# ADAPTIVE CONTROL SYSTEM FOR STRUCTURALLY UNDEFINED THERMAL POWER PLANT ON SET OF FUNCTIONING STATES

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**Abstract**

In this paper the synthesis of adaptive control law for thermal power plant with input delay with a step-by-step changing dynamics is considered. The plant operates under conditions of a priori parametric and structural uncertainties when measuring only output signal. The structure of control system with an implicit reference model includes a predictor-compensator, command and output correction filters.

**Key words**

structural and parametric priory uncertainty, thermal power plant, hyperstability criterion, predictor-compensator, correction filter.

## 1 Introduction

Thermal engineering is one of the industries where all stages of heat and electrical energy production are subject for automation. However, the problem of designing the effective automated control systems for heat and power facilities remains quite acute, for which the following are typical: complex dynamics of the control plant, changes in the parameters and structure of such systems, various types of delays, etc. [Eremin and Telichenko, 2009]. All these problems become much more critical if one takes into account the fact that a significant part of enterprises engaged in the production of heat and electricity operate on coal raw materials and may have a non-block structure [Pletnev, 2007], which entails great difficulties both in the analysis and in the construction of control systems for such plants [Eremin and Telichenko, 2009; Pletnev, 2007; Klyuev, Lebedev and Novikov, 1985; Rotach, Kuzishchin and Petrov, 2010; Smetana, 2009; Smirnov, Sabanin and Repin, 2007; Pikina, Kocharovsky, 2006].

As the control plant we discuss the pressure control system in the common steam line of the cogeneration station with cross-links [Eremin and Telichenko, 2011]. The main task of the considered pressure control loop is to ensure the required operating mode of the turbine units, regardless of their load, by changing the load on the boilers. The structure considered in the paper is not the only one [Pletnev, 2007], however, at coal-fired power plants with a common steam line it is a kind of standard. When solving the problem of obtaining a mathematical description for control loop, one should note its nontriviality: on the one hand, the control plant is, in fact, a generalized boiler model, which is difficult to describe in the form of simple dynamic links; on the other hand, using the Supervisory Control And Data Acquisition (SCADA) systems, it is possible to construct the dynamics of all possible control sections for sufficiently long time interval [Kositsin, Rybalev and Telichenko, 2013].

In the modern automatic control theory, the development of controllers based on analytical methods for control systems with delay and step-by-step changing dynamics is an urgent task. Changing dynamics is a characteristic for energy, mechanical, and chemical plants. Each of the control systems for such plants consists of a set of switched subsystems and switching law that determines the activity of the subsystem in a certain period of time [Aleksandrov and Platonov, 2008; Barseghian, 2002; Barseghian, 2012; Benzaouia, 2012; Chiou et. al., 2010; Daafouz et. al., 2002; García-Planas, 2020; Gorodetsky, Skobelev and Marik, 2020; Tsykunov, 2017]. In the present paper an adaptive control algorithm for plant with input delay is obtained. Authors take into account the switching of the parameters and the relative order of plant, i.e. the considered control plant is priori parametrically and structurally uncertain.

## 2 Mathematical Model of the Control System

We consider a priory parametrically and structurally uncertain plant with control delay. The dynamics of the plant changes $Q$ times at the time intervals $0 = t_0 < t_1 < ... < t_Q$. In operator form the considered plant model can be written like

$$
\begin{gathered}
a^{(q)}(p)y^{(q)}(t) = b^{(q)}(p)u^{q}(t-h), \\
p^i y^{(1)}(0) = y_{i0},\ u^{(1)}(\theta) = \varphi(\theta), \theta \in [-h;0]\,,
\end{gathered}
\tag{1}
$$

where $q = 1, 2, ..., Q$ is the number of time intervals $(t_q - t_{(q-1)})$; $y^{(q)}(t)$ is the composite output of the plant; $u^{(q)}(t)$ is the composite control signal; $h$ is known constant time delay.

The plant (1) operates under following conditions:

1. $a^{(q)}(p)$ and $b^{(q)}(p)$ are normalized polynomials, $\deg a^{(q)}(p) = n_q$, $\deg b^{(q)}(p) = m_q$, $p = d/dt$ is the differentiation operator; $b^{(q)}$ are Hurwitz polynomials, $a^{(q)}$ are polynomials with arbitrary roots;
2. the plant (1) is parametrically uncertain, coefficients of polynomials $a^{(q)}(p)$ are $b^{(q)}(p)$ are unknown numbers which depends on a set of unknown parameters $\xi$ belonging to know bounded set $\Xi$;
3. the plant (1) is structurally uncertain, polynomials $a^{(q)}(p), b^{(q)}(p)$ degrees $n_q$ and $m_q$ are unknown numbers but maximum degree $p$ of polynomials $a^{(q)}(p)$ and constant relative order $\rho > 1$ both known;
4. only scalar output $y^{(q)}(t)$ of the plant (1) is available for the direct measurement.

To eliminate structural uncertainty, we use the approach proposed in [Hodgson and Stoten, 1998]. We write the equation of the plant dynamics(1) in the following equivalent form:

$$
\begin{gathered}
(p+\upsilon_0)^{(n-n_q)}a^{(q)}(p)y^{(q)}(t) = \\
= (p+\upsilon_0)^{(n-n_q)}b^{(q)}(p)u^{(q)}(t-h), \\
\upsilon_0 = const > 0,\ q = 1, ..., Q. \\
c^{(q)}(p)y^{(q)}(t) = L^{(q)}(p)u^{(q)}(t-h), \\
a^{(q)}(p)\,(p+\upsilon_0)^{(n-n_q)} = c^{(q)}(p) = \\
= p^n + c_{(n-1)}p^{(n-1)} + ... + c_1 p + c_0, \\
b^{(q)}(p)\,(p+\upsilon_0)^{(n-n_q)} = b_q L^{(q)}(p) = \\
= b_q(p^{(n-\rho)} + l_{(n-\rho-1)}p^{(n-\rho-1)} + ... \\
+ l_1 p + l_0).
\end{gathered}
\tag{2}
$$

We can write the mathematical description of the equivalent plant (2) in state-space form as following composite system of differential equation:

$$
\begin{gathered}
\frac{dx^{(q)}(t)}{dt} = A^{(q)}x^{(q)}(t) + B^{(q)}u^{(q)}(t-h), \\
y^{(q)}(t) = \left(L^{(q)}\right)^T x^{(q)}(t),
\end{gathered}
\tag{3}
$$

where $q = 1, 2, ..., Q$; $x^{(q)}(t) = \left[x_1^{(q)}(t), x_2^{(q)}(t), ..., x_n^{(q)}(t)\right]^T$ is the composite vector of state variables; $A^{(q)}$ are arbitrary state matrices in the Frobenius form of $n \times n$ size; $B^{(q)} = [0, ..., 0, b_q]^T$ are vectors of $n$ size; $L^{(q)}$ are constant vectors of $(n-\rho)$ size. We assume that trajectories between the systems of equations (3) are joined: $x^{(q+1)}(t_q) = x^{(q)}(t_q)$, $q = 1, 2, ..., Q-1$ .

Since the relative order of the plant (3) transfer functions at each interval is greater than one we connect the output filter-corrector (*OFC*) to output of the plant (3):

$$
\begin{gathered}
y_f^{(q)}(s) = W_{OFC}(s)y^{(q)}(s) = \\
= \left(\frac{T_0 s + 1}{T_* s + 1}\right)^{(\rho-1)} y^{(q)}(s),
\end{gathered}
\tag{4}
$$

where $y_f^{(q)}(t)$ is the output of the filter-corrector; $T_0$, $T_*$ are time constants, $T_*$ is sufficiently small [Eremin, 2013; Eremin and Shelenok, 2018].

Serial connection of the plant (3) and the filter (4) we can represent as serial connection of the modified control plant (*MCP*) and the block of structural perturbation (*BSP*):

$$
\begin{gathered}
y_f^{(q)}(s) = W_{MCP}^{(q)}(s) \cdot W_{BSP}^{(q)}(s)e^{-sh}u^{(q)}(s) = \\
= \frac{b_q \tilde{L}^{(q)}(s)}{c^{(q)}(s)} \cdot \frac{1}{(T_* s + 1)^{(\rho-1)}} e^{-sh}u^{(q)}(s),
\end{gathered}
\tag{5}
$$

where $\tilde{L}(q)(s) = L^{(q)}(s)\,(T_0 s + 1)^{(\rho-1)}$, $\deg \tilde{L}^{(q)} = (n-1)$, $L^{(q)}$ is numerator of the plant (3) transfer function. Then the relative order of the *MCP* will be equal to one ($\rho_{MCP}^{(q)} = 1$) at any functioning interval.

In accordance with [Eremin, 2013; Eremin and Shelenok, 2018] we exclude the *BSP* from model (5) and rewrite mathematical description of the *MCP* in state space like

$$\frac{dx^{(q)}(t)}{dt} = A^{(q)}x^{(q)}(t) + B^{(q)}u^{(q)}(t-h), \quad \tilde{y}^{(q)}(t) = \left(\tilde{L}^{(q)}\right)^T x^{(q)}(t), \tag{6}$$

where $y^{(q)}(t) \in R$ is composite output signal of the *MCP*.

To compensate the control delay we connect the predictor-compensator [Eremin, Nikiforova, Pikul and Telichenko, 2019] in parallel to the plant with following mathematical model:

$$(p+\chi_{0q})\, y_k^{(q)}(t) = \chi_{0q}\left(u^{(q)}(t) - u^{(q)}(t-h)\right), \quad p^i y_k^{(1)}(0) = 0,\ q = 1, ..., Q, \tag{7}$$

where $\chi_{0q} = const > 0$ is the parameter of the predictor-compensator.

In accordance with the methodology of synthesis we extend the state space of the predictor-corrector (7) using the Hurwitz polynomial $\tilde{L}^{(q)}(p) = L^{(q)}(p)\,(T_0p+1)^{(\rho-1)}$ of $(n-1)$ degree:

$$\tilde{L}^{(q)}(p)\,(p+\chi_{0q})\, y_k^{(q)}(t) = = \chi_{0q}\tilde{L}^{(q)}(p)\left(u^{(q)}(t) - u^{(q)}(t-h)\right).$$

Then this equation we can rewrite in the state space like:

$$\frac{dx_k^{(q)}(t)}{dt} = A_*^{(q)}x_k^{(q)}(t) + B_*^{(q)}(u^{(q)}(t)- -u^{(q)}(t-h)),\ y_k^{(q)}(t) = \left(\tilde{L}^{(q)}\right)^T x_k^{(q)}(t), \tag{8}$$

where $q = 1, ..., Q$, $x_k^{(q)}(t) = \left[x_{k1}^{(q)}(t), x_{k2}^{(q)}(t), ..., x_{kn}^{(q)}(t)\right]^T$ is the states composite vector of the predictor-compensator; $A_*^{(q)}$ are Hurwitz matrices in the Frobenius form of $(n \times n)$ size; $B_*^{(q)} = [0, ..., 0, \chi_{0q}]^T$ are vectors of $n$ size; $y_k^{(q)}(t) \in R$ is the composite output.

The relative order of the predictor-corrector (8) transfer functions at each time interval is equal to one

$$y_k^{(q)}(s) = W_{PC}^{(q)}(s)\left(1-e^{-sh}\right)u^{(q)}(s) = = \left(\tilde{L}^{(q)}\right)^T\left(sE - A_*^{(q)}\right)^{-1} B_*\left(1-e^{-sh}\right) \times \times u^{(q)}(s) = \frac{\chi_{0q}\tilde{L}^{(q)}(s)}{(s+\chi_{0q})\,\tilde{L}^{(q)}(s)}\left(1-e^{-sh}\right) \times \times u^{(q)}(s). \tag{9}$$

## 2.1 Problem Statement

The main goal of heat-and-power plant control is to ensure the desired dynamics of outputs $y^q(t)$, which consists in high-quality processing of the given signals $r^{(q)}(t)$, i. e. in the fulfillment of the limit inequality

$$\lim_{t\to\infty}\left|r^{(q)}(t) - y^{(q)}(t)\right| \le \delta_{0q},\ q = 1, ..., Q, \tag{10}$$

where $\delta_{0q} = const > 0$ are required constant values.

For the main control loop we may get a view of the command signals by analogy with ([Eremin, 2018]) with the help of command filter-corrector (*CFC*)

$$\tilde{r}^{(q)}(s) = W_{CFC}(s)r^{(q)}(s) = = \left(\frac{T_0s+1}{T_*s+1}\right)^{(\rho-1)} r^{(q)}(s). \tag{11}$$

Then, for the modified control plant (6), operating in structural-parametric uncertainty, the additional control goal can be formulated as following. It is required to synthesize the explicit form of control law

$$u^{(q)}(t) = u\Big(\tilde{y}^{(q)}(t),\ \tilde{y}_k^{(q)}(t),\ \tilde{r}^{(q)}(t), u^{(q)}(t-h)\Big),\ q = 1, ..., Q, \tag{12}$$

so that at measuring only variables $y^{(q)}(t)$ and any initial conditions $y^{(1)}(0)$ the following inequality:

$$\lim_{t\to\infty}\left|y_*^{(q)}(t) - \tilde{y}^{(q)}(t)\right| \cong \cong \lim_{t\to\infty}\left|\tilde{r}^{(q)}(t) - \tilde{y}^{(q)}(t)\right| \le \delta_{1q}, \quad q = 1, ..., Q, \tag{13}$$

where $\delta_{1q} = const > 0$ are maximum permissible errors in steady state modes; $y_*^{(q)}(t)$ are output variables of the implicit reference model (*IRM*):

$$y_*^{(q)}(t) = \frac{\chi_{0q}}{p+\chi_{0q}}\tilde{r}^{(q)}(t). \tag{14}$$

It is well known [Fradkov, 1974] that if $\chi_{0q} \gg 0,\ q = 1, ..., Q$, then for (14) we can conclude following

$$y_*^{(q)}(t) \cong \tilde{r}^{(q)}(t). \tag{15}$$

Thus, if it is possible to ensure the existence of additional conditions (13), then the main control goal (10) also can be fulfilled due to the equivalence of the transfer functions in equations (4), (11). It should be noted that for the mathematical model of the plant (6) we use its equivalent analogue instead of *IRM* (14):

$$y_*^{(q)}(t) = \frac{\chi_{0q}}{(p+\chi_{0q})} \cdot \frac{\tilde{L}^{(q)}(p)}{\tilde{L}^{(q)}(p)} \tilde{r}^{(q)}(t),$$

which in the state space has the form like:

$$\begin{aligned} \frac{dx_*^{(q)}(t)}{dt} &= A_*^{(q)} x_*^{(q)}(t) + B_*^{(q)} \tilde{r}^{(q)}(t), \\ y_*^{(q)}(t) &= \left(\tilde{L}^{(q)}\right)^T x_*^{(q)}(t), \end{aligned} \tag{16}$$

where $q = 1, ..., Q$; $x_*^{(q)}(t) = \left[x_{*1}^q(t), x_{*2}^{(q)}(t), ..., x_{*n}^{(q)}(t)\right]^T$ is the reference composite state vector; $y_*^{(q)}(t) \in R$ is the reference composite output. Let for the reference model (16) and the control plant (3) following conditions of structural matching are satisfied:

$$\begin{aligned} A_*^{(q)} &= A^{(q)} - \chi_{0q} B_* \tilde{L}^{(q)}, \\ B^{(q)} &= B_*^{(q)} \left(1 + k_{0q}\right), \end{aligned}$$

where $\chi_{0q} = const > 0$ is quite a large numbers; $k_{0q} = const > 0$, $q = 1, ..., Q$.

If inequality (13) is satisfied, due to the equality of the transfer functions of the output (4) and command (11) filter-correctors, the main control goal (10) with respect to outputs of the control plant (1) will also be fulfilled.

## 3 Synthesis of the Adaptive Control Law

To determine the explicit form of the control law (13), we use the hyperstability criterion ([Landau, Lozano, and Saad, 1998]), following which we consider signals $e^{(q)}(t) = x_*^{(q)}(t) - \left(x^{(q)}(t) - x_k^{(q)}(t)\right)$ and equivalent mathematical description of the system (1), (4), (7), (10), (11):

$$\begin{aligned} \frac{de^{(q)}(t)}{dt} &= A_*^{(q)} e^{(q)}(t) + B_*^{(q)} \mu^{(q)}(t), \\ v^{(q)}(t) &= y_*^{(q)}(t) - \tilde{y}^{(q)}(t) - y_k^{(q)}(t), \\ \mu^{(q)}(t) &= -u^{(q)}(t) + r(t) - \\ &- \chi_{0q} \tilde{y}^{(q)}(t) - k_{0q} u^{(q)}(t-h). \end{aligned} \tag{17}$$

The requirement of the hyperstability criterion about the strict positive definiteness of the real frequency response of the linear part of system (17) can be written as an inequality:

$$\begin{aligned} \text{Re}\left[\left(\tilde{L}^{(q)}\right)^T \left(j\omega E - A_*^{(q)}\right)^{-1} B_*^{(q)}\right] > 0, \\ \forall \omega \geq 0, \end{aligned} \tag{18}$$

which always takes place on each time interval, since its transfer function, taking into account (8), coincides with the transfer function of the first-order aperiodic link:

$$W^{(q)}(s) = \left(\tilde{L}^{(q)}\right)^T \left(sE - A_*^{(q)}\right)^{-1} B_*^{(q)} = \frac{\chi_{0q}}{s + \chi_{0q}}.$$

The next requirement of the hyperstability criterion concerns the nonlinear nonstationary part of the equivalent system (17). Following ([Landau, Lozano, and Saad, 1998]), it must satisfy the integral inequality

$$\begin{aligned} \eta^{(q)}(0,t) = -\int_0^t \mu^{(q)}(\theta) v^{(q)}(\theta) d\theta > -\eta_0^{(q)}, \\ \eta_0^{(q)} = const, \ \forall t > 0. \end{aligned} \tag{19}$$

Substituting the nonlinear function $\mu^{(q)}(t)$ from (16) into (18), it is possible to determine following explicit form of the control algorithm which ensure inequality (20) satisfaction (see Appendix A):

$$\begin{aligned} u(t) = r(t) + h_1 \tilde{y}^{(q)}(t) \int_0^t \tilde{y}^{(q)}(\theta) v^{(q)}(\theta) d\theta + \\ + h_2 \left(\tilde{y}^{(q)}(t)\right)^2 v^{(q)}(t) + \\ + h_3 u^{(q)}(t-h) \int_0^t u^{(q)}(\theta - h) v^{(q)}(\theta) d\theta, \end{aligned} \tag{20}$$

where $h_i = const >)$, $i = 1, 2, 3$ are parameters of the law (21) the values of which are selected during the simulation.

By choosing a small parameter $T_*$, adaptive control law (20) guarantees system (6), (8), (10), (20) its *L*-dissipativity, and fulfillment of the control goal (13). Due to the fact that the transfer functions of the command and output filter-correctors are coincide, the fulfillment of the control goal (13) implies fulfillment of the control goal (15), i.e. system (1), (4), (7), (11), (20) will be *L*-dissipative.

## 4 Simulation Results

We obtain the mathematical description of the control plant according to practical data for boiler No. 1 of BKZ-420-140-7 type of the Blagoveshchensk Cogeneration Station. The plant has a classical structural uncertainty, dictated by the following factors:

1. fuel parameters (humidity, calorific value, etc.), operating modes of the boiler unit cannot be assessed with the help of classical methods;

2. disturbances acting from other boilers, the length of the steam line itself, operating modes of turbine units are objectively structural disturbances for such plant;
3. operating mode of the main regulator is not defined, since for various reasons the control systems included in its composition may be in different states (for example, loading of mills regulator for mills A and B in the automated mode; mills C and G in manual control; which can be dictated by the task of ensuring optimal combustion efficiency, since different mills affect different front of burners and these factors are a priori unknown).

By processing practically obtained data made it possible to obtain the following set of equations for the control plant object on the set of functioning states:

1. at the time interval $0 < t < t_1 = 13000$ (s):

$$\begin{aligned}&\left(210^2p^2 + 2\cdot 210\cdot 0.2p + 1\right)\times\\ &\times(201p+1)\,y^{(1)}(t) =\\ &= 0.148\,(p+1)\,u^{(1)}(t-116);\end{aligned} \tag{21}$$

2. at time interval $13000 < t < t_2 = 26000$ (s):

$$\begin{aligned}&\left(203.33^2p^2 + 2\cdot 203.33\cdot 0.383p + 1\right)\times\\ &\times y^{(2)}(t) = 0.209u^{(2)}(t-116);\end{aligned} \tag{22}$$

3. at time interval $26000 < t < t_3 = 38000$ (s):

$$\begin{aligned}&\left(200^2p^2 + 2\cdot 200\cdot 0.5p + 1\right)\times\\ &\times(100p+1)\,y^{(3)}(t) =\\ &= 0.187\,(p+1)\,u^{(3)}(t-116).\end{aligned} \tag{23}$$

In the course of simulation we studied the functioning of the plant with following non zero initial conditions: $y_i(0) = 1,\ i = 1,2,3$; at the first time interval third order plant (21) was considered, at the second time interval second order plant (22) was considered, at the third time interval third order plant (23) was considered.

Maximum degree of the considered thermal power plant equals to 3, relative order equals to 2.

Transfer function of the *CFC* and *OFC* has the form as follows:

$$W_{CFC}(s) = W_{OFC}(s) = \frac{0.1s+1}{0.001s+1}.$$

The command signals were:

$$r^{(1)}(t) = 0.3\left(1 + \frac{2}{3}\exp(-0.05t) - \frac{5}{3}\exp(-0.02t)\right),$$

$$\begin{aligned}r^{(2)}(t) = 0.8\Bigg(&1 + \frac{2}{3}\exp(-0.05(t-13000)) -\\ &-\frac{5}{3}\exp(-0.02(t-13000))\Bigg),\end{aligned}$$

$$\begin{aligned}r^{(3)}(t) = -0.4\Bigg(&1 + \frac{2}{3}\exp(-0.05(t-26000)) -\\ &-\frac{5}{3}\exp(-0.02(t-26000))\Bigg).\end{aligned}$$

Parameters of the predictor-compensator were: $\chi_{0q} = 150$.

After several stages of the system simulation the parameters of control law (20) were chosen like: $h_1 = 100$, $h_2 = 200$, $h_3 = 5$. Simulation results are presented at Fig. 1 and 2.

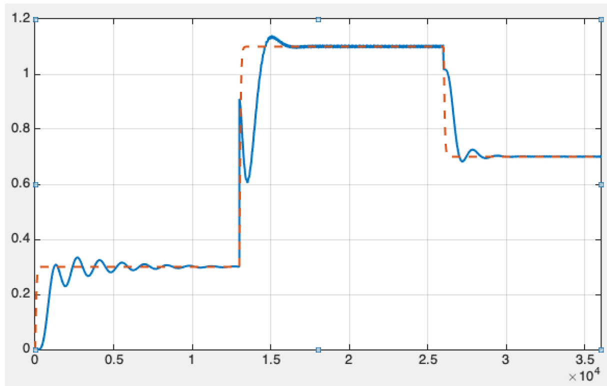

Figure 1. Dynamic processes of the control system: solid line is output of the control plant (1); dotted lone is the command signal $r^{(q)}(t)$

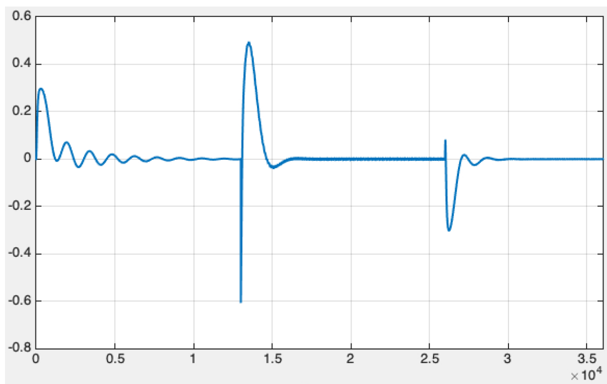

Figure 2. Signal of the error between the plant output (1) and the command signal $r^{(q)}(t)$

## 5 Conclusion

In the paper with the help of hyperstability criterion the adaptive control law for thermal power plant with control delay and step-by-step changing dynamics is synthesized. Considered plant functioning at priory parametric and structural uncertainties. In course of simulation of the obtained system values of the control errors in steady state does not exceed $0.5\%$. This circumstance indicates a quite enough operating quality of the control system.

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## References

Aleksandrov, A. Yu., and Platonov, A. V. (2008). On Absolute Stability of One Class of Nonlinear Switched Systems. *Autom. Remote Control*, **69** (2), pp. 1101–1116.

Barseghian, V. R. (2008). About a Problem of Optimal Control of Stepwise Changing Linear Systems with Phase States Restrictions in Intermediate Momente. *Proceedings of the YSU A: Physical and Mathematical Sciences*, (1), pp. 118–120. (in Russian).

Barseghian, V. R. (2012). A Constructive Approach to the Study of Control Problems of Linear Systems. *Control Sciences*, **2**, pp. 11–17. (in Russian)

Benzaouia, A. (2012). Introduction to Switched Systems. Saturated Switching Systems. *Lecture Notes in Control and Information Sciences*, **426**, Springer, London.

Chiou, J.-S., Wang, C.-J., Cheng, C.-M., and Wang C.-C. (2010). Analysis and synthesis of switched nonlinear systems using the T–S fuzzy model. *Applied Mathematical Modelling*, **34** (6), pp. 1467–1481.

Daafouz, J., Riedinger, P., and Iung, C. (2002). Stability Analysis and Control Synthesis for Switched Systems: a Switched Lyapunov Function Approach. *IEEE Transactions on Automatic Control*, **47** (11), pp. 1883–1887.

Eremin, E. L. (2013). *L*-dissipativity of Hyperstable Control System in Structural Indignation. IV. *Infromation Science and Control Systems*, **2** (36), pp. 100–106. (in Russian)

Eremin, E. L. (2018). Combined System with an Implicit Standard for a Class of Uncertain a Priory Single-Channel Non-Affine Control Plants on the Set of Operation States. *Infromation Science and Control Systems*, **3** (57), pp. 93–103. (in Russian).

Eremin, E. L., and Telichenko, D. A. (2009). *Adaptive and Robust Control of Heat Power Plants*. Amur state university, Blagoveshchensk. (in Russian).

Eremin, E. L., and Telichenko, D. A. (2011). The Automated System of Regulation of Fuel Consumption at Thermal Power Manufacture. *Information Science and Control Systems*, **2** (28), pp. 157–168. (in Russian).

Eremin, E. L., Nikiforova, L. V., Pikul, Z. D., and Telichenko, D. A. (2019). Combined Controller for Structurally and Parametrically Indeterminate Plant with Control Delay. *Sensors and Systems*, **10** (240), pp. 20–26. (in Russian).

Eremin, E. L., and Shelenok, E. A. (2018). Nonlinear-Periodic Control System for Non-Affine MIMO Plant with State Delay. *Cybernetics and Physics*, **7** (4), pp. 195–203.

Fradkov, A. L. (1974). Synthesis of Adaptive System of Stabilization for Linear Dynamic Plants. *Autom. Remote Control*, **35** (12), pp. 1960–1966.

García-Planas, M. I. (2020). Analyzing Control Properties of Multi-Agent Linear Systems. *Cybernetics and Physics*, **9** (2), pp. 81–85.

Gorodetsky, V., Skobelev, P., and Marik, V. (2020). System Engineering View on Multi-Agent Technology for Industrial Applications: Barriers and Prospects. *Cybernetics and Physics*, **9** (1), pp. 13–30.

Hodgson, S. P., and Stoten, D. P. (1998) Robustness of the Minimal Control Synthesis Algorithm with Minimal Phase Plant Dynamics of Unknown Order but Known Relative Degree. *International Journal of Control*, **71** (1), pp. 1–17.

Klyuev, A. S., Lebedev, A. T., and Novikov, S. I. (1985). *Adjustment of Automatic Control Systems for Drum Steam Boilers*. M.: Energoatomizdat. (in Russian).

Kositsin, V. Yu., Rybalev, A. N., and Telichenko, D. A. (2013). A System for Controlling the Boiler Heat Load. *Thermal Engineering*, **60** (2), pp. 130–136.

Landau, I. D., Lozano, R., and Saad M. M. (1998). *Adaptive Control*. L: Springer-Verlag.

Pikina, G. A., and Kocharovsky, D. N. (2006). Investigation of a System with a Maximally Rapid Response Prognostic Algorithm. *Thermal Engineering*, **53**, (10), pp. 819–823.

Pletnev, G. P. (2007). *Automation of Technological Processes and Production in Heat Power Engineering*. 4th ed., Stereo type, M .: Ed. House MEI. (in Rissian).

Rotach, V. Ya., Kuzishchin V. F., and Petrov S. V. (2010). Tunning of Industrial Controllers from the Transient Responses of Control Systems without Approximating them by Analytical Expressions. in *Thermal Engineering*, **57** (10), pp. 872–879.

Smetana A. Z. (2009). Modified Technique of Automatic and Automated Tunning of Thermal-Engineering Process Controllers. *Thermal Engineering*, **56** (4), pp. 312–315.

Smirnov, N. I., Sabanin, V. R., and Repin A. I. (2007). Robust Tunning of Two-Loop Automatic Control Systems. *Thermal Engineering*, **54** (7), pp. 563–571.

Tsykunov, A. M. (2017). Robust Control of Linear Plants With Switching. *Control Sciences*, **4**, pp. 2–7.

## Proof of the integral inequality (19) fulfillment

Let us show that synthesis of the control law in the form (20) allow us to fulfill the integral inequality (19).

The left part of the (19) respecting the form of $\mu^{(q)}(t)$ from equation (17) we can rewrite as follows:

$$\eta^{(q)}(0,t) = \int_0^t \left(u^{(q)}(\theta) - \tilde{r}^{(q)}(\theta)\right) v^{(q)}(\theta)d\theta + \\ + \chi_{0q}\int_0^t \tilde{y}^{(q)}(\theta)v^{(q)}(\theta)d\theta + \\ + k_{0q}\int_0^t u^{(q)}(\theta-h)v^{(q)}(\theta)d\theta. \tag{24}$$

It is appropriate to describe difference signal $\left(u^{(q)}(t) - \tilde{r}^{(q)}(t)\right)$ as a sum $\left(u_1^{(q)}(t) + u_2^{(q)}(t)\right)$. Then we have the signal like

$$u^{(q)}(t) = \tilde{r}^{(q)}(t) + u_1^{(q)}(t) + u_2^{(q)}(t), \tag{25}$$

where $u_1^{(q)}(t)$ and $u_2^{(q)}(t)$ are summands to be determined.

The integral (24) taking into account (25) it is possible to represent in the following form

$$\eta^{(q)}(0,t) = \eta_1^{(q)}(0,t) + \eta_2^{(q)}(0,t), \\ \eta_1^{(q)}(0,t) = \int_0^t u_1^{(q)}(\theta)v^{(q)}(\theta)d\theta + \\ + \chi_{0q}\int_0^t \tilde{y}^{(q)}(\theta)v^{(q)}(\theta)d\theta, \\ \eta_2^{(q)}(0,t) = \int_0^t u_2^{(q)}(\theta)v^{(q)}(\theta)d\theta + \\ + k_{0q}\int_0^t u^{(q)}(\theta-h)v^{(q)}(\theta)d\theta.$$

The component $u_1^{()q}(t)$ we synthesize as

$$u_1^{(q)}(t) = h_1\tilde{y}^{(q)}(t)\int_0^t \tilde{y}^{(q)}(\theta)v^{(q)}(\theta)d\theta + \\ + h_2\left(\tilde{y}^{(q)}(t)\right)^2 v^{(q)}(t),\ h_1,\ h_2 = const > 0. \tag{26}$$

Then the summand $\eta_1^{(q)}(0,t)$ we can estimate like

$$\eta_1^{(q)}(0,t) = \int_0^t u_1^{(q)}(\theta)v^{(q)}(\theta)d\theta + \\ + \chi_{0q}\int_0^t \tilde{y}^{(q)}(\theta)v^{(q)}d\theta = \\ = \int_0^t \Bigg(h_1\tilde{y}^{(q)}(\theta)\int_0^\theta \tilde{y}^{(q)}(\vartheta)v^{(q)}(\vartheta)d\vartheta + \\ + h_2\left(\tilde{y}^{(q)}(\theta)\right)^2 v^{(q)}(\theta)\Bigg)v^{(q)}(\theta)d\theta + \\ + \chi_{0q}\int_0^t \tilde{y}^{(q)}(\theta)v^{(q)}(\theta)d\theta = \\ = h_1\int_0^t \tilde{y}^{(q)}(\theta)v^{(q)}(\theta)\int_0^\theta \tilde{y}^{(q)}(\vartheta)v^{(q)}(\vartheta)\times \\ \times d\vartheta d\theta + h_2\int_0^t \left(\tilde{y}^{(q)}(\theta)v^{(q)}(\theta)\right)^2 d\theta + \tag{27}$$

If we synthesize the component $u_2^{(q)}(t)$ as follows

$$u_2^{(q)}(t) = h_3u^{(q)}(t-h)\int_0^t u^{(q)}(\theta-h)\times \\ \times v^{(q)}(\theta)d\theta,\ h_3 = const > 0, \tag{28}$$

then for the summand $\eta_2^{(q)}(0,t)$ we can obtain following estimate:

$$\eta_2^{(q)}(0,t) = \int_0^t u_2^{(q)}(\theta)v^{(q)}(\theta)d\theta + \\ + k_{0q}\int_0^t u^{(q)}(\theta-h)v^{(q)}(\theta)d\theta = \\ = \int_0^t h_3u^{(q)}(\theta-h)\int_0^\theta u^{(q)}(\vartheta-h)\times \\ \times v^{(q)}(\vartheta)d\vartheta\theta + k_{0q}\int_0^t u^{(q)}(\theta-h)v^{(q)}(\theta)d\theta \geq \\ \geq \frac{1}{2}\Bigg(h_3\left(\int_0^t u^{(q)}(\theta-h)v^{(q)}(\theta)d\theta\right)^2 + \\ + 2k_{0q}\int_0^t u^{(q)}(\theta-h)v^{(q)}(\theta)d\theta + \frac{k_{0q}^2}{2h_3}\Bigg) - \\ - \frac{k_{0q}^2}{2h_3} \geq -\frac{k_{0q}^2}{2h_3} = -\eta_{02}^{(q)}, \\ \eta_{02}^{(q)} = const,\ \forall t > 0. \tag{29}$$

We take into account obtained estimates (27) and (29) and obtain the relation

$$\eta^{(q)}(0,t) \geq -\eta_{01}^{(q)} - \eta_{02}^{(q)} = -\eta_0^{(2)} = const, \forall t > 0,$$

that means the integral inequality (19) fulfillment.

Thus the synthesized control law that has general form (12) with respect to (25), (26) and (28) we should write like (20).