t) + \widetilde{D}_{21}\,w(t) + \widetilde{D}_{22}\,\widetilde{u}(t))^T\widetilde{u} \\ &+2\beta Tr\{\delta(t)K^T(t)\} - \gamma^2 w(t)^T w(t) \\ &+(\widetilde{C}_1\,x(t) + \widetilde{D}_{11}\,w(t) + \widetilde{D}_{12}\,\widetilde{u}(t))^T \\ &(\widetilde{C}_1\,x(t) + \widetilde{D}_{11}\,w(t) + \widetilde{D}_{12}\,\widetilde{u}(t))\end{aligned} \tag{20}$$

where use is made of the fact that $tr(AB) = tr(BA)$. Define

$$\mathcal{S} = \lambda_1(t) + \lambda_2(t)$$

where:

$$\lambda_1(t) = \begin{bmatrix} x^T(t) & \widetilde{u}^T(t) & w^T(t) \end{bmatrix} \Gamma \begin{bmatrix} x(t) \\ \widetilde{u}(t) \\ w(t) \end{bmatrix}$$

$$\lambda_2(t) = 2\beta Tr\{\delta(t)K^T(t)\}$$

and where

$$\Gamma \triangleq \begin{bmatrix} E_{11} & P\widetilde{B}_2 - \widetilde{C}_2^{\,T} + \widetilde{C}_1^T\widetilde{D}_{12} & P\widetilde{B}_1 + \widetilde{C}_1^T\widetilde{D}_{11} \\ * & -\widetilde{D_{22}} - \widetilde{D_{22}}^T + \widetilde{D}_{12}^T\widetilde{D}_{12} & \widetilde{D}_{21} + \widetilde{D}_{12}^T\widetilde{D}_{11} \\ * & * & -\gamma^2 + \widetilde{D}_{11}^T\widetilde{D}_{11} \end{bmatrix} \leq 0 \tag{21}$$

$$E_{11} = \widetilde{A}^T P + P\widetilde{A} + \widetilde{C}_1^T\widetilde{C}_1.$$

For $\beta = 0$ we obtain that $\lambda_2(t) = 0$ and it is easy to show that (21) may be rewritten as (14).

We next show that the system states and gains are bounded also if $\beta > 0$. To this end, note that if (14) is satisfied then $\lambda_1(t) \leq 0$ and that

$$\lambda_2(t) = -2\beta Tr\{K(t)K^T(t)\} + 2\beta Tr\{K_e K^T(t)\}$$

is not definite. But, $\lambda_2(t)$ (the second term of $\dot{V}(t)$) is quadratic in $K(t)$. Since $K(t)$ may steadily increase when $y(t) \neq 0$, the term $-Tr\{K(t)K^T(t)\}$ becomes dominant in $\lambda_2(t)$), hence $\dot{V}(t)$ becomes negative. This guarantees that all adaptation variables are bounded (LaSalle's Theorem). QED

Remark 3. It follows from Theorem 1 that the addressed SAC scheme is bounded even without the β-term in (10). The β-term maintains the adaptive gains small and prevents attaining undesirable high gains.

In the case where $C_1 = 0$, $D_{11} = 0$ and $D_{12} = 0$ (no objective vector) it is easy to show that (14) may be rewritten as

$$\Gamma \triangleq \begin{bmatrix} \widetilde{E}_{11} & P\widetilde{B}_2 - \widetilde{C}_2^T + \gamma^{-2}P\widetilde{B}_1\widetilde{D}_{21}^T \\ * & -\widetilde{D}_{22} - \widetilde{D}_{22}^T + \gamma^{-2}\widetilde{D}_{21}\widetilde{D}_{21}^T \end{bmatrix} \leq 0 \quad (22)$$

where

$$\widetilde{E}_{11} = \widetilde{A}^T P + P\widetilde{A} + \gamma^{-2}P\widetilde{B}_1\widetilde{B}_1^T P \quad (23)$$

It follows from (22) that, under the existence of exogenous disturbance $w(t)$, bounded gains and states are guaranteed if the system satisfies a condition which is more conservative than the ASPR condition but reduces to the ASPR condition in the limit where γ tends to infinity. Moreover, note that stability in the presence of nonzero measurement noise (i.e. nonzero D_{21} and finite γ) requires a large enough $D_{22} > 0$.

4. ROBUST SIMPLIFIED ADAPTIVE CONTROL WITH UNCERTAINTIES AND DISTURBANCE

We next extend the results of Theorem 1 to the case where the A , B_1 and B_2 of the system (1) are not exactly known. Denoting

$$\Omega = \left\{ A \; B_1 \; B_2 \right\} \quad (24)$$

where $\Omega \in Co\{\Omega_i, \; i = 1, ...N\}$, namely,

$$\Omega = \sum_{i=1}^{N} f_i \Omega_i \quad \text{for some} \quad 0 \leq f_i \leq 1, \; \sum_{i=1}^{N} f_i = 1 \quad (25)$$

where the vertices of the polytope are described by

$$\Omega_i = \left\{ A^{(i)} \; B_1^{(i)} \; B_2^{(i)} \right\}, \quad i = 1, 2..., N. \quad (26)$$

Next theorem describes conditions which assure that the closed-loop system (1) is not only stable but it also has a H_∞ disturbance attenuation level γ over $Co\{\Omega_i\}$.

Theorem 2. For an ASPR plant, the addressed SAC scheme has a disturbance attenuation level γ for any $\beta \geq 0$ over $Co\{\Omega_i\}$ if the following BMI holds:

$$\Phi \triangleq \begin{bmatrix} \widehat{E}_{11} & P\widetilde{B_2}^{(i)} - \widetilde{C_2}^{(i)^T} & P\widetilde{B_1}^{(i)} & \widetilde{C}_1^T \\ * & -\widetilde{D_{22}}^{(i)} - \widetilde{D_{22}}^{(i)^T} & \widetilde{D}_{21} & \widetilde{D}_{12}^T \\ * & * & -\gamma^2 & \widetilde{D}_{11}^T \\ * & * & * & -I \end{bmatrix} \leq 0 \quad (27)$$

where

$$\widehat{E}_{11} = \widetilde{A}^{(i)^T} P + P\widetilde{A}^{(i)} \quad (28)$$

Proof : The latter is affine in $A^{(i)}$ and $B_1^{(i)}$ and $B_2^{(i)}$. We thus readily obtain by multiplying (27) by f_i and summing over $i = 1, 2, ..., N$ that $\Phi \leq 0$ is satisfied over Ω.

Remark 4. It follows from the equivalence between the ASPR property and the minimum-phase (MP) property (Kaufman et al., 1998) that, under the existence of an exogenous disturbance $w(t)$, bounded gains and states are guaranteed if the plant is MP at all the vertices. Namely, one can establish equivalence between the ASPR property of (1), which can be verified by the BMI (22) where γ tends to infinity, and its minimum-phase property which can be verified by the LMI

$$(H^{(i)})^T P + PH^{(i)} < 0 \, , i = 1, 2, ..., N \quad (29)$$

where

$$H^{(i)} = (A^{(i)} - B_2^{(i)} D_{22}^{-1} C_2) \quad (30)$$

5. NUMERICAL EXAMPLE

In this section we present a numerical example to demonstrate the application of the theory developed above. we consider a modified version of the angle of attack/pitch-rate dynamics example of (Gahinet et al., 1995). This example describes the short period dynamics of a missile and was used in (Gahinet et al., 1995) to study gain scheduled control. The state-vector is $x = [\alpha, q]^T$ where α is the angle of attack and q is the pitch rate. The plant input is the elevator angle δ_e, and the plant output is the pitch-rate plus $0.01u_k$ where the latter term was added in order to assure the ASPR property of the open-loop system. It should also be noted that the nonzero but small D_{22} is chosen which is no particular physical significance. The plant is described by continuous time state-space model for $N = 4$, where:

$$A = \begin{bmatrix} -Z_{\alpha_j} & 1 \\ -M_{\alpha_j} & 0 \end{bmatrix}, \; B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \text{and} \quad D_{22} = 0.01$$

$$D_{11} = 0, D_{21} = 0.1, D_{12} = 0.01$$

and where the parameters of the four vertices are $Z_\alpha \in \{0.5, 0.5, 4, 4\}$ and $M_\alpha \in \{6, 106, 6, 106\}$.

We note that the open-loop system is not SPR throughout the convex combinations of these vertices and that the first and third vertices correspond to plants that are not even asymptotically stable. Using (29) and Matlab's LMI Toolbox, we find that the plant is MP at all the four vertices where a single

$$P = \begin{bmatrix} 0.2196 & 0.0014 \\ 0.0014 & 0.0039 \end{bmatrix} \tag{31}$$

was used to verify $(A - BD^{-1}C)^T P + P(A - BD^{-1}C) < 0$ at all vertices (or, equivalently, verifying the quadratic stability of the zero dynamics of the plant). Our aim is to regulate the states of this plant when

$$w(t) = \sin(3t)e^{-0.01t}.$$

Simulation results are given in Fig. 1. The initial conditions are $\alpha = 10°$ and $q = 5°/sec$. Fig. 1 describes the states versus time at all four operating points. Evidently, all the states are regulated to zero by the proposed control law (8) and gain adaptation formula (10).

Fig. 2 describes the momentary minimum disturbance attenuation level γ as a function of the (scalar, in our example) gain $\widehat{K}_e$. Note that since $D_{22} = 0.01$, we obtain (using (11))

$$0 < \widehat{K}_e \leq 100.$$

It can be seen from Fig. 2 that γ sharply grows at low adaptive gain ($\widehat{K}_e < 15$) and at high adaptive gain. In fact, γ tends to infinity when $\widehat{K}_e$ tends to D_{22}^{-1}. The best γ is 0.82, and is achieved when $\widehat{K}_e = 23$. Thus, if good H_∞ performance is desired, for example $\gamma < 2$, one may adopt the practice of initializing the gain adaptation by $\widehat{K}(t) \approx 15$ and limit it to $\widehat{K}(t) < 87$.

6. CONCLUSIONS

In this paper the existing theory of Simplified Adaptive Control has been generalized to systems with uncertainties and H_∞ disturbance attenuation requirements. The results assure closed-loop stability and some disturbance attenuation level γ under the requirement of Almost-strictly-positive-realness of the systems (or, equivalently, minimum phase). A similar condition, which is verified using Linear Matrix Inequalities, is shown to be valid also for system with polytopic uncertainties.

This results are illustrated via an example, taken from the field of flight control. This results encourage further research, such as simplified adaptive control with exogenous disturbance and measurement noise for discrete-time systems.

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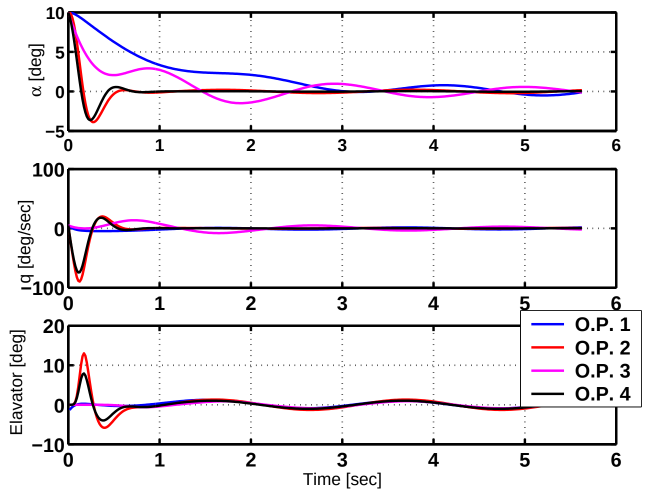

Fig. 1. Simulation Results at the 4 vertices

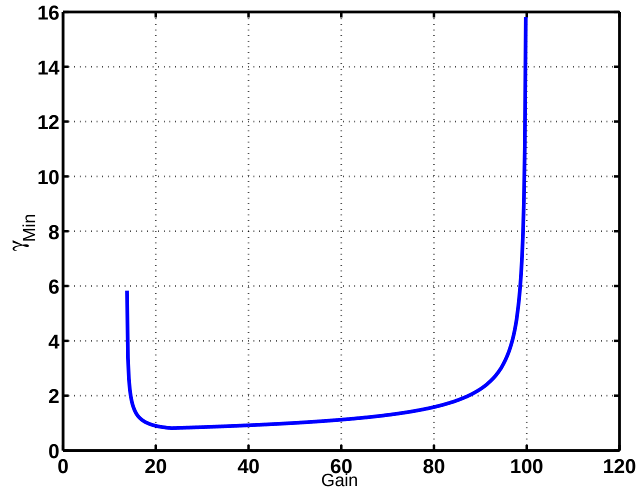

Fig. 2. Minimum disturbance attenuation level γ as function of the gain $\widehat{K}_e$