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PARAMETRIC-RESONANCE-BASED CONTROL OF BUOY CARDANIC SYSTEM FOR A BEACON SIGNALIZATION

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***Abstract*—This work aims at the control of nonlinear oscillations in a multibody dynamics composed by a moored buoy and a gimbal-beacon system that can swing like a physical pendulum restrained to a planar motion on a moving plane. We define a control objective based on parametric resonance on the pendulum employing a sliding mass in the underside of the rod. Additionally we analyze the mechanical energy transference on the pendulum oscillation due to buoy motions like tilts and yaw motion. The control performance in the presence of buoy-induced perturbations on the controlled oscillations stays as the main issue of the paper.**

Keywords - Moored buoys - Physical pendulum - Gimbal system - Nonlinear oscillations - Parametric resonance - Oscillation control

I. INTRODUCTION

Nondamping oscillations are present in various physical systems. Particularly physical, simple pendulums as well as pendulum-like oscillating systems are found as components in many engineering systems with oscillatory elements, cranes transporting goods, tower crane, quadrotors with suspended load, robotics, robotic walking, fluid in containers, autonomous aerial vehicles, among other things, [2], [3].

Often, undesired oscillations are subject to control with the end of effectively damping or completely suppressing them.

Nondamping oscillations can be energized or de-energized with a periodic variation of some system parameter to which the system motion is sensitive. This occurs with periodic changes of this parameters and the system dynamics manifests the phenomenon of parametric resonance. For instance a mass-spring system with periodic modulation (exactly twice the system natural frequency) of the elastic coefficient, it manifests a progressively increasing oscillation. This simple system is modelled by the fundamental differential equation of Mathieu-Hill. For small amplitudes one can apply the Theory of Floquet for linear and periodic-varying dynamics (Berkey, 1976). Also almost periodic nonlinear oscillation can be simply be approximated by arithmetic-geometric mean [1]. Also a simplified representation of the perturbed pendulum dynamic leads to bifurcations of Duffing equation included strange attractors. In [9] the harmonic modification of the pivot in a pendulum gives rise to chaotic dynamics. Also the appearance of bifurcations in the nonlinear dynamics is studied in [2] when the system is excited with periodic displacements of the pivot and is reflected in Poincaré maps.

The control of the forced nonlinear oscillations in pendulum systems has been investigated extensively. The most widespread principle of control of a pending weight in tower cranes is based on displacements of the pivot (both horizontally or vertical) with information of the pendulum angle and mass position [3]. In [4] damped oscillations are obtained by decreasing the mass at a constant rate. A real-time time-optimal control problem for a cart-and-pendulum system is considered in [5]. In [7] the parametric resonance was leveraged to damp induced nonlinear oscillations by shifting a mass across the rod in accordance to certain cinematic rules.

In this paper, we analyze a pendulum-gimbal system like in [6] which is mounted on a oceanographic buoy. Moored buoy dynamics is extensively studied, see for instance [8]. They are used extensively in access channels to safely guide traffic of ships to and from ports. Beacons mounted at the top of buoys generally emit a sequence of light flashes or acoustic and eventually electromagnetic signals in order to alert for a geographic position and/or possible dangerous obstacles or to indicate limits of sure depths.

When buoys are excited by currents, wind and waves they may experiment a wide irregular behavior, so that under circumstances, signalization sequences may be received corruptly by the ship. Specially visual and acoustic signals are programmed to give pulses at regular periods subject to specific codes. Also for regular waves, the forced motion of the a moored buoy may be characterized by complex bifurcated behaviors such as oscillations with long periods or chaotic displacements and inclinations. In all these cases the

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partial loss of visual signals is a potential source of bad ship maneuvers that puts at stake the navigation.

In this problematic we will focus on an experimental beacon device illustrated in Fig. 1 in the framework of control dynamics with perturbations. The control problem herein is to damp the oscillations of the beacon induced by the moored buoy movements when it is excited by waves. The goal is, in broad outlines, to keep the beacon as vertically as possible so as it can be observed from the distance without discontinuity. Clearly, to achieve this end the beacon can be not fixed to the buoy but instead on a gimbal which has two degrees of freedom of rotational modes and another degree for axial movements along the rod.

The device will attempt to reduce the oscillations by properly sliding a mass on the underside of the rod up and down. Control strategies are described in [7] and employ the effect of the parametric resonance in order to suck the energy of the pendulum oscillation gradually.

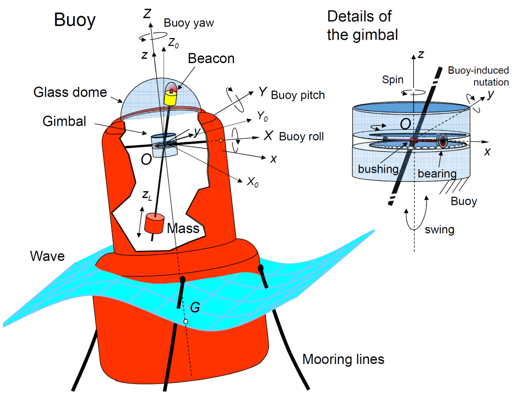

Figure 1 - Wave-excited buoy with an embedded gimbal system for beacon signalization

For the analysis we chose three coordinate frames, first a world-fixed frame coordinates $X_0Y_0Z_0$ with center at O, which is the pendulum pivot, then a buoy-fixed frame XYZ also with center on O and a third frame xyz fixed to the gimbal with the same center O.

The beacon is rigidly installed at the top of the rod in opposition to the mass which lies on its underside. The mass distribution meets a design condition that the mass center of the divice lies well below O. The pendulum pivot holds on a bushing at O and the turn physical axis is coincident with x and rests on a ball bearing whose motion is a spin about z.

The main typical turn motions of the buoy are pitch, roll and yaw, while the movements of the device are the spin about z, the swing about x and the mass slide z_L along the rod (see Fig. 1).

A characteristic of the assembly is that the axes Z and z are always coincident what it does not mean that both may have the same turn. Another particularity is that the rod can freely swing only in the plane yz about x and this plane can also rotate about z when the buoy motion or inner dynamic moments generate a moment about z which drives the bearing balls to roll. Certainly, these moments can be generated either directly by buoy yaw motions or also by buoy tilts (nutations) which indirectly induce relatively strong gyroscopic moments about z. Likewise, buoy-generated moments about x drag the bushing axis to turn and this way forcing the pendulum to swing.

The way the buoy transfer energy to the device depends on static and kinematic rolling moments at the bearing and bushing. If for instance the static rolling resistance of the bearing is not overcome by all moments applied about z, then the device is dragged by the buoy movement with the same buoy yaw rate. On the contrary, when the static moment is exceeded, the device drag, even when it keeps on existing, decreases dramatically, releasing the device to move with its own dynamics. Moreover, the buoy tilts strictly impose device nutations. Thus the device dynamics in this mode is completely suppressed and determined by the buoy tilts as we will shown thereafter.

Buoy-induced perturbations acting on the device dynamics constrain the performance of the control system in an intricate way that we will make more transparent. This is one of the matters of this paper.

Buoy and device have interactuating dynamics as every multibody system. A particularity of this conjunction is that the buoy mass is much greater than the device mass, so the movements of the buoy forces the movements of the device system without reciprocity. Moreover, it is known that moored buoys may behave in a complex manner under sea wave excitations, even in the case of simple harmonic waves, wherein long-periods movements and chaotic responses may occur frequently. Thus, the device may be excited in an unpredictable way.

The paper is organized as follows. First a summary of the system in the swing plane yz along with the basic control and a more sophisticated adaptive control based on knowledge are presented. Further on, we deduce the dynamics of the system in the rotation modes and interpret the relations about forced motion and induced gyroscopic moments. We will assess the nonlinear dynamics of the gimbal system and its potential to attenuate complex perturbations on the controlled pendulum. Finally, simulation a comparison of control performance is made between the control approaches and the system behavior without control.

II. Control of oscillations

A. Basic control

The dynamics of this device system on the plane xy is (Jordan and Bonitatibus, 2005)

$$\ddot{\alpha} = -\frac{1}{I_b + I_0 + I_m + m z_L^2(t)} \Bigg(\delta_a \dot{\alpha} + g \sin\alpha (1 - \frac{d(t)}{L_0}) \\ (L_b M_b/2 + L_0 M_0/2 + m z_L(t)) + 2 m \dot{\alpha} z_L(t) \dot{z}_L(t) \Bigg), \quad (1)$$

where α is the rod angle, z_L the sliding mass distance from O along the rod, d is the pivot displacement, I_b and I_0 are the inertia moments of the rod and beacon respectively with respect to rotation axis, L_0 the rod length, L_b the beacon length, m the sliding mass, M_0 the rod mass, M_b the beacon mass, g the gravity acceleration, δ_a a friction coefficient due to air, $\dot{\alpha}$ and $\dot{z}_L$ are rates of the oscillation and the slide velocity, respectively. For control purposes let us define $z_L(t)$ as the control action and $d(t)$ as the pivot perturbation due to a buoy translation on the plane yz.

The basic control algorithm works in the following way (Jordan and Bonitatibus, 2005). During a complete period of the pendulum oscillation, the sliding mass motion is synchronized to ascend and descend two times completing a cycle. In the basic control strategy, the mass descends when the rod is at one of its maximal inclination and ascends when the rod is passing by the vertical line in one or other direction. The synchronization of the complete control cycle occurs at 4 time points, referred to as t_1 up to t_4. One pair (t_1 and t_3) corresponds to the instants when the mass begins its descents (twice per cycle), and the other pair (t_2 and t_4) when the mass begins its ascents (twice per cycle).

By leading the mass in the other direction, the pendulum will be excited instead and gains energy. This is an unstable control (*cf.,* parametric resonance). Mathematically, the basic control law is

$$\begin{cases} z_L(t) = z_{L_1} + \int_{t_1}^{t_2} v(t)\ dt, \text{ from } t_1 \text{ up to } t_2, \\ \qquad \text{where } t_1,\, t_2 \text{ fulfill: } \dot{\alpha}(t_1) = 0 \text{ and } z_L(t_2) = z_{L_0} \\ z_L(t) = z_{L_0} - \int_{t_3}^{t_4} v(t)\ dt, \text{ from } t_3 \text{ up to } t_4, \\ \qquad \text{where } t_3,\, t_4 \text{ fulfill: } \alpha(t_3) = 0 \text{ and } z_L(t_4) = z_{L_1} \\ z_L(t) = \text{constant from } t_2 \text{ to } t_3 \text{ and from } t_4 \text{ to } t_1\ . \end{cases} \tag{2}$$

There exist complex relations between the values of the control parameter set $\{z_{L_0}, z_{L_1}, \frac{z_{L_0}+z_{L_1}}{2}, v, d\}$ and the synchronism of the control system, it is the set $\{t_1, t_2, t_3, t_4\}$. The relations simplify when $d \equiv 0$ at any t. For instance, given a span and a midpoint of it, there exists a critical sliding velocity to maintain the synchronism. This is defined indirectly by

$$t_2\text{-}t_1 = \frac{z_{L_0}\text{-}z_{L_1}}{\bar{v}} \leq \frac{T_c}{4} \text{ and } t_4\text{-}t_3 = \frac{z_{L_0}\text{-}z_{L_1}}{\bar{v}} \leq \frac{T_c}{4}, \tag{3}$$

where T_c is the period of the controlled oscillation and $\bar{v}$ the mean mass velocity which is considered equal in both ascent and descent.

An outstanding feature of this synchronization is that the frequency of the mass motion in a cycle is twice the frequency of the oscillation.

There are many ways by which the basic control law can be optimized in the sense that the loss of mechanical energy per cycle is maximal. This is accomplished if the areas enclosed by the mass path in each cycle are maximized. More precisely for a given pair of levels z_{L_0} and z_{L_1} the control law is optimized according to

$$\max_{\{t_1, t_3, z_{L_0}, z_{L_1}\}} \oint_{y\text{-}z} \text{sign}(\alpha)\ A((z_L, v)\ \ ds \tag{4}$$

where the integral is the Green integral along the path $x(t)$-$z(t)$ and A is the area enclosed. Hence an optimal set $\{t_1, t_3, z_{L_0}, z_{L_1}\}$ is found according to the provided v and taking into account conditions (3). The implementation of the optimization (3) is not really feasible because the optimal switching time points change with wave frequency.

A much more practical way to influence to some extent the speed of the energy leakage per cycle is a control strategy based on knowledge. By way of example we will illustrate this control strategy employing the pivot frequency of the excitation as the main dimension with mass range and mass velocity as codimensions.

B. Knowledge-based control

As in many mechanical nonlinear systems, the pendulum system subject to an harmonic perturbation of the pivot, may set off bifurcations, quasi-periodic or chaotic oscillations. One of the most useful results of the previous work in [6] are depicted in spectrum of the controlled variable of the sliding mass like in Fig. 2. Herein, the powers (labelled as P_0) of the spectra of the induced swing motion in steady state for a wave of frequency ω_0 were obtained.

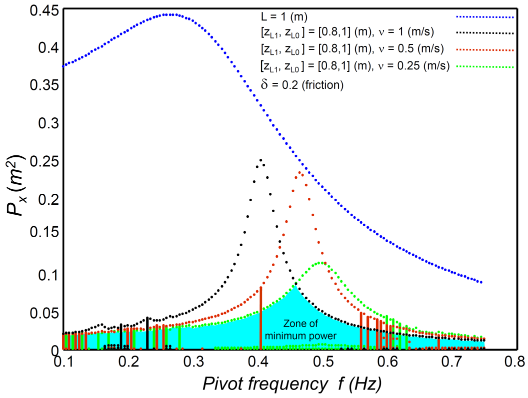

Figure 2 - Pendulum spectrum energy and bifurcations for variable velocity to pull up and down of the mass

Moreover, large periods in the oscillation are obtained using expansion in a Fourier series. The occurrence of subharmonics are indicated with point bars for two specific parameter in the codimensions, namely the range of the sliding interval and the speed to pull up and down the mass along the rod.

First, the uncontrolled swing with static mass at the underside of the rod leads to large forced oscillations. It is noticing that a variation of the range or of the mass speed will produce a significant reduction of the power of the swing. Furthermore, the presence of large periods like $P2$, $P3$, $P4$ and chaotic response are common in all cases when the mass is driven.

For instance, keeping the mass interval fixed and changing the speed the zone of minimum powered highlighted in the figure yield significantly reduced.

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Herein it is seen that bifurcation phenomena are common for lower as for higher frequencies. Additionally, one notes that there is a split frequency $\omega_0 = 0.46Hz$ wherein low speeds are more profitable than higher speeds for $\omega < 0.46Hz$. On the contrary, for higher frequencies, the situation is inverted, *i.e.*, a high rate produces a more efficient power reduction of the controlled swing.

In broad outlines, based on this knowledge, the best way to maintain the energy of the orbit as low as possible is to regulate the mass rate appropriately. Accordingly, the frequency $\omega_c = 0.46$ (Hz) is chosen as the critical frequency for switching rates to obtained optimal properties of the control performance.

The switching control obtains permanently information of the fundamental frequency ω_0 via FFT and identify it on small periods of the evolution of $\alpha(t)$ to finally detect the crossing point about $\omega_c = 0.46Hz$. The results have shown a drastic reduction of the energy of the oscillation in comparison with previous cases without switching.

We are stating now, that the pendulum dynamics may become much more complex when mechanical energy is allowed to input the system in other cross modes of oscillations whilst the described planar controls are working. This is the matter of the next section.

III. Control dynamics with perturbation

We think henceforth in a pendulum-gimbal system mounted at the top of a buoy like in Fig. 1, which is excited by a sea wave actuating on some unknown direction with respect to the x axis.

Henceforth for the analysis, the notation for vectors is in bold text and for scalars in normal text. Besides, subindices in lowercase letter are referred to a rotating frame and in uppercase letter to a buoy-fixed frame. Finally, derivatives of temporal variables are represented using dots over them.

We postulate the angular velocity $\mathbf{\Omega}$ as the vectorial sum of the device spin $\dot{\phi}$ and the pitch $\dot{\theta}$ about Y. Similarly, we define an angular velocity $\boldsymbol{\omega}$ as the vectorial sum of the device spin $\dot{\psi}$ about z and the pendulum swing $\dot{\alpha}$ about x and the nutation $\dot{\theta}$ about y. As the device has mass symmetry in the x and y axes, but not with respect to z, Then we postulate a rotating frame xyz

Both $\mathbf{\Omega}$ and $\boldsymbol{\omega}$ are represented by their projections in the frame xyz. Additionally the directions of the frame xyz are reflected in its unit vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$.

As x and z are t this stage, we can directly apply the theory of Euler for the temporal change of the absolute momentum $\mathbf{L}$ to the system and relate it to a total moment $\mathbf{M}$ both related to $(\mathbf{i}, \mathbf{j}, \mathbf{k})$.

To this end we have $\dot{\mathbf{L}} = d(L_x\mathbf{i} + L_y\mathbf{j} + L_z\mathbf{k})/dt$. Since $(L_x, L_y, L_z) = (I_x\omega_x, I_y\omega_y, I_z\omega_z)$ and $d\mathbf{i}/dt = \mathbf{\Omega} \times \mathbf{i}$, $d\mathbf{j}/dt = \mathbf{\Omega} \times \mathbf{j}$ and $d\mathbf{k}/dt = \mathbf{\Omega} \times \mathbf{k}$. Thus

$$M_x = I_x\dot{\omega}_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z + \dot{I}_x\omega_x \quad (5)$$
$$M_y = I_y\dot{\omega}_y - I_z\Omega_x\omega_z + I_x\Omega_z\omega_x + \dot{I}_y\omega_y \quad (6)$$
$$M_z = I_z\dot{\omega}_z - I_x\Omega_y\omega_x + I_y\Omega_x\omega_y, \quad (7)$$

where $\dot{I}_x$ and $\dot{I}_y$ in (5)-(6) correspond to derivatives of the time-varying inertia moments owing to the sliding mass motion. Taking into account the mass position $-z_L$ in (2), it holds in accordance to (1)

$$I_x = I_b + I_0 + I_m + m\ z_L^2 \quad (8)$$
$$I_y = I_b + I_0 + I_m + m\ z_L^2 \quad (9)$$
$$\dot{I}_x = \dot{I}_y = 2m\ z_L\ \dot{z}_L. \quad (10)$$

Moreover, from the definition of $\mathbf{\Omega}$ and $\boldsymbol{\omega}$ it follows after some intricate deductions

$$\mathbf{\Omega} = -\dot{\phi}\sin\theta\ \mathbf{i} + \dot{\theta}\ \mathbf{j} + \dot{\phi}\cos\theta\ \mathbf{k} \quad (11)$$
$$\boldsymbol{\omega} = (\dot{\alpha} - \dot{\phi}\sin\theta)\ \mathbf{i} + \dot{\theta}\ \mathbf{j} + \dot{\phi}\cos\theta\ \mathbf{k} \quad (12)$$

We consider the external moment $\mathbf{M}$ as the one generated by the buoy motion, namely the moment about Z designated as M_Z and tilt moments M_X and M_Y about X and Y, respectively. Their projections in the frame xyz are the external moment components

$$M_{e_x} = M_X\cos(\phi - \phi_b)\cos\theta + M_Y\sin(\phi - \phi_b)\cos\theta - M_Z\sin\theta \quad (13)$$
$$M_{e_y} = -M_X\sin(\phi - \phi_b) + M_Y\cos(\phi - \phi_b) \quad (14)$$
$$M_{e_z} = M_X\cos(\phi - \phi_b)\sin\theta + M_Y\sin(\phi - \phi_b)\sin\theta + M_Z\cos\theta. \quad (15)$$

where ϕ_b is the buoy yaw motion. It is worth noticing that M_{e_z} is resisted by the friction on the bearing, which can drag the conjunction to follow the buoy rotation.

The external moments include beyond the applied moments, also friction moments at the ball bearing seats and bushing axis, along with the weight of the beacon-rod-mass. Its components in the xyz frame are

$$M_x = M_{e_x} - \delta_a\omega_x + M_{bu} - (w_0 + mz_L)g\sin\alpha\cos\theta \quad (16)$$
$$M_y = M_{e_y} - \delta_a\omega_y + R_{bu} - (w_0 + mz_L)g\cos\alpha\sin\theta \quad (17)$$
$$M_z = M_{e_z} - \delta_{a_z}\omega_z + M_{be} - (w_0 + mz_L)g\sin\alpha\sin\theta \quad (18)$$

where R_{bu} is the reaction moment of the bushing to a any moment about y which precludes any possibility of the pendulum rotation as a spherical pendulum, $w_0 = M_0L_0/2 - M_bL_b/2$, δ_a is a friction coefficient due to air resistance in the rod, mass and beacon which is assumed to have approximately the same value because of the similar effective area of the pendulum for rotations about x, y and z, M_{bu} is a friction resistance moment about x at the bushing and M_{be} is a rolling resistance moment about Z at the bearing.

Both, friction moments provide in general a relatively low resistance because they involve a type of lubricated friction.However, by rather slow motions this kind of friction may increase substantially. Moreover, both resistance moments are proportional each one to normal forces aligned with z for the bearing and the direction of the rod for the bushing. The moments about x and z will force the bushing and ball bearing to rotate inasmuch as their modules do not exceed constant moments due to static friction. From there on, the friction diminishes drastically until it is limited by much more small moments owing to the low kinetic friction.

Accordingly, the friction on the bushing causes the moment M_{bu} which can be expressed in an IF-conditional form as

$$\text{IF } M_{bu_0} > \begin{array}{l}|M_{e_x} - I_x\dot{\omega}_x + I_y\Omega_z\omega_y - I_z\Omega_y\omega_z - \dot{I}_x\omega - \\ -\delta_a\omega_x - (w_0 + mz_L)g\sin\alpha\cos\theta|\end{array}$$
$$\text{AND } |\dot{\alpha}| \le \varepsilon_{\dot{\alpha}} \quad \text{THEN } M_{e_x} = -M_{bu} \quad (19)$$
$$\text{ELSE} \quad M_{bu} = -\delta_{bu}n_r r\,\mathrm{sgn}(\dot{\alpha}) \quad \text{ENDIF}$$

where M_{bu_0} is the modulus of the friction moment due to the static friction at the bushing, $\varepsilon_{\dot{\alpha}}$ represents an upper bound for delimiting slow motion of $\dot{\alpha}$, δ_{bu} a dimensionless friction coefficient of the bushing, n_r the normal force on the bushing in the direction of the rod and r is the radius of its axis.

Taken into account the weights of the rod, beacon and mass, and their centrifugal forces, the resulting force along the rod is calculated through the second derivative of their position vectors. The unit vector along the rod is

$$\mathbf{r} = (\sin\alpha\sin\phi + \cos\alpha\sin\theta\cos\phi)\ \mathbf{i} + (\sin\alpha\cos\phi - \cos\alpha\sin\theta\sin\phi)\ \mathbf{j} + \cos\alpha\cos\theta\ \mathbf{k}, \quad (20)$$

and all forces acting in this direction, namely centripetal force and projected weights, determine n_r. Thus

$$n_r = (M_0 + M_b + m)g\cos\alpha\cos\theta - (w_0 + mz_L)\mathbf{r}.\ddot{\mathbf{r}} \quad (21)$$

with the dot between vectors meaning the scalar product. Similarly, the friction on the bearing causes the moment M_{be} which is accounted for by

$$\text{IF } M_{be_0} > |M_{e_z} - I_z\dot{\omega}_z + I_x\Omega_y\omega_x - I_y\Omega_x\omega_y - -\delta_a\omega_z - (w_0 + mz_L)g\sin\alpha\sin\theta| \quad (22)$$

$$\text{THEN } \phi{=}\phi(t_1){+}\phi_b,\ \dot{\phi}{=}\dot{\phi}_b{=}0,\ \ddot{\phi}{=}\ddot{\phi}_b{=}0 \text{ AND } f_{be} = 1$$

$$\text{ELSE } M_{be}{=} -\delta_{be}n_z R\,\mathrm{sgn}(\dot{\phi}) \text{ AND } f_{be} = 0 \text{ ENDIF}$$

where f_{be} is binary variable state of the friction at bearing, meaning static when $f_{be} = 1$ and cinematic when $f_{be} = 0$, t_1 is the instant when f_{be} changes from 0 to 1, M_{be_0} is the modulus of the reaction moment due to static friction which resists the inertia moment about Z, δ_{be} is a dimensionless rolling resistance coefficient of the bearing balls, n_z the normal force on the ball bearing in the direction z and R is the radius of either of them. As we will see further down, the third sentence of the IF-condition will imply simply that the buoy motion impose completely the bearing seat position and angular rate.

Moreover n_z is represented by

$$n_z = (M_0 + M_b + m)g\cos\theta - \cos\alpha(w_0 + mz_L)\mathbf{r}.\ddot{\mathbf{r}} \quad (23)$$

On account of the enormous disproportion between masses of the buoy and gimbal-pendulum system, it is remarked that any tilt of the buoy will force the system in a nutation θ.

Another remark is related to physical aspects of the bushing at the pivot which represents an holonomic restraint which blocks any movement of the sliding mass out of the motion plane yz. On one side, this restraint is already considered in the orthogonality between $\dot{\alpha}$ and $\dot{\theta}$. On the other side this restraint imposes that the moment M_y is entirely resisted by a bushing reaction moment. In consequence the pendulum can not oscillate like a spherical pendulum and the nutation of the gimbal is caused by buoy tilts only but not by the inherent dynamics of the device gimbal-pendulum self.

As a result, the ODE (6) for the absolute change of the angular momentum in the $\mathbf{j}$ direction is cancelled and the expression (14) of the external moment M_{e_y} is replaced by

$$\dot{\theta} = -\dot{\theta}_X \sin(\phi - \phi_b) + \dot{\theta}_Y \cos(\phi - \phi_b), \quad (24)$$

where $\dot{\theta}_X$ and $\dot{\theta}_Y$ are the buoy tilts about X and Y, respectively, which have their maximal effect on the gimbal when $\phi = 3/4\pi + i\pi$ with $i = 0, 1, 2....$

Considering the dynamics equations (5)-(7) with the expressions for the inertia moments (8)-(9) and their derivatives (10), the angular velocities (11)-(12), the buoy-induced perturbation moments (13)-(15), the resistance moments with their normals (22)-(23) and (19)-(21), and finally the total moment components in (16)-(18) with the expressions for air friction moment and pendulum weight and equation (24) with the attached remarks, one can draw out the following complete ODEs of the system dynamics

$$M_{e_x} = I_x(\ddot{\alpha} - \ddot{\phi}\sin\theta - \dot{\phi}\dot{\theta}\cos\theta) - I_y\dot{\phi}\dot{\theta}\cos(\theta) + +I_z\dot{\phi}\dot{\theta}\cos\theta + 2m\ z_L\dot{z}_L(\dot{\alpha} - \dot{\phi}\sin\theta) + \delta_a(\dot{\alpha} - -\dot{\phi}\sin\theta) - M_{bu} + (w_0 + mz_L)g\sin\alpha\cos\theta \quad (25)$$

IF $f_{be} = 0$ THEN

$$M_{e_z} = I_z(\ddot{\phi}\cos\theta - \dot{\phi}\dot{\theta}\sin\theta) - I_x\dot{\theta}(\dot{\alpha} - \dot{\phi}\sin\theta) - -I_y\dot{\phi}\dot{\theta}\sin\theta + \delta_a\dot{\phi}\cos\theta - M_{be} + (w_0 + +mz_L)g\sin\theta\sin\alpha \text{ with IC: } \phi(t_0) \text{ and } \dot{\phi}(t_0) \quad (26)$$

$$\text{ELSE } \phi = \phi(t_1) + \int_0^t \dot{\phi}_b d\tau,\ \dot{\phi}{=}\dot{\phi}_b \text{ and } \ddot{\phi}{=}\ddot{\phi}_b \text{ ENDIF} \quad (27)$$

where t_0 is the instant when f_{be} changes to 0, t_1 is the instant when f_{be} changes to 1 and IC means initial conditions. It is remarking that the condition (22) is always being assessed from (26), nevertheless whether ϕ is generated by the ODE (22) or calculated from (27).

The ODE systems (25)-(26) or (25)-(27) are accounted for by the rotational behavior of the system dynamics, wherein the moments M_{e_x} and M_{e_z} and the angles ϕ_b and θ_X and θ_Y are external perturbations which are specified in our study as temporal functions emulating the buoy behavior with harmonic either long-periodic cycling as well as chaotic motions.

Besides z_L contains the control action onto the plane yz given by the basic law (2) or a more sophisticated law like for a knowledge-based control.

As seen in our approach, the control performance is aimed to assess only perturbation moments about O without displacements of that point. Indeed, this study pursues the isolation of rotations in every mode that ultimately will submit the controlled system to interactive oscillations like pendulum swing with buoy-induced excitation. This will, in turn, allow us to understand better the role played by gyroscopic forces and the ability of the device to counterbalance control performance and disturbance rejection.

IV. Case Study

In the following, some selected simulations are portrayed aiming to illustrate the main outcomes of this study. Results regarding the controlled and uncontrolled cases (with acronyms CC and UC, respectively) are contrasted in the same figure. We illustrates the basic control algorithm along with the estimation of the fundamental wave frequency to optimally regulate the sliding interval and mass speed.

General settings are: $l = 1m$, $m = 0.5kg$, $R = 0.003m$ and $r = 0.003m$, $\delta_{a_x} = \delta_{a_y} = 0.01Nms/rad$, $\delta_{a_z} = 0.001Nms/rad$, $\delta_{be} = 0.0025Nms/rad, \delta_{bu} = 0.1$, $\varepsilon_{\dot{\alpha}} = 0.02rad/s$, $M_{be_0} = 0.00001Nm$, $M_{bu_0} = 0.07Nm$, $\alpha(0) = 45°$, $z_L(0) = 1m$. Buoy-induced and external moments along with buoy tilts are indicated in the figures.

Fig. 3 reproduces the CC and UC when the wave is exciting the buoy in stationary state along y causing a continuous pendulum swing by dragging on the bushing axis on account of the static and kinetic friction moments. A while later, a strong pulsating rotation of the buoy about Z changes the orientation of the oscillation plane yz in both directions. However the swing motion results unperturbed by this turn because there is not any buoy tilt. The controlled swing behavior seems to be much more damped in the transient than the uncontrolled case, and much softened in permanent state.

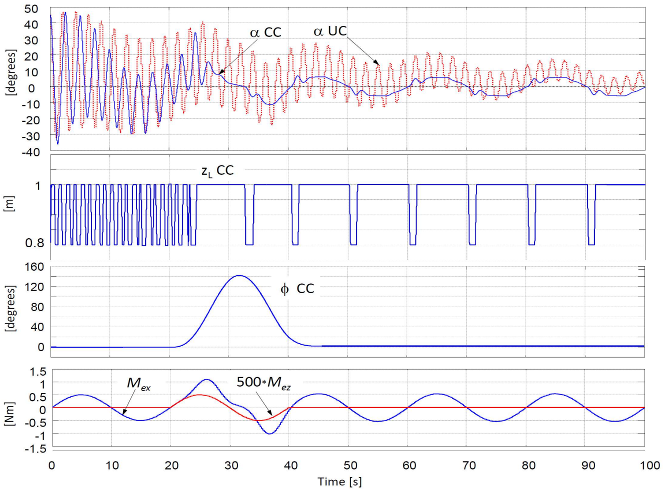

Figure 3 - Case study 1: Perturbation of the beacon-gimbal system through buoy yaw and pitch motions

Fig 4. illustrates the complex scenario wherein a wave induces a pendulum swing through a buoy pitch and simultaneously changing buoy tilts affect the orientation of the motion plane yz. It is noticed that the control behavior is strongly affected by generated gyroscopic moments M_{e_z} which produced many complete yaw rotations of the buoy. The buoy tilts create three pseudo-moments about z in (26), namely $-I_z\dot{\phi}\dot{\theta}\sin\theta$, $-I_x\dot{\theta}(\dot{\alpha}-\dot{\phi}\sin\theta)$, $-I_y\dot{\phi}\dot{\theta}\sin\theta$, which are null when $\theta = 0$. Even though the control seems to have comparable response as in the UC, this can reach a superior performance when the pseudo moments remain smooth due to a small $\dot{\phi}$.

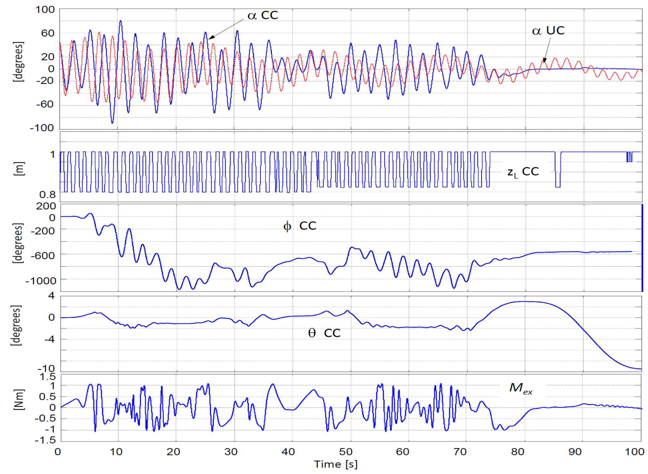

Figure 4 - Case study 1: Perturbation of the beacon-gimbal system through buoy roll and pitch motions

V. Conclusions

Based on previous works of the authors associated to an optimal oscillation control on a plane with knowledge-based parameter settings, this paper attempts to analyze eventual drops in the control performance under a wide spectrum of complex disturbances. Here the work has dealt with an holonomic-constrained control of a pendulum in a 3D space owing to its particular design. The controlled system is presented as a stabilized beacon-gimbal-mass device of three DOF mounted on a buoy for port signalization. It manipulates the position of a sliding mass on the rod extreme employing a parametric resonance effect to attenuate the oscillations. The general dynamics of this multibody is described wherein the buoy movements are assumed as perturbations of the device.

In broad outlines, the performance is outstanding when the motion plane of the pendulum is aligned with the wave direction. By pure yaw rotations of the buoy the control performance is barely influenced. Here, a tilt of the buoy aligned with the wave direction does not affect the control.

This situation can dramatically change when tilts of the buoy appear in different directions because they may induce strong gyroscopic moments that jeopardize the control end. As the device can not be aligned by self according to the wave direction, buoy tilts can produce many turns of the device about the vertical axis. Nevertheless the by-control damping of the oscillations are more effective in the CC than in the UC.

Some selected simulations attempt to illustrate the performance of our the oscillation control.

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