Successive Passage Of Atoms In A Cavity In Interacting Fock Space

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Abstract

After passing two V-type atoms successively through a single mode interacting field in a cavity we arrive at a state which has been analyzed to study nonclassicality of the evolved state of the system. In the process we plan to study Mandel's Q-parameter and normal squeezing of the resulting field.

Key words:

V-type three level atom, nonclassical states, Mandel's Q-parameter, squeezing.

1 Introduction

A manifold of nonclassical features of quantum light delivered by interaction of an electromagnetic field with an atom is a central topic in quantum optics[9,10]. Jaynes-Cummings model helps us to understand interaction of a single atom with a high-quality cavity yielding many important results. Interaction of an atom and a laser beam in a cavity performed close to one of the atomic resonances leads to light emission from the atom with a rich set of spectral and temporal properties.Temporally,the light emitted will show antibunching with a second order correlation which has a minimum for zero time delay. Spectrally, with the increase of laser intensity light emitted will have symmetric side lobes around the central excitation frequency which is called Mollow triplet. By employing nonclassical light sources the performances of optical technology such as metrology, communication and imaging can be improved beyond the limitation of classical physics. Various schemes have been proposed in the context of cavity QED to generate Fock states and superpositions of Fock states using resonant interactions of two-level and three-level atoms one at a time with a cavity mode followed by measurement of the atomic states. The production of two-photon state has been reported recently by single atom in a high-Q cavity .

In another aspect, the preparation of quantum entangled states through cavity QED is a subject of intense theoretical and experimental studies. Studying of such states evokes insight into the fundamentals of quantum mechanics.They are also useful in quantum information processing. Manipulation of a light field at the single-photon level provides a basis for important applications in quantum information science. A desired field state can be obtained by two elementary operations on a single-mode field. For example, photon addition or subtraction is known to create a nonclassical state from any classical state and both the photon-subtracted[12] and photon-added squeezed states were suggested to improve fidelity of continuous variable teleportation.

In this paper, we consider an interacting one-mode field which interacts in a cavity with the atom by letting two V-type atoms successively passing through it. After tracing out the atomic parts from the generated atom-field system we get the field left in the cavity and explore the nonclassical properties of the field.

In the beginning, we describe the basic idea of one-mode interacting Fock space[1-8]. Then we give the time-dependent state of the system containing a V-type three-level atom[9,10] which interacts with a single mode of interacting field successively . In subsequent sections we show nonclassicality of the evolved state with the help of Mandel's Q paarameter and the initial coherent state loses its coherence and become a squeezed state due to interaction of field and successive passage of atom in the cavity. Lastly, we give a conclusion.

2 Basic preliminaries and notations

As a vector space [1] one mode interacting Fock space $\Gamma(\mathbb{C})$ is defined by

$$\Gamma(\mathbb{C}) = \bigoplus_{n=0}^{\infty} \mathbb{C}|n\rangle \tag{1}$$

for any $n \in \mathbb{N}$ where $\mathbb{C}|n\rangle$ is called the n-particle subspace. The different n-particle subspaces are orthogonal, that is, the sum in (1) is orthogonal. The square of the seminorm of the vector $|n\rangle$ is given by

$$\langle n|n\rangle = \lambda_n \tag{2}$$

where $\lambda_n \geq 0$ for each $n \in I\!\!N$ and if for some n we have $\lambda_n = 0$, then $\lambda_m = 0$ for all $m \geq n$. After taking quotient, the seminorm in (2) becomes a norm which makes $\Gamma(\mathbb{C})$ a pre-Hilbert space.In the following we will consider its completion, which, with an abouse of notation, will be denoted by $\Gamma(\mathbb{C})$.

An arbitrary vector f in $\Gamma(\mathbb{C})$ is given by

$$f \equiv c_0|0\rangle + c_1|1\rangle + c_2|2\rangle + \ldots + c_n|n\rangle + \ldots \tag{3}$$

for any $n \in I\!\!N$ with $\|f\| = (\sum_{n=0}^{\infty} |c_n|^2\lambda_n)^{1/2} < \infty$.

We now consider the following actions on $\Gamma(\mathbb{C})$:

$$\begin{aligned} A^\dagger|n\rangle &= |n+1\rangle \\ A|n+1\rangle &= \frac{\lambda_{n+1}}{\lambda_n}|n\rangle \end{aligned} \tag{4}$$

$A^\dagger$ is called the *creation operator* and its adjoint A is called the *annihilation operator*.

The commutation relation takes the form

$$[A, A^\dagger] = \frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} \tag{5}$$

where N is the number operator defined by $N|n\rangle = n|n\rangle$.

In a recent paper [6] we have proved that the set $\{\frac{|n\rangle}{\sqrt{\lambda_n}}, n = 0, 1, 2, 3, \ldots\}$ forms a complete orthonormal set and the solution of the following eigenvalue equation

$$Af_\alpha = \alpha f_\alpha \tag{6}$$

is given by

$$f_\alpha = \psi(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\lambda_n}|n\rangle \tag{7}$$

where $\psi(|\alpha|^2) = \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{\lambda_n}$. We call f_α a **coherent vector** in $\Gamma(\mathbb{C})$.

Now, we observe that

$$AA^\dagger = \frac{\lambda_{N+1}}{\lambda_N}\, ,\, A^\dagger A = \frac{\lambda_N}{\lambda_{N-1}}$$

We further observe that $(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}})$ commutes with both $A^\dagger A$ and $AA^\dagger$.

3 Time Evolution of State Vector

The scheme of the **V-type** three-level atomic system consists of two allowed transitions

$$|a\rangle \leftrightarrow |c\rangle \text{and } |b\rangle \leftrightarrow |c\rangle$$

where $|a\rangle, |b\rangle$ and $|c\rangle$ are excited state, intermediate state and ground state respectively. Each interaction has a different mode of the field. In the rotating-wave approximation, its Hamiltonian is described by

$$H = H_0 + H_1, \tag{8}$$

where

$$H_0 = \omega_a|a\rangle\langle a| + \omega_b|b\rangle\langle b| + \omega_c|c\rangle\langle c| + \gamma A^\dagger A \;(\hbar = 1), \tag{9}$$

and

$$H_1 = g_1 A|a\rangle\langle c| + g_1 A^\dagger|c\rangle\langle a| + g_2 A|b\rangle\langle c| + g_2 A^\dagger|c\rangle\langle b|. \tag{10}$$

Here $A^\dagger$ and A are, respectively, the creation and annihilation operators for the field of frequency γ. $|i\rangle (i = a, b, c)$ is the eigenstate of the atom with eigenfrequency ω_i, and g is the corresponding coupling constant. We assume the coupling constants to be real throughout the paper.

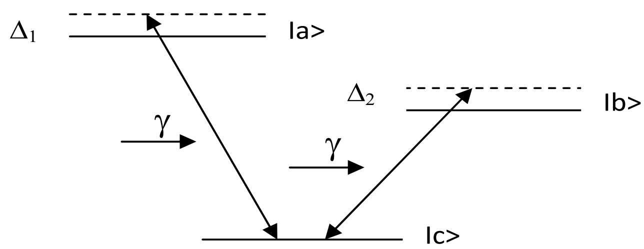

Figure 1: Energy diagram of a vee-configuration three-level atom interacting with one quantized cavity mode.

In the interaction picture, the state vector of this atom-field coupling system at time t can be described by

$$|\psi(t)\rangle = \sum_n (C_{a,n}|a, \frac{n}{\sqrt{\lambda_n}}\rangle + C_{b,n}|b, \frac{n}{\sqrt{\lambda_n}}\rangle + C_{c,n}|c, \frac{n}{\sqrt{\lambda_n}}\rangle). \tag{11}$$

The Hamiltonian in the interaction picture is given by

$$\begin{aligned} V \;=\; & g_1 e^{i\triangle_1 t} A|a\rangle\langle c| + g_1 A^\dagger e^{-i\triangle_1 t}|c\rangle\langle a| \\ & + g_2 e^{i\triangle_2 t} A|b\rangle\langle c| + g_2 A^\dagger e^{-i\triangle_2 t}|c\rangle\langle b|. \end{aligned} \tag{12}$$

where

$$\Delta_1 = (\omega_a - \omega_c - \gamma(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}))$$

and

$$\Delta_2 = (\omega_b - \omega_c - \gamma(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}})).$$

On solving the Schrödinger equation we get the equations of motion for probability amplitudes as

$$\dot{C}_{a,n-1} = -ig_1\sqrt{\frac{\lambda_n}{\lambda_{n-1}}} e^{i\triangle' t} C_{c,n} \tag{13}$$

$$\dot{C}_{b,n-1} = -ig_2\sqrt{\frac{\lambda_n}{\lambda_{n-1}}} e^{i\triangle' t} C_{c,n} \tag{14}$$

$$\begin{aligned} \dot{C}_{c,n} &= (-ig_1)\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}e^{-i\triangle' t}C_{a,n-1} \\ &\quad +(-ig_2)\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}e^{-i\triangle' t}C_{b,n-1} \end{aligned} \tag{15}$$

where we assume

$$\begin{aligned} \Delta' &\equiv (\omega_a-\omega_c-\gamma(\tfrac{\lambda_{n+1}}{\lambda_n}-\tfrac{\lambda_n}{\lambda_{n-1}})) \\ &= (\omega_b-\omega_c-\gamma(\tfrac{\lambda_{n+1}}{\lambda_n}-\tfrac{\lambda_n}{\lambda_{n-1}})) \end{aligned} \tag{16}$$

If the atom is initially in the state $|\psi_A(0)\rangle$,

$$|\psi_A(0)\rangle = \cos\frac{\alpha}{2}|a\rangle + \sin\frac{\alpha}{2}e^{-i\psi}|b\rangle \tag{17}$$

which means that the atom is in the coherent superposition state of its eigenkets $|a\rangle$ and $|b\rangle$, and the field is in the superposition of the photon number states at time $t=0$

$$|\psi_f(0)\rangle = \sum_n F_n|\frac{n}{\sqrt{\lambda_n}}\rangle, \tag{18}$$

where $\sum_n |F_n|^2 = 1$, then the state vector of the total system at $t=0$ can be described as

$$\begin{aligned} |\psi(0)\rangle &= \textstyle\sum_n[\cos\frac{\alpha}{2}F_{n-1}|a,\frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle \\ &\quad +\sin\frac{\alpha}{2}e^{-i\psi}F_{n-1}|b,\frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle \end{aligned} \tag{19}$$

With this initial condition we get

$$C_{c,n}(t) = B_1\{e^{-i(\triangle'/2+\beta)t} - e^{-i(\triangle'/2-\beta)t}\} \tag{20}$$

where B_1 is given by

$$\begin{aligned} B_1 &= (g_1\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}\cos\tfrac{\alpha}{2}F_{n-1} \\ &\quad +g_2\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}\sin\tfrac{\alpha}{2}e^{-i\psi}F_{n-1})/2\beta \end{aligned} \tag{21}$$

Similarly we get

$$\begin{aligned} &C_{a,n-1}(t) \\ &= -g_1\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}B_1[\tfrac{e^{i(\Delta'/2+\beta)t}-1}{(\Delta'/2+\beta)} - \tfrac{e^{i(\Delta'/2-\beta)t}-1}{(\Delta'/2-\beta)}] \\ &\quad +\cos\tfrac{\alpha}{2}F_{n-1} \end{aligned} \tag{22}$$

and

$$\begin{aligned} &C_{b,n-1}(t) \\ &= -g_2\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}B_1[\tfrac{e^{i(\Delta'/2+\beta)t}-1}{(\Delta'/2+\beta)} - \tfrac{e^{i(\Delta'/2-\beta)t}-1}{(\Delta'/2-\beta)}] \\ &\quad +\sin\tfrac{\alpha}{2}e^{-i\psi}F_{n-1} \end{aligned} \tag{23}$$

Substituting the values of $C_{c,n}(t)$, $C_{a,n-1}(t)$ and $C_{b,n-1}(t)$ from (20), (22) and (23) respectively in equation (11) we can obtain the state vector of the system at time t in the interaction picture.

At this stage we assume that

$$\alpha = 90^0 \text{ and } \psi = 0.\text{Also } F_n \approx F_{n-1}.$$

This reduces the state vector (11) with

$$C_{c,n}(t) = B_1\{e^{-i(\triangle'/2+\beta)t} - e^{-i(\triangle'/2-\beta)t}\} \tag{24}$$

$$\begin{aligned} &C_{a,n-1}(t) \\ &= -g_1\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}B_1[\tfrac{e^{i(\Delta'/2+\beta)t}-1}{(\Delta'/2+\beta)} - \tfrac{e^{i(\Delta'/2-\beta)t}-1}{(\Delta'/2-\beta)}] \\ &\quad +\tfrac{1}{\sqrt{2}}F_n \end{aligned} \tag{25}$$

$$\begin{aligned} &C_{b,n-1}(t) \\ &= -g_2\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}B_1[\tfrac{e^{i(\Delta'/2+\beta)t}-1}{(\Delta'/2+\beta)} - \tfrac{e^{i(\Delta'/2-\beta)t}-1}{(\Delta'/2-\beta)}] \\ &\quad +\tfrac{1}{\sqrt{2}}F_n \end{aligned} \tag{26}$$

with

$$\begin{aligned} B_1 &= (g_1\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}\tfrac{F_n}{\sqrt{2}} + g_2\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}\tfrac{F_n}{\sqrt{2}})/2\beta \\ &= \frac{\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}F_n(g_1+g_2)}{2\sqrt{2}\beta} \end{aligned} \tag{27}$$

We assume that the atom enters the cavity with the initial state

$$|\psi(0)\rangle = \sum_n \frac{1}{\sqrt{2}}F_n(|a,\frac{n}{\sqrt{\lambda_n}}\rangle + |b,\frac{n}{\sqrt{\lambda_n}}\rangle) \tag{28}$$

and after the evolution, for time t_1, the state vector of the considered atom-field system becomes

$$\begin{aligned} &|\psi(t_1)\rangle \\ &= \textstyle\sum_n(C_{a,n-1}(t_1)|a,\frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle + C_{b,n-1}(t_1)|b,\frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle \\ &\quad +C_{c,n}(t_1)|c,\frac{n}{\sqrt{\lambda_n}}\rangle). \end{aligned} \tag{29}$$

where from (24), (25) and (26) we have

$$C_{c,n}(t_1) = B_1\{e^{-i(\triangle'/2+\beta)t_1} - e^{-i(\triangle'/2-\beta)t_1}\} \tag{30}$$

$$\begin{aligned} &C_{a,n-1}(t_1) \\ &= -g_1\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}B_1[\tfrac{e^{i(\Delta'/2+\beta)t_1}-1}{(\Delta'/2+\beta)} - \tfrac{e^{i(\Delta'/2-\beta)t_1}-1}{(\Delta'/2-\beta)}] \\ &\quad +\tfrac{1}{\sqrt{2}}F_n \end{aligned} \tag{31}$$

$$\begin{aligned} &C_{b,n-1}(t_1) \\ &= -g_2\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}B_1[\tfrac{e^{i(\Delta'/2+\beta)t_1}-1}{(\Delta'/2+\beta)} - \tfrac{e^{i(\Delta'/2-\beta)t_1}-1}{(\Delta'/2-\beta)}] \\ &\quad +\tfrac{1}{\sqrt{2}}F_n \end{aligned} \tag{32}$$

with

$$\begin{aligned} B_1 &= (g_1\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}\tfrac{F_n}{\sqrt{2}} + g_2\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}\tfrac{F_n}{\sqrt{2}})/2\beta \\ &= \frac{\sqrt{\tfrac{\lambda_n}{\lambda_{n-1}}}F_n(g_1+g_2)}{2\sqrt{2}\beta} \end{aligned} \tag{33}$$

and

$$\beta^2 = \Delta'^2/4 + (g_1^2+g_2^2)\frac{\lambda_n}{\lambda_{n-1}}. \tag{34}$$

Now after the interaction with the field if we detect the atom in the ground state $|c\rangle$ after time t_1 then effectively atom absorbs no photon but projects the cavity field into the state

$$|\psi_1\rangle = \frac{1}{\eta}\sum_n C_{c,n}(t_1)|\frac{n}{\sqrt{\lambda_n}}\rangle \tag{35}$$

where,from (30)

$$C_{c,n}(t_1) = -2B_1 i e^{-i\frac{\triangle' t_1}{2}} \sin\beta t_1 \tag{36}$$

with B_1 given by (33).

If we now consider the passage of a second identical atom through the cavity[6, 7], then the field inside the cavity becomes

$$|\psi(t)\rangle = \sum_n (D_{a,n}|a, \frac{n}{\sqrt{\lambda_n}}\rangle + D_{b,n}|b, \frac{n}{\sqrt{\lambda_n}}\rangle + D_{c,n}|c, \frac{n}{\sqrt{\lambda_n}}\rangle). \tag{37}$$

On solving, as in the previous case, we see that the second identical atom transits through the cavity for time t_2 and for $g_1 = g_2 = g$ with zero detuning, the system evolves to

$$\begin{aligned} & |\psi(t_2)\rangle \\ = \; & \textstyle\sum_n (D_{a,n-1}(t_2)|e, \frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle + D_{b,n-1}(t_2)|i, \frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle \\ & +D_{c,n}(t_2)|c, \frac{n}{\sqrt{\lambda_n}}\rangle) \end{aligned} \tag{38}$$

where

$$D_{c,n}(t_2) = -\frac{F_n}{\eta} \sin\sqrt{2\frac{\lambda_n}{\lambda_{n-1}}} g t_1 \sin\sqrt{2\frac{\lambda_n}{\lambda_{n-1}}} g t_2 \tag{39}$$

$$D_{a,n-1}(t_2) = \frac{1}{\eta\sqrt{2}} C_{c,n}(t_1) \cos\sqrt{2\frac{\lambda_n}{\lambda_{n-1}}} g t_2 \tag{40}$$

$$D_{b,n-1}(t_2) = \frac{1}{\eta\sqrt{2}} C_{c,n}(t_1) \cos\sqrt{2\frac{\lambda_n}{\lambda_{n-1}}} g t_2 \tag{41}$$

Now, we assume $\Delta' = 0$ to get

$$C_{c,n}(t_1) = -iF_n \sin\beta t_1 \tag{42}$$

From (39), (40), (41) and (42) we now have

$$D_{c,n}(t_2) = -\frac{F_n}{\eta} \sin\beta t_1 \sin\beta t_2 \tag{43}$$

$$D_{a,n-1}(t_2) = -\frac{i}{\eta\sqrt{2}} F_n \sin\beta t_1 \cos\beta t_2 \tag{44}$$

$$D_{b,n-1}(t_2) = -\frac{i}{\eta\sqrt{2}} F_n \sin\beta t_1 \cos\beta t_2 \tag{45}$$

The state vector $|\psi(t_2)\rangle$ (38) describes the time evolution of the whole atom-field system but we now concentrate on some statistical properties of the single-mode field. The field inside the cavity after departing the second atom is obtained by tracing out the atomic part of $\rho(t_2) = |\psi(t_2)\rangle\langle\psi(t_2)|$ as

$$\rho_f(t_2) = Tr_a[\rho(t_2)], \tag{46}$$

where we have used the subscript $a(f)$ to denote the atom(field).

This $\rho_f(t_2)$ will be of consideration throughout the next section to determine the statistical properties of the field left into the cavity.

Now, from (38), we get

$$\begin{aligned} & \rho(t_2) \\ = \; & |\psi(t_2)\rangle\langle\psi(t_2)| \\ = \; & \textstyle\sum_{m,n} \{ D_{a,n-1}(t_2)\bar{D}_{a,m-1}(t_2)|a, \frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle\langle a, \frac{m-1}{\sqrt{\lambda_{m-1}}}| \\ & +D_{b,n-1}(t_2)\bar{D}_{b,m-1}(t_2)|b, \frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle\langle b, \frac{m-1}{\sqrt{\lambda_{m-1}}}| \\ & +D_{c,n}(t_2)\bar{D}_{c,m}(t_2)|c, \frac{n}{\sqrt{\lambda_n}}\rangle\langle c, \frac{m}{\sqrt{\lambda_m}}| \\ & +D_{a,n-1}(t_2)\bar{D}_{b,m-1}(t_2)|a, \frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle\langle b, \frac{m-1}{\sqrt{\lambda_{m-1}}}| \\ & +D_{a,n-1}(t_2)\bar{D}_{c,m}(t_2)|a, \frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle\langle c, \frac{m}{\sqrt{\lambda_m}}| \\ & +D_{b,n-1}(t_2)\bar{D}_{a,m-1}(t_2)|b, \frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle\langle a, \frac{m-1}{\sqrt{\lambda_{m-1}}}| \\ & +D_{b,n-1}(t_2)\bar{D}_{c,m}(t_2)|b, \frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle\langle c, \frac{m}{\sqrt{\lambda_m}}| \\ & +D_{c,n}(t_2)\bar{D}_{a,m-1}(t_2)|c, \frac{n}{\sqrt{\lambda_n}}\rangle\langle a, \frac{m-1}{\sqrt{\lambda_{m-1}}}| \\ & +D_{c,n}(t_2)\bar{D}_{b,m-1}(t_2)|c, \frac{n}{\sqrt{\lambda_n}}\rangle\langle b, \frac{m-1}{\sqrt{\lambda_{m-1}}}| \} \end{aligned} \tag{47}$$

From (47) we have

$$\begin{aligned} & \rho_f(t_2) \\ = \; & Tr_a[\rho(t_2)] \\ = \; & \textstyle\sum_{m,n} [D_{a,n-1}(t_2)\bar{D}_{a,m-1}(t_2)|\frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle\langle\frac{m-1}{\sqrt{\lambda_{m-1}}}| \\ & +D_{b,n-1}(t_2)\bar{D}_{b,m-1}(t_2)|\frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle\langle\frac{m-1}{\sqrt{\lambda_{m-1}}}| \\ & +D_{c,n}(t_2)\bar{D}_{c,m}(t_2)|\frac{n}{\sqrt{\lambda_n}}\rangle\langle\frac{m}{\sqrt{\lambda_m}}|] \end{aligned} \tag{48}$$

4 Statistical properties of the radiation field

In this section we investigate two nonclassical effects, namely, sub-Poissonian photon statistics and quadrature squeezing.

4.1 Sub-Poissonian photon statistics

The simplest criterion [11] for a single-mode radiation to be a nonclassical state is

$$Q^M \equiv \frac{\langle n^{(2)}\rangle}{\langle n\rangle} - \langle n\rangle < 0. \tag{49}$$

where

$$\langle n^{(2)}\rangle = \langle\psi|A^\dagger A^\dagger A A|\psi\rangle \tag{50}$$

and

$$\langle n\rangle = \langle\psi|A^\dagger A|\psi\rangle \tag{51}$$

Before we proceed for the calculation proper we obtain the following results:

$$A^\dagger A|\frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle = \frac{\lambda_{n-1}}{\lambda_{n-2}}|\frac{n-1}{\sqrt{\lambda_{n-1}}}\rangle \tag{52}$$

and

$$A^\dagger A|\frac{n}{\sqrt{\lambda_n}}\rangle = \frac{\lambda_n}{\lambda_{n-1}}|\frac{n}{\sqrt{\lambda_n}}\rangle \tag{53}$$

From (52) and (53) we now have

$$\langle A^\dagger A\rangle = \sum_n 2D_{a,n-1}(t_2)\bar{D}_{a,n-1}(t_2)\tfrac{\lambda_{n-1}}{\lambda_{n-2}} + \sum_n D_{c,n}(t_2)\bar{D}_{c,n}(t_2)\tfrac{\lambda_n}{\lambda_{n-1}} \quad (54)$$

and

$$\langle A^\dagger A^\dagger AA\rangle = \sum_n 2D_{a,n-1}(t_2)\bar{D}_{a,n-1}(t_2)\tfrac{\lambda_{n-1}}{\lambda_{n-3}} + \sum_n D_{c,n}(t_2)\bar{D}_{c,n}(t_2)\tfrac{\lambda_n}{\lambda_{n-2}} \quad (55)$$

Incorporating calculations (54) and (55) we get

$$Q^M = \tfrac{A+B}{C+D} - [\sum_n \tfrac{1}{\eta^2}|F_n|^2 \sin^2\beta t_1 \cos^2\beta t_2 \tfrac{\lambda_{n-1}}{\lambda_{n-2}} + \sum_n \tfrac{1}{\eta^2}|F_n|^2 \sin^2\beta t_1 \sin^2\beta t_2 \tfrac{\lambda_n}{\lambda_{n-1}}] \quad (56)$$

where

$$A = \sum_n \frac{1}{\eta^2}|F_n|^2 \sin^2\beta t_1 \cos^2\beta t_2 \frac{\lambda_{n-1}}{\lambda_{n-3}}$$

$$B = \sum_n \frac{1}{\eta^2}|F_n|^2 \sin^2\beta t_1 \sin^2\beta t_2 \frac{\lambda_n}{\lambda_{n-2}}$$

$$C = \sum_n \frac{1}{\eta^2}|F_n|^2 \sin^2\beta t_1 \cos^2\beta t_2 \frac{\lambda_{n-1}}{\lambda_{n-2}}$$

$$D = \sum_n \frac{1}{\eta^2}|F_n|^2 \sin^2\beta t_1 \sin^2\beta t_2 \frac{\lambda_n}{\lambda_{n-1}}$$

If the radiation field [10] is initially in a coherent state, then

$$F_n(0) = \exp(-\bar{n}/2)\frac{\bar{n}^{n/2}e^{i\zeta n}}{\sqrt{n!}}, \quad (57)$$

Hence

$$|F_n(0)|^2 = \exp(-\bar{n})\frac{\bar{n}^n}{n!}, \quad (58)$$

Substituting the value of $|F_n(0)|^2$ from (58) and assuming

$$\beta t_1 \equiv \theta_1; \beta t_2 \equiv \theta_2$$

and finally taking $t_1 = t_2 = t$ so that $\theta_1 = \theta_2 = \theta$ with

$$\theta = \sqrt{2\frac{\lambda_n}{\lambda_{n-1}}}gt$$

we get

$$Q^M = \tfrac{A'+B'}{C'+D'} - \tfrac{1}{\eta^2}[\sum_n \tfrac{\lambda_{n-1}}{\lambda_{n-2}}\exp(-\bar{n})\tfrac{\bar{n}^n}{n!}\sin^2\theta\cos^2\theta + \sum_n \tfrac{\lambda_n}{\lambda_{n-1}}\exp(-\bar{n})\tfrac{\bar{n}^n}{n!}\sin^2\theta\sin^2\theta] \quad (59)$$

where

$$A' = \sum_n \frac{\lambda_{n-1}}{\lambda_{n-3}}\exp(-\bar{n})\frac{\bar{n}^n}{n!}\sin^2\theta\cos^2\theta$$

$$B' = \sum_n \frac{\lambda_n}{\lambda_{n-2}}\exp(-\bar{n})\frac{\bar{n}^n}{n!}\sin^2\theta\sin^2\theta$$

$$C' = \sum_n \frac{\lambda_{n-1}}{\lambda_{n-2}}\exp(-\bar{n})\frac{\bar{n}^n}{n!}\sin^2\theta\cos^2\theta$$

$$D' = \sum_n \frac{\lambda_n}{\lambda_{n-1}}\exp(-\bar{n})\frac{\bar{n}^n}{n!}\sin^2\theta\sin^2\theta$$

To draw the graph of Q^M against gt we assume $gt = x, \eta = 1$ where we choose $\lambda_n \sim n!, (n!)^2$ and $[n]$. Here $[n] = (1-q^n)/(1-q), 0 < q < 1$.

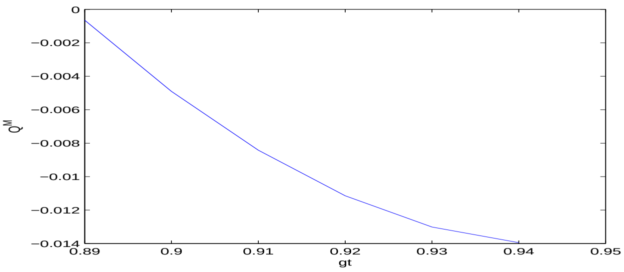

Figure 2: Mandel's Q^M as a function of gt for a coherent state input and $\lambda_n \sim n!$.

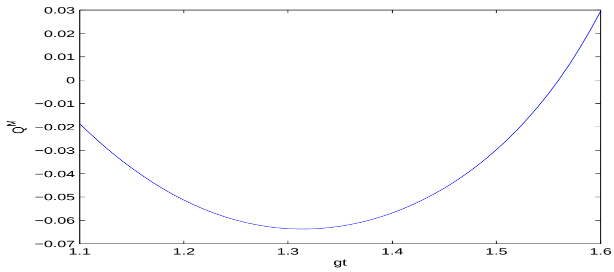

Figure 3: Mandel's Q^M as a function of gt for a coherent state input and $\lambda_n \sim (n!)^2$.

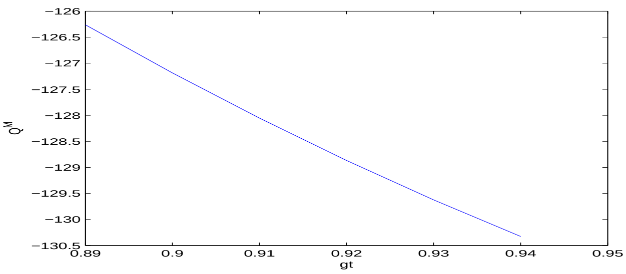

Figure 4: Mandel's Q^M as a function of gt for a coherent state input and $\lambda_n \sim [n]!$.

4.2 Squeezing properties of the radiation field

To analyze the squeezing properties of the radiation field[12,13] we introduce two hermitian quadrature operators

$$X = A + A^\dagger, \quad Y = -i(A - A^\dagger) \quad (60)$$

These two quadrature operators satisfy the commutation relation

$$[X,Y] = 2i(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}). \tag{61}$$

As a result the quadrature operators in (60) satisfy the uncertainty relation

$$\langle(\Delta X)^2\rangle\langle(\Delta Y)^2\rangle \geq (\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}})^2. \tag{62}$$

A state is said to be squeezed if either $\langle(\Delta X)^2\rangle$ or $\langle(\Delta Y)^2\rangle$ is less than $(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}})$.

To review the principle of squeezing, we define an appropriate quadrature operator

$$X_\theta = X\cos\theta + Y\sin\theta = Ae^{-i\theta} + A^\dagger e^{i\theta} \tag{63}$$

Then we get

$$\Delta X_\theta = X_\theta - \langle X_\theta\rangle = \Delta A e^{-i\theta} + \Delta A^\dagger e^{i\theta} \tag{64}$$

And hence

$$\begin{aligned}\langle(\Delta X_\theta)^2\rangle &= \langle(\Delta A)^2\rangle e^{-2i\theta} + \langle(\Delta A^\dagger)^2\rangle e^{2i\theta} \\ &\quad + \langle(\Delta A)(\Delta A^\dagger)\rangle + \langle(\Delta A^\dagger)(\Delta A)\rangle\end{aligned} \tag{65}$$

After some simplification we get

$$\begin{aligned}&\langle(\Delta X_\theta)^2\rangle \\ =\ &(\langle A^2\rangle - \langle A\rangle^2)e^{-2i\theta} + (\langle {A^\dagger}^2\rangle - \langle A^\dagger\rangle^2)e^{2i\theta} + \langle AA^\dagger\rangle \\ &+\langle AA^\dagger\rangle - \langle A\rangle\langle A^\dagger\rangle + \langle A^\dagger A\rangle - \langle A^\dagger\rangle\langle A\rangle\end{aligned} \tag{66}$$

From (66) we get, for operators in their normal orders,

$$\langle: (\Delta X_\theta)^2 :\rangle = \bar{\zeta}e^{-2i\theta} + \zeta e^{2i\theta} + 2\langle A^\dagger A\rangle - 2\langle A\rangle\langle A^\dagger\rangle \tag{67}$$

where $\zeta = \langle {A^\dagger}^2\rangle - \langle A^\dagger\rangle^2$. After observing that $\langle A\rangle = \overline{\langle A^\dagger\rangle}$ we get

$$\langle: (\Delta X_\theta)^2 :\rangle = \bar{\zeta}e^{-2i\theta} + \zeta e^{2i\theta} + 2\langle A^\dagger A\rangle - 2|\langle A^\dagger\rangle|^2 \tag{68}$$

To minimize (68) over whole angle θ we observe the following fact: we take $\zeta = |\zeta|e^{2i\eta}$ to obtain

$$\begin{aligned}[\bar{\zeta}e^{-2i\theta} + \zeta e^{2i\theta}]_{\min} &= [|\zeta|e^{-2i(\theta+\eta)} + |\zeta|e^{2i(\theta+\eta)}]_{\min} \\ &= [2|\zeta|\cos(2\theta + 2\eta)]_{\min} \\ &= -2|\zeta|\end{aligned} \tag{69}$$

Finally, from (68) and (69), we have

$$\begin{aligned}S_{opt} &= \langle: (\Delta X_\theta)^2 :\rangle_{\min} \\ &= -2|\langle {A^\dagger}^2\rangle - \langle A^\dagger\rangle^2| + 2\langle A^\dagger A\rangle - 2|\langle A^\dagger\rangle|^2\end{aligned} \tag{70}$$

We now calculate

$$\langle {A^\dagger}^2\rangle = \sum_n 2D_{e,n-1}\bar{D}_{e,n-1}\sqrt{\frac{\lambda_{n+1}}{\lambda_{n-1}}} + \sum_n D_{g,n}\bar{D}_{g,n}\sqrt{\frac{\lambda_{n+2}}{\lambda_n}} \tag{71}$$

and

$$\langle A^\dagger\rangle = 2\sum_n\sqrt{\frac{\lambda_n}{\lambda_{n-1}}}D_{e,n-1}\bar{D}_{e,n} + \sum_n\sqrt{\frac{\lambda_{n+1}}{\lambda_n}}D_{g,n+1}\bar{D}_{g,n} \tag{72}$$

For the problem under consideration, $\langle A^\dagger A\rangle$ has been derived in (54) and the other terms are given by (71) and (72). Substituting the above expectation values in equation (70) we obtain an expression of S_{opt} for initial coherent $F_n = e^{-|\alpha_0|^2/2}\frac{\alpha_0^n}{\sqrt{n!}}$. We draw the plot of S_{opt} as a function of $|\alpha_0|$ for different values of gt where we choose $\lambda_n \sim n!, (n!)^2$ and $[n]!$. Here $[n] = (1-q^n)/(1-q), 0 < q < 1$.

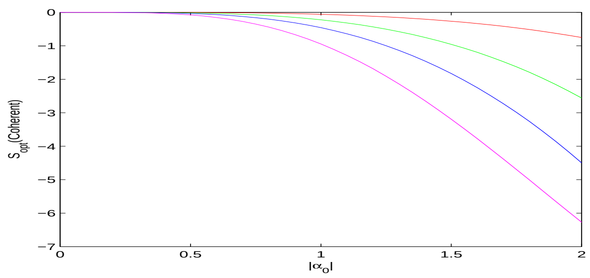

Figure 5: Plot for S_{opt} as a function of $|\alpha_0|$ with coherent state input and $\lambda_n \sim n!$.

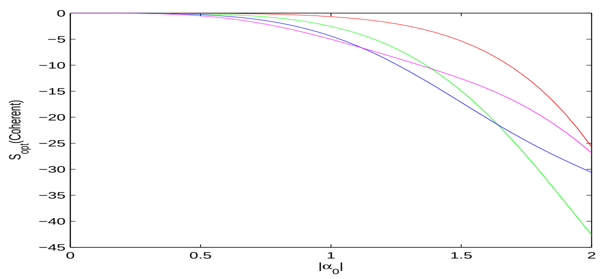

Figure 6: Plot for S_{opt} as a function of $|\alpha_0|$ with coherent state input and $\lambda_n \sim (n!)^2$.

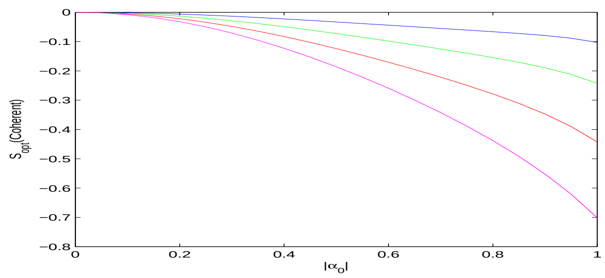

Figure 7: Plot for S_{opt} as a function of $|\alpha_0|$ with coherent state input and $\lambda_n \sim [n]!$.

5 Conclusion

We have thus investigated the effect of passage of a three-level atom one after another successively in a cavity containing an one-mode interacting field. In the process we observed a coherent state loses its coherence and the field after interaction becomes a nonclassical state which is evident from the study of Mandel's Q-parmeter and squeezing of the field inside the cavity. Mandel's parameter clearly shows a negative portion for $\lambda_n \sim n!, (n!)^2, [n]!$ which shows nonclassicality of the field inside the cavity. To further support the nonclassical nature of the field we observed the squeezing effect in the figure 5, figure 6 and figure 7. In figure 5 we see that maximum squeezing occurs for $gt = 0.5$ whereas in figure 6 the maximum squeezing occurs for $gt = 0.5$ and in figure 7 maximum squeezing occurs for $gt = 0.2$. Thus the successive injection of two three-level atom reduces the coherent field inside the cavity into a nonclassical state.

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