Iterative Methods of Ballistic Schemes Optimization of Interplanetary Mission with Low Thrust

O. L. Starinova *, V.V. Salmin *, K.V. Petrukhina*

**Samara State Aerospace University, Samara, Russia (e-mail: sputnik@ssaul.ru)*

Abstract: Researches of planets, interplanetary and circumsolar space give the chance to receive answers to many fundamental questions and to use astronautics achievements by working out of practically unlimited resources of Solar system. Programs of studying of space demand the big material inputs and do not give fast return. Use of perspective electro propulsion systems (EPS) allows to raise efficiency of the created and developed space technology. In the decision of a problem of increase scientific and economic efficiency of space researches the important role is played by the complex optimization of mission including definition of optimum structure and parameters of space vehicles (SV) systems, control optimum programs and movement trajectories corresponding to them. Given article is devoted this problem.

Keywords: Computer-aided method, nonlinear systems, modeling, spacecraft

1. STATEMENT OF OPTIMIZATION PROBLEM

Let's formulate the general statement of interplanetary mission optimization problem. Into consideration are entered: $\bar{x}(t)\in X$ - the vector of phase coordinates SV submitting to boundary conditions, defined by the purpose of flight and possible restrictions, where X - set of admissible phase coordinates; $\bar{u}(t)\in U$ - a vector of control functions on which components the restrictions connected with design features of the space vehicle and propulsion system (PS) where U - set of admissible controls are imposed; $\bar{p}\in P$ - the vector of optimised design parameters SV, is limited by set of admissible design parameters P; $\bar{b}\in B$ - the vector of ballistic parameters depending on the purpose of mission.

Depending on the chosen model of movement, the vector of phase co-ordinates of the space vehicle contains $\bar{x}(t)=(M,\bar{r},\bar{V},Rad(\bar{r}),\bar{r}_i)^{\mathrm{T}}$, where M – current mass of SV; $\bar{r}$ - radius-vector of SV in the chosen frame; $\bar{V}$ - a vector of SV speed; $Rad(\bar{r})$ - current intensity of radiating irradiation of SV; $\bar{r}_i$ - radius-vector of SV concerning others gravitating bodies.

The composition of control functions vector changes depending on the chosen movement model and criterion of an optimality $\bar{u}(t)=(\bar{e},\delta,\vartheta)^{\mathrm{T}}$, where $\bar{e}$ - the direction of operating acceleration defined by angles: λ_1 – between projection of acceleration and orbital plane, λ_2 – between acceleration and orbital plane; δ – function of switching; ϑ - a angle between a normal to a plane solar sails (SS) and a direction on the Sun.

The vector of design parameters SV depending on chosen design model SV can contain components $\bar{p}=(a_0,\ c_0,\ \alpha_i,\ \gamma_i,\ \chi(t,\bar{r},\bar{e}))^{\mathrm{T}}$, where a_0 – nominal acceleration of propulsion system; c_0 – rated speed of the expiration of a propellant; α_i, γ_i – specific mass characteristics of components SV on thrust and power; $\chi(t,\bar{r},\bar{e})$ – function of change of propulsion system thrust depending on phase co-ordinates of system and the chosen control.

The composition of parameters vector of the flight ballistic scheme depends on the purpose and the ballistic scheme of mission $\bar{b}=(D_0,T_i,\Delta\bar{V}_i)^T$, where D_0 – date of mission start; T_i – duration of parts of the flight, defining position of planets of appointment, finish and intermediate gravitational maneuvers; $\Delta\bar{V}_i$ – hyperbolic excesses of speed on borders of movement parts, an increment of speeds during the moments of gravitational maneuvers or engines inclusions of the high thrust.

By optimization of interplanetary missions with low thrust following an optimality criteria of mission are used: the minimum starting mass M_0 of SV, the maximum mass of a payload M_P and the minimum duration of mission (T for SV with the SS). In the first case the problem of design-ballistic optimization of mission is formulated as follows. To define vector $\bar{p}\in P$, $\bar{u}(t)\in U$, and $\bar{b}\in$B, delivering minimum M_0 and providing the mission performance purposes $\bar{x}\in X$ at set M_P and mission duration T :

$$M_0=\min_{\bar{u}(t)\in U,\,\bar{p}\in P,\,\bar{b}\in B} M_0\left(M_P,T=fixe,\bar{x}\in X,\bar{u}(t),\bar{p},\bar{b}\right). \qquad (1)$$

The symmetric statement of the problem providing a maximum of payload at the fixed starting mass of SV and mission duration, looks like:

$$M_P=\max_{\bar{u}(t)\in U,\,\bar{p}\in P,\,\bar{b}\in B} M_P\left(M_0,T=fixe,\bar{x}\in X,\bar{u}(t),\bar{p},\bar{b}\right). \qquad (2)$$

For optimization of the missions which are carried out SV from the SS, statement of a problem of optimization will become :

$$T = \min_{\bar{u}(t)\in U,\, \bar{p}\in P,\, \bar{b}\in B} T\left(M_0 = fixe, \bar{x} \in X, \bar{u}(t), \bar{p}, \bar{b}\right). \quad (3)$$

The most essential simplification of problems (1 – 3) is division of a problem of optimization into design and ballistic parts. Ballistic optimization of mission consists in definition of optimum vectors of control $\bar{u}_{opt}(t) \in U$ and parameters of the ballistic scheme of mission $\bar{b}_{opt} \in B$, delivering an extremum to optimality criterion at the fixed design parameters. Design optimization of mission consists in a choice of the optimum design parameters SV $\bar{p} \in P$ delivering an extremum to optimality criterion. Dependences $\bar{u}_{opt}(t, \bar{p}), \bar{b}_{opt}(\bar{p})$ received during ballistic optimization are thus used.

Generally in statements (1 - 3) the decision of a dynamic problem does not possess invariancy in relation to design parameters SV and consequently strict division into design and ballistic parts is impossible.

For the SV with EPS power of energy source and thrust of PS depend on phase coordinates (distance of SV from the Sun, a angle of rotation and possible shading of solar batteries, work duration of a nuclear reactor, etc.). As intermediate criterion of an optimality it is convenient to use the resulted operating time of engines $T_\mu^*(T) = \int_{t_0}^{T} \chi(\bar{x})\delta dt$ where $\chi(\bar{x}) = \beta(\bar{x})/\beta_0$ - defines dependence of propellant second expense $\beta(\bar{x})$ on phase coordinates. With using these designations expression for mass required propellant becomes M_R (Salmin et al., 2006) :

$$M_R(T) = \int_{t_0}^{T} \beta(\bar{x})\delta\ dt = \frac{P_0}{c_0}\int_{t_0}^{T} \chi(\bar{x})\delta\ dt = \frac{P_0}{c_0}\cdot T_\mu^*(T). \quad (4)$$

The function form depends $\chi(\bar{x})$ on used models of functioning of system energy source. For SV with solar energy source usually consider $\chi(\bar{x}) = r^{-k}$, $k \approx 1{,}7 \ldots 2$ and for nuclear system energy source it is possible to consider $\chi(\bar{x}) \equiv 1$.

Problems of mission design-ballistic optimization can be decided with various degree of accuracy depending on the chosen movement models and design shape SV. The following approach to a choice of models is offered. At first the problem is formulated in most general statement taking into account a full set of communications and the restrictions adequately describing design shape SV and physical features of its movement. Then some restrictions and the communications are eliminated, the division of a problem described above into design and ballistic parts, decomposition of a trajectory into parts according to the theory of action spheres, etc. As a result formed sequence of specified models of design shape and controlled movement of the SV.

2. MODELLING AND OPTIMIZATION METHODS

If the considered ballistic scheme of flight provides realisation planetocentric maneuvers of a parabolic speed increase, braking and formation of a operating orbit with engines of low thrust for calculation of planetocentric movement parts are used the models resulted in the Tab. 1.

For the description planetocentric movements SV with EPS or the SS without perturbations is used model M 2.i. Movement of SV is described by system of the differential equations in planar polar system of coordinates (SC) in the central field of an attraction. At modelling of movement SV with EPS it is considered, that operating acceleration is directed tangentially, for SV from the SS the locally-optimum law of control is used, at which the sail creates the greatest tangential acceleration ($\vartheta \approx 35{,}7°$).

In Fig. 1 dependences of maneuver duration of a of parabolic speed increases in sphere of action of the Earth (height of a starting orbit of 500 km) from design parameters SV are shown. Continuous lines the results received on M 2.i, by a dotted line – on M1.i are shown. Comparison of maneuvers durations has shown high accuracy of approximately-analytical dependences for SV with EPS (does not exceed 2 percent for $c \in [10;100]$ km/c, $a_0 \in [0{,}1;100]$ mm/s^2). Model M 3.i is applied to calculation and to optimize non-coplanar planetocentric movement and the account of perturbations. Movement is described in combined frame, supplementing planar polar frame by an inclination and a longitude of the ascending node of a current orbit, and perturbations from non-centrality of a gravitational field of a planet, gravitation of the Sun and satellites, resistance of an upper atmosphere and current conditions of shadow/ illuminance in an orbit are considered. Strict optimization of maneuvers taking into account all perturbation factors was not spent. For modelling of spatial maneuver of speed increase the law of control offered by V.N. Lebedev at which the projection of acting acceleration to an orbit plane coincides with a tangential direction was used and deviates an instant plane of movement on a angle providing $\lambda_2(t)$ during the final moment of time demanded values of phase co-ordinates.

In Tab. 2 design-ballistic characteristics of parabolic speed increases maneuver in sphere of action of the Earth for SV with solar EPS ($M_0 = 5500$ kg, $c = 70$ km/s, $P = 4$ H, height of an initial circular orbit of 500 km) are resulted. Results of modelling show, that the account conditions of shadow / illuminance in an orbit, perturbations and transition to calculation of spatial movement makes essential impact on maneuver characteristics: its duration increases and heliocentric co-ordinates SV at the moment of an exit from planet action sphere change.

Movement SV with solar EPS in areas of raised radiation is accompanied by degradation of solar batteries, and leads also to strengthening of requirements to radiating protection of payload, that considerably increases mass of a construction. For calculation of radiation dose received at geocentric movement SV of a the M4.i, including the equation describing intensity of radiation depending on phase coordinates was used $Rad(\bar{x}, t)$. In Fig. 2 change of radiation intensity is shown at passage of radiating belts of the Earth during maneuver of speed increase. In common integrating the equations of movement and the equation for it is possible to define a total dose of radiation and to estimate

Table 1. The sequence of models used by optimization planetocentric of maneuvers of a parabolic speed increase, braking and formation of the set orbit

Model	Phase coordinates	Criterion of an optimality and control	Assumptions	Model used
M 1.i	$\bar{x}^1 = \begin{pmatrix} r \\ \varphi \\ m \end{pmatrix}$	Tangential direction of acceleration, $T \to \min$	- Design-ballistic parameters of maneuvers pay off on the approached dependences received in work of V.N.Lebedev	The approached calculations of propellant expense and duration of maneuvers
M 2.i	$\bar{x}^2 = \begin{pmatrix} \bar{x}^1 \\ V_r \\ V_\varphi \end{pmatrix}$	Tangential direction of acceleration, $T \to \min$	- The problem is planar. - The engine works without cutoffs. - The engines thrust magnitude is constant.	Initial approach for M 3.i. The approached calculation of propellant expense and duration
M 3.i	$\bar{x}^3 = \begin{pmatrix} \bar{x}^2 \\ i \\ \Omega \end{pmatrix}$	Simple adaptive laws of control	- Spatial problem. - Perturbations from non-central field of gravitation, atmosphere, conditions of shadow/ illuminance in an orbit are considered.	Initial approach for model M 4.i. Calculation of propellant expense and duration
M 4.i	$\bar{x}^4 = \begin{pmatrix} \bar{x}^3 \\ Rad \\ \psi \end{pmatrix}$	Simple adaptive laws of control	- The problem is spatial. - Perturbations from non-central field of gravitation, atmosphere are considered. - Conditions of shadow/illuminance in an orbit and radiation influence on energy source are considered	Testing modeling, specification of propellant expense and duration of maneuvers. Calculation of a dose of radiation

Table 2. Results of parabolic speed increase maneuver modeling in sphere of action of Earth SV with solar EPS

Model and the considered perturbations	T, day	M_R, kg	Coordinates in heliocentric frame					
			r, 10^6 km	φ, deg	V_r, km/s	V_φ, km/s	i, deg	Ω, deg
M 2.i, without perturbations	106,1	550	150,86	346,3	-1,01	30,60	0,004	-0,16
M 2.i, taking into account a shade, without perturbations	122,1	550	149,64	2,031	-0,41	28,41	0,007	0,07
M 2.i, taking into account a shade and gravitational perturbations from nonsphericity of the Earth	117,9	550	150,75	357,9	0,368	30,49	0,014	-0,17
M 2.i, the shade, atmosphere and nonsphericity of the Earth, perturbation from the Moon, the Sun are considered	117,9	550	149,92	357,9	-1,44	29,44	0,030	-0,95
M 3.i, with change of an inclination from 51,6 to 23,45 degrees, without perturbations	108,3	560	150,62	348,4	0,372	29,07	0,004	-0,16
M 3.i, with change of an inclination from 51,6 to 23,45 degrees, all perturbations are considered	126,5	560	149,01	6,307	-1,42	29,04	0,020	-0,03

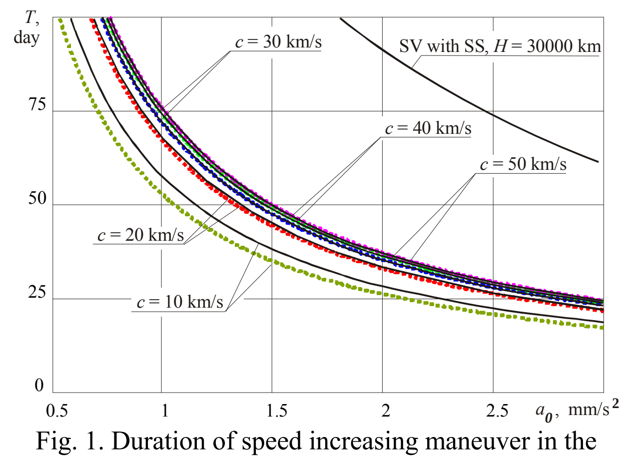

Fig. 1. Duration of speed increasing maneuver in the Earth sphere of action, received on *M 1.i*, *M2.i*

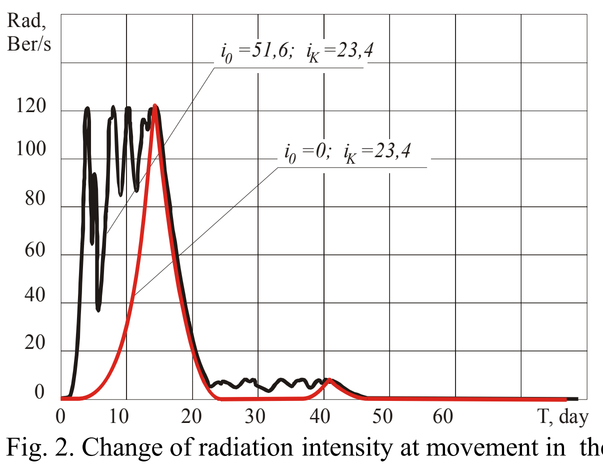

Fig. 2. Change of radiation intensity at movement in the Earth action sphere

Table 3. The sequence of becoming complicated models used by optimization of heliocentric movement (phase coordinates – dimensionless, are carried to radius and circular speed of a characteristic orbit)

Model	Phase coordinates	Criterion of an optimality and control	Assumptions	Model use
M i.1	$\bar{x}^1 = \begin{pmatrix} r \\ \varphi \\ V_r \\ V_\varphi \end{pmatrix}$	$\lambda_1 = const$, $T \to \min$	- The problem is planar. - The engine works without cutoffs. - Control - is constant. - Propellant expense is not considered	Initial approach for *M i.2*. A bottom border estimation of flights duration
M i.2	$\bar{x}^2 = \bar{x}^2$	$\lambda_1(t) = opt$, $T \to \min$	- The engine works without cutoffs. - Propellant expense is not considered	Initial approach for *M i.3*. An estimation of flights duration
M i.3	$\bar{x}^3 = \begin{pmatrix} \bar{x}^2 \\ m \end{pmatrix}$	$\lambda_1(t) = opt$, $T \to \min$, $m - unfixe$	- The problem is planar. - The engine works without cutoffs. - Propellant expense is considered	Initial approach for *M i.4*. Exact bottom border of flights duration.
M i.4	$\bar{x}^4 = \bar{x}^3$	$\lambda_1(t) = opt$, $\delta(t) = opt$, $T = fixe$, $m \to \min$	- The problem is planar. - Power of energy source decreases with removal of SV from the Sun. - Restrictions on phase co-ordinates are considered	Initial approach for *M i.5*, an estimation of propellant expense for the given flight duration
M i.5	$\bar{x}^5 = \begin{pmatrix} \bar{x}^3 \\ i \\ \Omega \end{pmatrix}$	$\lambda_1(t), \lambda_2(t) = opt$, $\delta(t) = opt$, $T = fixe$, $m \to \min$	- The problem is spatial. - Restrictions on phase co-ordinates are considered - Passing gravitational maneuvers pay off	Calculation of optimum control and corresponding movement trajectory. Calculation of propellant expense and flight duration
M i.6	$\bar{x}^6 = \begin{pmatrix} \bar{x}^5 \\ Rad \end{pmatrix}$	$\lambda_1(t), \lambda_2(t) = opt$, $\delta(t) = opt$, $T = fixe$, $m \to \min$	- The problem is spatial. - The radiation total dose pays off, degradation of panels of batteries is considered. - Restrictions on phase coordinates are considered	Calculation of optimum control taking into account features energy source. Testing calculation of propellant expense and flight duration

degradation of energy sources. Results of calculations allow to give recommendations about designing of payload protection and about change of the mission ballistic scheme on more safe protection.

For calculation of movement heliocentric parts the sequence of specified models presented in Tab. 3 is used. On heliocentric parts of flight optimum control is defined with use of Pontryagin's maximum principle formalism. We will write down the movement equations of SV in a vector kind:

$$\frac{d\bar{r}}{dt} = \bar{V}, \ \frac{d\bar{V}}{dt} = \frac{a_0}{1-m}\chi(\bar{r},\bar{e})\delta\bar{e} + \bar{g}, \ \frac{dm}{dt} = \frac{a_0}{c_0}\chi(\bar{r},\bar{e})\delta. \quad (10)$$

Also we will enter a vector of the co-variables $P = \left(\bar{P}_r, \ \bar{P}_V, \ P_m\right)^T$. Hamiltonian of system will look like:

$$H = \left(\bar{P}_r \cdot \bar{V}\right) + \frac{a_0}{1-m}\chi(\bar{r},\bar{e})\delta\left(\bar{P}_V \cdot \bar{e}\right) + \left(\bar{P}_V \cdot \bar{g}\right) + \frac{a_0}{c_0}\chi(\bar{r},\bar{e})\delta P_m. \quad (11)$$

The problem of optimum control design consists in definition of change programs $\delta(t)$, $\bar{e}(t)$ and delivering a minimum to an optimality criterion and flight boundary conditions providing performance.

If thrust change function does not depend on control (for SV with EPS) from conditions of Hamiltonian maximum on control it is found:

$$\bar{e} = \frac{\bar{P}_V}{\left|\bar{P}_V\right|}, \ \delta = \begin{cases} 1, & \Delta > 0 \\ 0, & \Delta \le 0 \end{cases}, \text{ where } \Delta = \frac{\left|\bar{P}_V\right|}{1-m} + \frac{P_m}{c_0}. \quad (12)$$

For SV with the SS function $\chi(\bar{r},\bar{e})$ depends on control, and necessary conditions for optimum control are not defined under formulas (12). The co-system of the equations for (12) looks like (Starinova, 2007):

$$\frac{d\bar{P}_r}{dt} = -\frac{\partial H}{\partial \bar{r}} = -\left(\frac{a_0}{1-m}\delta\left(\bar{P}_V \cdot \bar{e}\right) + \frac{a_0}{c_0}\delta P_m\right)\frac{\partial\chi(\bar{r},\bar{e})}{\partial \bar{r}} - \left(\bar{P}_V \cdot \frac{\partial \bar{g}}{\partial \bar{r}}\right),$$

$$\frac{d\bar{P}_V}{dt} = -\frac{\partial H}{\partial \bar{V}} = -\bar{P}_r, \quad (13)$$

$$\frac{dP_m}{dt} = -\frac{\partial H}{\partial m} = \frac{a_0}{(1-m)^2}\chi(\bar{r},\bar{e}) \ \delta \left(\bar{P}_V \cdot \bar{e}\right).$$

Performance of flight boundary conditions is reached at the expense of an entry conditions appropriate choice at integration of systems (10, 13) taking into account parities (12). This problem is a point-to-point boundary value problem of the optimum control theory. Well-known, that at the big dimension the boundary value problems possess bad convergence if initial approach for the co-multipliers are far from the decision. In the given work the approach based on procedure of "moving" on sequence of movement models from simple to difficult is offered.

In model Mi.1 power of energy source inversely proportional to a square of distance from the Sun, the engine works without cut off, speed of the expiration so big, that propellant expense can be neglected in comparison with mass of the SV. Acceleration from thrust of engines and a vector of speed SV have a constant direction concerning a radius-vector (it is defined by angles α and λ accordingly). Use of these assumptions allows to receive the particular analytical decision of system, not resorting to Pontryagin's maximum principle:

$$r = r_0 e^{\varphi \cdot ctg\alpha}, \quad \frac{V_\varphi}{V_r} = ctg\alpha, \quad T = \frac{2\left(r_k^{3/2} - r_0^{3/2}\right)}{3\cos\alpha\sqrt{1 - a_0\frac{\sin(\alpha-\lambda)}{\sin\alpha}}}. \qquad (14)$$

The analytical decision (14) exists, if parameters α and λ satisfy to the communication equations:

$$\frac{\cos\alpha\cdot\sin\alpha}{2-\cos^2\alpha} = \frac{a_0\sin\lambda}{1-a_0\cos\lambda}, \quad a_0 \le \frac{1}{3\sin\left(\lambda+\tilde{\lambda}\right)}, \quad \tilde{\lambda} = \arcsin\frac{1}{3}. \qquad (15)$$

For definition of the control optimum law $\lambda_{opt}(t)$ it is required to pass to model Mi.2, i.e. to a variation problem about optimum flights on speed. Assumptions concerning propellant expense and power changes energy source remain the same, as in the first model. The problem about optimum flight on speed between coplanar orbits without mass change is reduced to a two-parametrical point-to-point boundary value problem. Received for various accelerations ($r_k = 0{,}5$) control optimum laws are shown in Fig. 3. At small accelerations ($a_0 \le 0{,}02$) the control optimum angle fluctuates concerning the value received on model of Mi.1. With reduction of acceleration the amplitude of these fluctuations decreases. For the high levels of acceleration the optimum control program and a movement trajectory are close to the pulse decision for the high thrust (Figure 4). Use of initial approach (14, 15) and "moving" procedures on parameter a_0 at the decision of boundary value problems on Mi.2 provides good convergence of optimization numerical process. For the account of SV mass change influence the model Mi.3, considering optimum flight on speed between coplanar orbits of SV with final speed of propellant expiration is entered. As $P_m(t) \ge 0$ then $\delta \equiv 1$. The order of a boundary value problem increases to three, as initial values the results received on model Mi.2 are used. The error of calculation of the flight duration, connected with mass change, makes from 5 to 15 percent, however optimum control and a corresponding movement trajectory practically do not change. For calculation optimum trajectories under propellant expense of the fixed duration the model Mi.4 is entered. The equations for the phase and co-coordinates coincides with Mi.3, but boundary conditions for propellant expense and corresponding co-multiplier change: $m(t_0) = 0$. $m(T) \to \min$ $P_m(T) < 0$. On a trajectory there are parts of movement with switched off PS. Use as initial approaches the decisions received on model Mi.3, has allowed to receive set optimum under propellant expense of movement trajectories with various duration. For the account noncomplanar movements model Mi.5, describing movement in combined frame is used. The variation problem about optimum noncomplanar heliocentric flights is reduced to a six-parametrical boundary value problem. As initial approach for its decision results of optimization on model Mi.4 for values of the co-multipliers $P_\varphi(t_0) = 0$, $P_i(t_0) = 0$, $P_\Omega(t_0) = 0$ were used.

Process of mission optimization demands the repeated decision of variation problems on optimum heliocentric flights at various values of design and ballistic parameters.

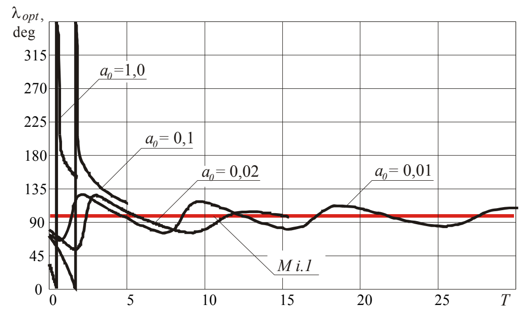

Fig. 3. Optimum controls for various accelerations

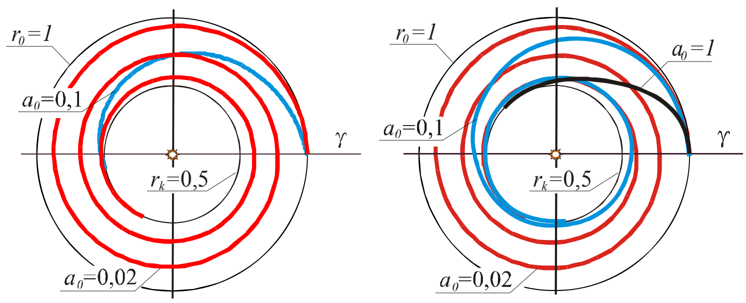

Fig. 4. Trajectories of flight with r_k =0,5 a.e. At the left - model M*i.1,* on the right - model M*i.2*

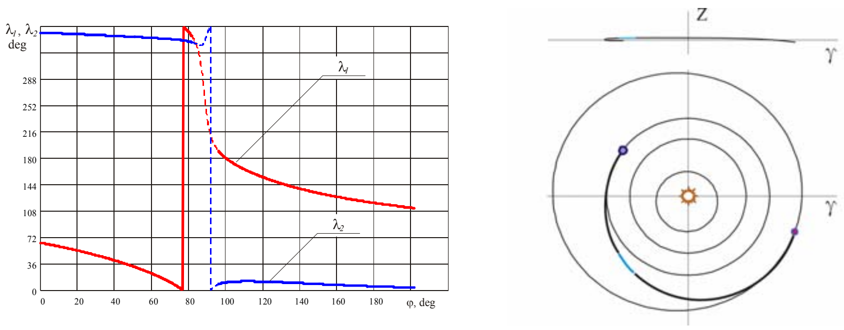

Fig. 5. Optimum control and a projection of trajectory on a polar plane for flight Earth–Mars, start date - 10.02.2016

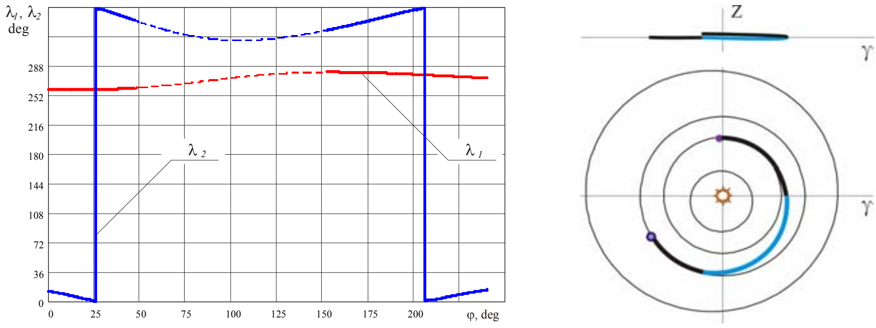

Fig.6. The optimum control and a projection of an optimum trajectory to a polar plane and ecliptic for flight the Earth–Venus. Start date - 22.04.2015

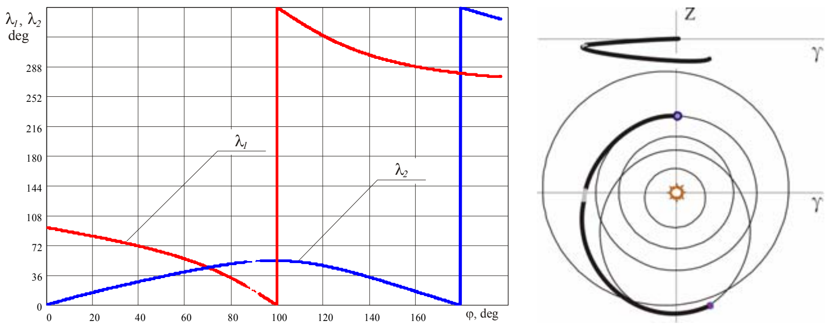

Fig. 7. The optimum control and a projection of an optimum trajectory to a polar plane and ecliptic for flight the Earth–Kastalya (4769). Start date - 20.12.2015

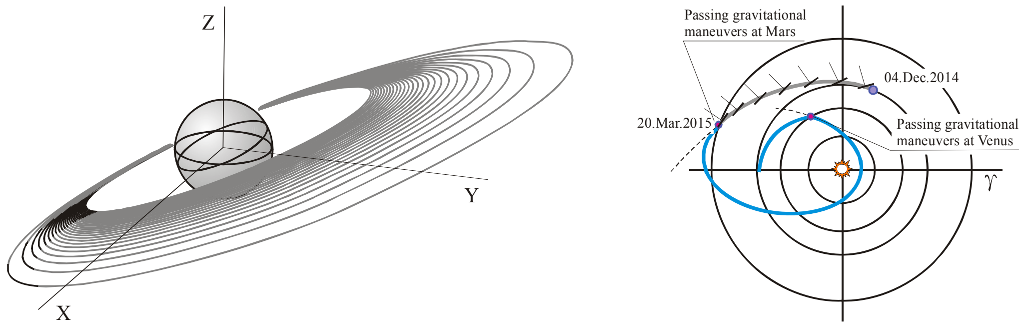

Fig. 8. Maneuvers of speed increase and heliocentric flight for SV with a solar sail

Table 4. Results of iterative optimization of piloted mission the Earth-Mars-Earth

The used models	T_{Σ}, day	Mass SV, tones			Optimum parameters of mission				
					The ballistic			The design	
		Start	Finish	The expense	Start date	Finish date	τ	P, H	c, km/s
M 1.1 (planeto - and heliocentric movement on the approached dependences)	770	371,0	130,0	181,0	6.7.2017	5.8.2019	0,456	350,2	72
M 2.4 (planetocentric movement without perturbations, heliocentric – coplanar)	770	361,8	129,9	171,9	8.7.2017	7.8.2019	0,456	350,1	70
M 3.4 (planetocentric movement taking into account perturbations, without optimum joining of parts)	770	316,8	129,9	126,9	8.7.2017	7.8.2019	0,454	350,0	70
M 4.5 (the spatial, perturbation movement with optimum joining of parts)	770	309,7	129,9	119,8	9.7.2017	8.8.2019	0,456	350,0	70
M 5.7 – testing calculation (the account of degradation of energy sources, a problem of three bodies)	772	309,7	129,9	119,8	9.7.2017	10.8.2019	-	350,0	70

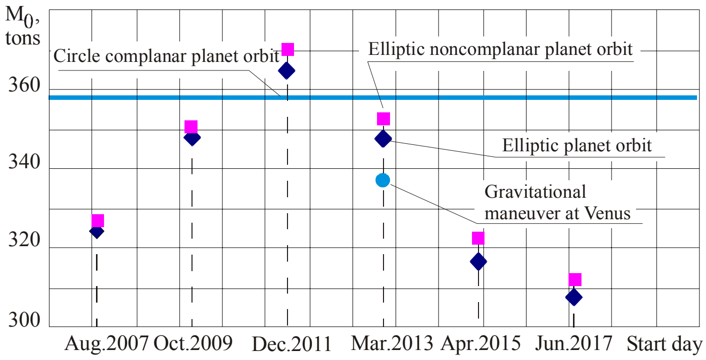

Fig. 9. Dependence of starting mass on start date, received by model *M4.6*

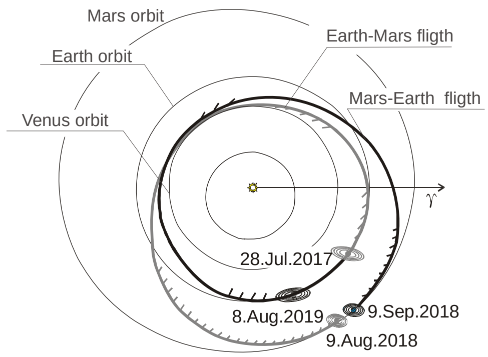

Fig. 10. The optimum ballistic scheme of mission the Earth-Mars-Earth (model *M4.6*)

Existing methods of the decision of these problems demand from the executor of heuristic approaches, to high qualification in the field of flight dynamics, great volume of calculations and do not give the guaranteed results. The offered iterative method, is based on sequence of becoming complicated models and realized in the information system intended for support of missions optimization process. shadow/illuminance.

3. RESULTS OF OPTIMIZATION.

With use of the described technique the optimization results of the interplanetary missions which are not demanding returning to the Earth and carried out SV with solar EPS and the SS are received. In Figs. 7 - 9 optimum control programs and movement trajectory for flights the Earth–Mars, the Earth-Venus, the Earth–Kastalya (an asteroid 4769) are shown, at optimum start dates received on M4.5.

Results of ballistic optimization of interplanetary missions SV from the SS have been received. In the Figure 10 are shown optimum on speed of a trajectory of speed increase in action sphere of the Earth and heliocentric flight the Earth-Mars (windage 50 m^2/kg) with use of passing gravitational maneuvers at Mars and Venus (are received on M4.6).

Results of interplanetary expedition optimization the Earth-Mars-Earth with solar EPS are received. By optimization of this mission restrictions on the minimum heliocentric distance, duration of expedition and a total radiation dose were considered. Given tables 4 show, weak influence of the chosen model of movement on optimum design and ballistic parameters of mission, however criterion of optimization (starting mass of SV) changes at specification of movement models considerably (decreases approximately on 60 tons).

Therefore at a stage of preoutline designing when possibility of the fast analysis is important, optimization is more favourable for spending on simple models, and at following design stages to use for optimization more exact models yielding results differing by a more concrete definition. The best dates of start and ballistic schemes of flights (Figs. 9, 10) are defined. The estimation of possibility of passing gravitational maneuvers in action sphere of Venus is spent (at start in 2013), allowing to lower starting mass of SV and to raise scientific efficiency of mission. The total radiation dose received SV that allows to choose radiating protection parameters of panels SB and payload pays off.

4. CONCLUSIONS

The developed method of iterative optimization of interplanetary missions with the low thrust, using sequence of movement specified mathematical models and design shape SV, is realised in system of formation support of missions ballistic schemes. On the basis of Pontryagin's maximum principle the necessary conditions of a control laws optimality for all models of sequence are received. The new particular analytical decision describing planar movement of SV with solar EPS is described, allowing to construct initial approach in the iterative scheme of optimization. Developed a method of modelling and optimization of interplanetary missions ballistic schemes of SV with the low thrust, based on a combination of Pontryagin's maximum principle formalism conditions of transversality and methods of the mathematical programming, allowing to consider restrictions characteristic for concrete interplanetary missions. Recommendations for choice design-ballistic parameters of the interplanetary missions SV, received taking into account features nuclear and solar EPS for missions on delivery of a payload to orbits of Mars, Venus, asteroid Kastalya interplanetary flights with a solar sail, expeditions the Earth–Mars–Earth are received.

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