ON AN APPLICATION OF THE FILTERING THEORY FOR DELAYED DYNAMIC SYSTEMS TO PLANTS WITHOUT DELAY

A.I. Matasov

Faculty of Mechanics and Mathematics, Moscow Lomonosov State University
Leninskie Gory 1, Main MSU Building
Moscow, GSP-2, 119992, Russia;
Institute for Space Research, Russian Academy of Sciences
Profsoyuznaya ul., 84/32
Moscow, GSP-7, 117997, Russia
E-mail: amatasov@iki.rssi.ru

In various dynamic systems, the future state of a process depends not only on a current state but also on the past states of the process. Delayed equations describe a wide class of automatic systems; they are used for modeling mechanical processes which contain pneumatic and hydraulic circuits, the motion of a solid in liquid media, etc. (see, e.g. [1, 2, 3]). The optimal filtering problem for linear delayed dynamic systems was solved in [4, 5] by Kolmanovskii and Maisenberg under the assumption that the initial state of the system is zero almost everywhere. However, this assumption does not always hold. In the present work, the result of [4, 5] is extended to the case of *nonzero* initial conditions. Note that this extension does not follow directly from [4, 5]. The central part of the solution is the same system of functional differential equations as in [4, 5] but with another boundary conditions. The main result is proved by a method developed in [8], which differs from that used in [4, 5]; it is convenient for a further analysis of variational problems. The generalization obtained in this work renders the classical result of [4, 5] more final. For an approximate solution of the estimation problem, the influence of the initial conditions was studied in [6, 7].

Consider a classical filtering problem for a continuous systems without delay but with arbitrarily correlated plant noise (with a given correlation function) and continuous measurements. For the case of discrete measurements, this problem was investigated by Nazarenko and Markova in [9] with the help of Bayes approach, i.e. by the maximization of conditional density. We construct an auxiliary delayed system where the delay is equal to the length of the observation time and the term with delay is equal to the term with correlated plant noise. Then it turns out that the filtering problem with correlated plant noise is a special case of the corresponding estimation problem for delayed systems. It can be shown that the solution of the filtering problem with correlated plant noise is defined by a Kalman-type dynamic filter with an additional integral term. The gains in this filter are calculated via the solution of a system of functional differential equations that generalize the Riccati equation. Since the plant noise may have an arbitrary correlation function, our problem statement allows us to consider the plant noises that are modeled by *nonlinear* transformations of stochastic processes.

Note that, as early as 1967, Kwakernaak [10] studied an even more general setting of the estimation problem. Along with nonzero initial conditions, he investigated the smoothing problem for several constant delays. Using the Wiener-Hopf equation Kwakernaak obtained a similar system of functional differential covariance equations and a similar optimal filter. However, in [10], the existence and uniqueness of solutions to the covariance equations was not considered, and the relationship between the problem under consideration and the filtering problem with arbitrarily correlated plant noise was not studied.

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