# ELECTROMECHANICAL AND MATHEMATICAL MODELS OF SYNCHRONOUS ELECTRICAL MACHINES

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**Abstract**
Two electromechanical models of synchronous machines are considered. For these models the differential equations with angular coordinates are obtained.

**Key words**
Stability, synchronous machines, rotor, stator, differential equations.

## 1 Introduction

The rotating magnetic field, generated by alternate current in stator windings, is one of the basic elements of synchronous and asynchronous ac motors [Gorev, 1985; Yanko-Trinitskii, 1958; Kononenko, Sipailov and Khor'kov, 1975; Vazhnov, 1969; Adkins, 1960; Vol'dek, 1980; Ivanovo-Smolenskii, 1980; Venikov, 1985]. For the first time such a field was obtained by N. Tesla and G. Ferraris in 1888 year.

In this case a natural step is the study of a rotor motion of synchronous machine in the rotating coordinates connected with a rotating magnetic field, generated by currents in stator windings. The present work is devoted to the derivation of differential equations for rotor motion in such coordinate system.

The generation of equations for different types of rotors turns out rather simple and natural. In addition, for electromechanical models of T-rotor (Fig. 1) such equations coincide with well-known equations, considered in the works [Szego, Olech, and Cellina, 1968; Tricomi, 1931; Tricomi, 1933; Andronov, Vitt, and Khaikin, 1959; Barbashin, Tabueva, 1969; Fagiuoli, Szegö, 1970].

These equations are highly distinct from the equations obtained here also in the rotating coordinates for differential equations of electromechanical model of salient-pole rotor (Fig. 2). For such a model the problems on a static stability of synchronous machines are also considered.

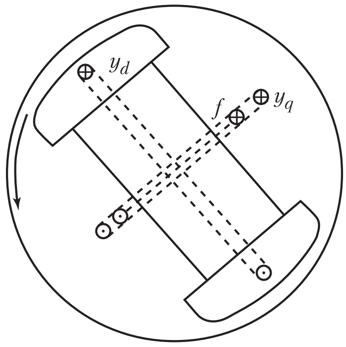


Figure 1. $f$ is an exciting winding. $y_q$ and $y_d$ are damper windings.

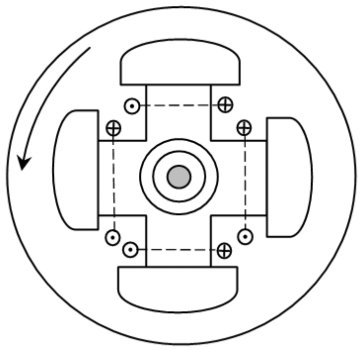

Figure 2. Salient-pole rotor. Exciting windings are shown.

## 2 T-rotor

Consider first the motion of one wind of winding in the rotating coordinates, rigidly connected with magnetic vector. Suppose that the constant voltage $e$ is applied to the wind. The current $i(t)$ in winding is determined with regard to Ohm's law and the law of electromagnetic induction.

$$L \cdot \frac{di(t)}{dt} + R \cdot i(t) = e + nSB(\sin \theta(t)) \cdot \dot{\theta}(t) \quad (1)$$

In this case the equation of motion of rotor with the located on it exciting winding with respect to rotating magnetic field takes the form

$$I\ddot{\theta} = -\beta nSBi(t)\sin\theta(t) - M. \tag{2}$$

In equations (1), (2) $R$ is a resistance $L$ is a winding inductance, $S$ is an area of one wind of winding, $n$ is a number of winds in winding, $B$ is a magnetic intensity $\theta(t)$ is the angle included between the plane of wind with current $i_1$ and the plane, which is perpendicular to magnetic vector; $I$ is a moment of inertia of rotor, $\beta$ is a coefficient of proportionality, $M$ is a moment of external load.

The change of variables $\theta := -\theta$, $\dot{\theta} = -\eta$, $z = i(t) + \frac{nSB}{L}\cos\theta$ makes it possible to reduce system (1), (2) to the third order system

$$\begin{aligned} &\dot{\theta} = \eta, \\ &\dot{\eta} = a_1 - a_2 z\sin\theta + a_3\sin\theta\cos\theta, \\ &\dot{z} = a_4 - a_5 z + a_6\cos\theta, \end{aligned} \tag{3}$$

which as a model of synchronous machine was studied in [Szego, Olech, and Cellina, 1968].

Here $a_1 = \frac{M}{I}$, $a_2 = \frac{\beta nSB}{I}$, $a_3 = \frac{\beta(nSB)^2}{IL}$, $a_4 = \frac{e}{L}$, $a_5 = \frac{R}{L}$, $a_6 = \frac{nRSB}{L^2}$.

Suppose, in rotor slots there are two perpendicular to each other windings. They are schematically shown in Fig. 3. The constant voltage $e$ is applied (usually by electromotor brushes) to winding with the current $i_1(t)$. The damping winding with the current $i_2(t)$ is short-circuited.

Now we consider, as before, the motion of two winds of winding in the rotating coordinates rigidly connected with magnetic vector [Leonov, 2006; Leonov, 2006a]. The parameters $R, L, S$ of windings are assumed to be the same. In this case if the mutual induction of windings can be neglected, then the currents $i_1(t)$ and $i_2(t)$ in windings are given by formulas

$$\begin{aligned} &L\,\frac{di_1(t)}{dt} + Ri_1(t) = e + nSB(\sin\theta(t))\,\dot{\theta}(t), \\ &L\,\frac{di_2(t)}{dt} + Ri_2(t) = nSB(\cos\theta(t))\,\dot{\theta}(t). \end{aligned} \tag{4}$$

Here $\theta(t)$ is the angle included between the plane of a wind of winding with the current $i_1$ and the plane, which is perpendicular to magnetic vector.

The equation of motion of rotor with the located on it two windings with respect to rotating magnetic field is as follows

$$I\ddot{\theta} = -\beta nSB(i_1(t)\sin\theta + i_2(t)\cos\theta) - M. \tag{5}$$

We assume first that $L = 0$. Then, substituting (4) in (5), we obtain

$$I\ddot{\theta} = -\frac{\beta(nSB)^2}{R}\,\dot{\theta} - \frac{\beta nSBe}{R}\,\sin\theta - M.$$

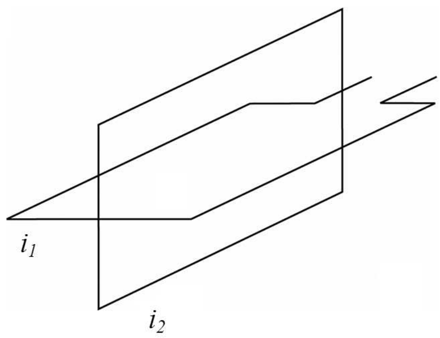


Figure 3. Scheme of two windings. Here it is shown a wind of exciting winding with the current $i_1(t)$ and a wind of damping winding with the current $i_2(t)$.

By the change $\theta \to -\theta$ we can represent this equation in the form

$$\ddot{\theta} + k\theta + b\sin\theta = \gamma. \tag{6}$$

Here

$$k = \frac{\beta(nSB)^2}{IR}, \qquad b = \frac{\beta nSBe}{IR}, \qquad \gamma = \frac{M}{I}.$$

Without loss of generality, suppose $b = 1$. Equation (6) can be reduced to the equation of above type by making a change in the time: $\tau = t \cdot \sqrt{b}$.

Let us change from equation (7) to the equivalent system

$$\begin{aligned} &\dot{\theta} = \eta, \\ &\dot{\eta} = -\alpha\eta - \sin\theta + \gamma. \end{aligned} \tag{7}$$

To the synchronous operating mode of synchronous machine corresponds the equilibrium state of system (7):

$$\theta(t) \equiv \theta_0, \quad \eta(t) \equiv 0,$$

where $\theta_0$ satisfies the relations

$$\sin\theta_0 = \gamma, \quad \cos\theta_0 > 0.$$

This operating mode occurs for $\gamma < 1$. In this case it is locally stable.

The equilibrium states of system (7)

$$\theta(t) \equiv \theta_1, \quad \eta(t) \equiv 0,$$

where $\theta_1$ satisfies the relations

$$\sin\theta_1 = \gamma, \quad \cos\theta_1 < 0,$$

are unstable and correspond to physically nonrealizable operating mode of synchronous machine.

In the case $\gamma > 1$ system (7) does not have equilibrium states (its stationary set is empty).

Note that the stationary sets of system (4), (5) for $L = 0$ and for $L > 0$ coincide. Also, a local stability and instability of stationary solutions in passing from equation (6) (the case $L = 0$) to system (4), (5) (the case $L \geq 0$) are preserved.

Note that the angle $\theta$ is called an operational angle of synchronous motor, $\eta$ is a motor slip.

Consider further the electromechanical model of three windings, shown in Fig. 4.

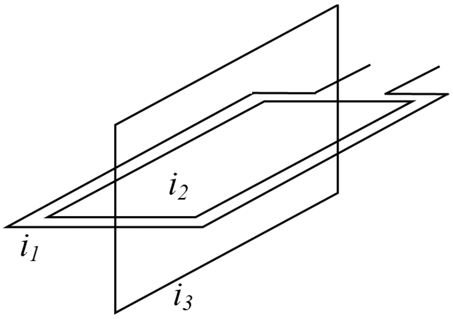


Figure 4. Scheme of three windings. Her it is shown two parallel winds: a wind of exciting winding with the current $i_1(t)$ and a transverse damping winding with the current $i_2(t)$ and the orthogonal to them wind of longitudinal damping winding with the current $i_3(t)$.

Consider, as previously, the motion of windings in the rotate coordinates, rigidly connected with the magnetic vector $B$. We introduce the parameters of windings: $S_1, S_2, S_3$ are areas of every wind, $L_1, L_2, L_3$ are the inductance of windings, $R_1, R_2, R_3$ are the resistance of windings. With neglect of mutual induction of windings we obtain the following system for the currents $i_1(t), i_2(t), i_3(t)$:

$$
\begin{aligned}
&L_1 \cdot \frac{di_1(t)}{dt} + R_1 \cdot i_1(t) = n_1 S_1 B \cdot \sin\theta(t) \cdot \dot{\theta}(t) + e,\\
&L_2 \cdot \frac{di_2(t)}{dt} + R_2 \cdot i_2(t) = n_2 S_2 B \cdot \sin\theta(t) \cdot \dot{\theta}(t),\\
&L_3 \cdot \frac{di_3(t)}{dt} + R_3 \cdot i_3(t) == n_3 S_3 B \cdot \cos\theta(t) \cdot \dot{\theta}(t).
\end{aligned}
\tag{8}
$$

For the dynamics of rotor with the located on it three windings in rotating magnetic field we have the following equation

$$
\begin{aligned}
I\ddot{\theta} = &-\beta B(n_1 S_1 i_1(t)\sin\theta + n_2 S_2 i_2(t)\sin\theta +\\
&+ n_3 S_3 i_3(t)\cos\theta) - M.
\end{aligned}
\tag{9}
$$

Here $n_1, n_2, n_3$ are the number of winds in each winding.

Neglecting the inductance of damping windings, i.e. assuming in system (8), (9) $L_2 = L_3 = 0$ and making the transformation of coordinates: $\theta = -\theta$, $\eta = -\dot{\theta}$, $z = i_1(t) - e/R_1$, we can reduce system (8), (9) to the form

$$
\begin{aligned}
&\dot{\theta} = \eta,\\
&\dot{\eta} = -(a_1 + a_2\cos 2\theta)\eta - a_3 z\sin\theta - (a_4\sin\theta - a_5),\\
&\dot{z} = -a_6 z + a_7\eta\sin\theta.
\end{aligned}
\tag{10}
$$

Here $a_1 = \frac{\beta B^2}{2I}\left(\frac{n_2^2 S_2^2}{R_2} + \frac{n_3^2 S_3^2}{R_3}\right)$, $a_2 = \frac{\beta B^2}{2I}\left(\frac{n_3^3 S_3^2}{R_3} - \frac{n_2^2 S_2^2}{R_2}\right)$, $a_3 = \frac{\beta n_1 B S_1}{I}$, $a_4 = \frac{\beta n_1 S_1 B e}{I R_1}$, $a_5 = \frac{M}{I}$, $a_6 = \frac{R_1}{L_1}$, $a_7 = \frac{n_1 S_1 B}{L_1}$.

In the treatise [Yanko-Trinitskii, 1958] system (10) is represented as the equations of nonsalient-pole synchronous motor in the case of approximate account of damping windings.

Having performed the change of variables: $\theta := -\theta$, $\eta = -\dot{\theta}$, $y_1 = i_1 + \frac{n_1 S_1 B}{L_1}\cos\theta$, $y_2 = i_2 + \frac{n_2 S_2 B}{L_2}\cos\theta$, $y_3 = -i_3 + \frac{n_3 S_3 B}{L_3}\sin\theta$ we can represent system (8), (9) in the form

$$
\begin{aligned}
&\dot{\theta} = \eta,\\
&\dot{\eta} = (-b_1 y_1 - b_2 y_2)\sin\theta - b_3 y_3\cos\theta +\\
&\quad + b_4\sin\theta\cos\theta + b_5\,,\\
&\dot{y}_1 = b_6 - b_7 y_1 + b_8\cos\theta,\\
&\dot{y}_2 = -b_9 y_2 + b_{10}\cos\theta,\\
&\dot{y}_3 = -b_{11} y_3 - b_{12}\sin\theta,
\end{aligned}
\tag{11}
$$

with the positive parameters $b_j$ $(j = 1, 2, \dots, 13)$.

In the work [Fagiuoli, Szegö, 1970] the following system, which is close to system (11) and has the form

$$
\begin{aligned}
&\dot{\theta} = \eta,\\
&\dot{\eta} = (-a_1 x_1 - a_2 x_2)\sin\theta - a_3 x_3\cos\theta +\\
&\quad + a_4\sin\theta\cos\theta,\\
&\dot{x}_1 = a_5 - a_6 x_1 + a_7 x_2 + a_8\cos\theta,\\
&\dot{x}_2 = a_9 x_1 - a_{10} x_2 + a_{11}\cos\theta,\\
&\dot{x}_3 = -a_{12} x_3 - a_{13}\sin\theta
\end{aligned}
\tag{12}
$$

with the positive parameters $a_i$ $(i = 1, 2, \dots, 13)$ such that $a_6 a_{10} - a_7 a_9 > 0$, is regarded as the system of equations of synchronous motor with zero load.

By the nonsingular linear transformation of variables $y_1, y_2, y_3$, system (11) can be reduced to system (12).

All the derived here equations are obtained due to a general approach when the coordinate system, rigidly connected with rotating magnetic field, and the motion of electromechanical model of synchronous machine in this coordinate system are used. Such consideration is rather demonstrative and it simplifies the obtaining of well-known equations of Gorev–Park [Gorev, 1985; Yanko-Trinitskii, 1958; Kononenko, Sipailov and Khor'kov, 1975; Vazhnov, 1969; Adkins, 1960; Vol'dek, 1980; Ivanovo-Smolenskii, 1980] under above conditions.

## 3 Salient-pole rotor

Suppose now that in the rotor slots there is located an exciting winding only. It is shown schematically in Fig. 5.

By assumption all four winds of winding are the same. As in the case of two windings we consider the motion of two pairs of parallel winds of windings in the coordinate system rigidly connected with magnetic vector. Then the currents $i_1(t), i_2(t)$ and $i_3(t), i_4(t)$ in the first and second pairs of winds of windings are given by the following equations

$$L\frac{di_1(t)}{dt} + Ri_1(t) = e + (2nlB\sin\alpha)\sin\theta(t)\cdot\dot\theta(t), \tag{13}$$

$$L\frac{di_2(t)}{dt} + Ri_2(t) = e + (2nlB\sin\alpha)\sin\theta(t)\cdot\dot\theta(t), \tag{14}$$

$$L\frac{di_3(t)}{dt} + Ri_3(t) = e + (2nlB\sin\alpha)\cos\theta(t)\cdot\dot\theta(t), \tag{15}$$

$$L\frac{di_4(t)}{dt} + Ri_4(t) = e + (2nlB\sin\alpha)\cos\theta(t)\cdot\dot\theta(t), \tag{16}$$

Here the parameters $R, L, B, e$ and the variable angle $\theta(t)$ are the same as before.

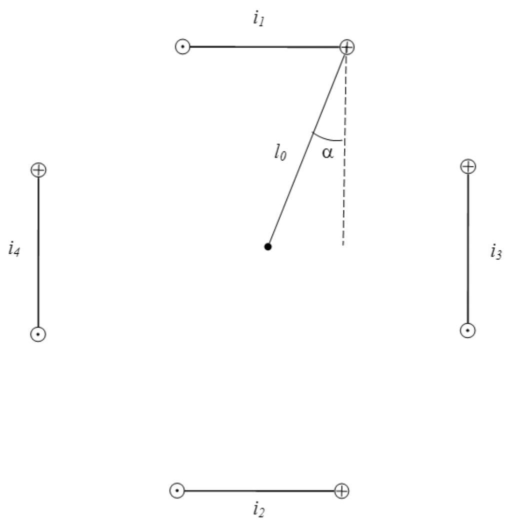


Figure 5. Scheme of four windings. Here it is shown two orthogonal pairs of parallel winds of exciting winding (a pair with the currents $i_1(t)$ and $i_2(t)$ and a pair with the currents $i_3(t)$ and $i_4(t)$)

The equation of motion of rotor with the located on it two pairs of parallel winds of exciting winding with respect to rotating magnetic field is as follows

$$\begin{aligned} I\ddot\theta = -2\beta nll_0\sin\alpha[(i_1(t)+i_2(t))\sin\theta + \\ +(i_3(t)+i_4(t))\cos\theta] - M. \end{aligned} \tag{17}$$

Here $l$ is a length of one side of wind, $l_0$ is a length of rotation radius-vector, $n$ is a number of winds in each winding, $\beta$ is a coefficient of proportionality.

Equations (13)–(17) are the equations of salient-pole synchronous machine.

Assuming $L = 0$, substituting (13)–(16) in (17), and replacing $\theta$ by $-\theta$, we obtain

$$I\ddot\theta + k\dot\theta + b\sin\left(\theta - \frac{\pi}{4}\right) = \gamma. \tag{18}$$

Here $k = 8\beta l_0 nSB^2\sin^2\alpha/(IR)$, $b = 4\sqrt{2}\,\beta neBll_0\sin\alpha/(IR)$, $\gamma = M/I$.

By the change of variables $\theta = \theta - \frac{\pi}{4}$ equation (18) can be reduced to equation (6).

Note that a damping moment occurs in the case of orthogonal disposition of windings. Therefore in a salient-pole machine this moment is supplied by exciting windings.

The following notation are needed for the sequel:

$$a_\alpha = \frac{2nlB}{L}\sin\alpha, \qquad b_\alpha = \frac{2\beta nBll_0}{I}\sin\alpha.$$

By the change in system (13)–(17) from $\theta$ to $-\theta$ equation (17) takes the form

$$\begin{aligned} &\dot\theta = \eta, \\ &\dot\eta = -b_\alpha\big(i_1(t)+i_2(t)\big)\sin\theta - b_\alpha\big(i_3(t)+i_4(t)\big)\cos\theta - \gamma. \end{aligned} \tag{19}$$

The equilibrium states of system (13)–(16), (19) under the condition

$$\gamma < \frac{2\sqrt{2}\,eb_\alpha}{R} \tag{20}$$

are the points $\theta = \theta_i + 2n\pi$ ($n$ is an integer number), $\eta = 0$, $i_1(t) = i_2(t) = i_3(t) = i_4(t) = e/R$. Here $\theta_i$ $(i = 0, 1)$ are the roots of the following equation

$$\varphi(\theta) = \gamma, \tag{21}$$

where

$$\varphi(\theta) = \frac{2\sqrt{2}\,eb_\alpha}{R}\sin\left(\theta - \frac{\pi}{4}\right). \tag{22}$$

In this case we have

$$\varphi'(\theta_0) > 0, \quad \varphi'(\theta_1) < 0.$$

A stationary set of system (19), (13)–(16) is empty under the condition

$$\gamma > \frac{2\sqrt{2}\, eb_\alpha}{R}. \tag{23}$$

Stability or instability of its characteristic polynomial, namely

$$f(p) = \left(\frac{R}{L} + p\right)^3 \left[p^3 + \frac{R}{L}\,p^2 + \right.$$
$$\left. + \ (2a_\alpha b_\alpha + \varphi'(\theta_i))p + \frac{R}{L}\,\varphi'(\theta_i)\right],$$

is defined by stability or instability of the third order polynomial in square brackets.

It is known that necessary and sufficient conditions of the polynomial

$$p^3 + a_2p^2 + a_1p + a_0$$

are the following

$$a_2 > 0,\ \ a_1 > 0,\ \ a_0 > 0,\ \ a_2a_1 - a_0 > 0, \tag{24}$$

These conditions are called, sometimes, the conditions of Vyshnegradsky [11].

It is easily seen that the characteristic polynomial $f(p)$ is stable under the condition $\varphi'(\theta_i) > 0$ and is unstable under the condition $\varphi'(\theta_i) < 0$. Therefore the equilibrium states $\theta = \theta_0 + 2n\pi$, $\eta = 0$, $i_1(t) = e/R$, $i_2(t) = e/R$, $i_3(t) = e/R$, $i_4(t) = e/R$ are stable. They correspond to operating modes of synchronous motor. The equilibrium states $\theta = \theta_1 + 2n\pi$, $\eta = 0$, $i_1(t) = e/R$, $i_2(t) = e/R$, $i_3(t) = e/R$, $i_4(t) = e/R$ are unstable, i.e. physically unrealizable.

## 4 Salient-pole rotor with damping windings

We assume further that besides an exciting winding in the slots of salient-pole rotor there are two orthogonal short-circuited damping windings with the currents $i_5(t), i_6(t)$.

Consider the electromechanical model of six windings. It consists of two orthogonal pairs of parallel winds of exciting winding (a pair with the currents $i_1(t)$ and $i_2(t)$ and a pair with the currents $i_3(t)$ and $i_4(t)$) and centered orthogonal pairs of damping windings with the currents $i_5(t)$ and $i_6(t)$.

We also assume that the parameters of damping windings: $L$ being inductance, $R$ being armature resistance, $S$ being an area of one wind of winding, coincide with the same parameters of exciting winding.

In the coordinate system, rigidly connected with the magnetic vector $B$, the exciting currents $i_1(t), i_2(t), i_3(t), i_4(t)$ and the damping currents $i_5(t), i_6(t)$ are given by the following equations

$$L\,\frac{di_1(t)}{dt} + Ri_1(t) = e + (2nlB\sin\alpha)\sin\theta(t)\cdot\dot\theta(t), \tag{25}$$

$$L\,\frac{di_2(t)}{dt} + Ri_2(t) = e + (2nlB\sin\alpha)\sin\theta(t)\cdot\dot\theta(t), \tag{26}$$

$$L\,\frac{di_3(t)}{dt} + Ri_3(t) = e + (2nlB\sin\alpha)\cos\theta(t)\cdot\dot\theta(t), \tag{27}$$

$$L\,\frac{di_4(t)}{dt} + Ri_4(t) = e + (2nlB\sin\alpha)\cos\theta(t)\cdot\dot\theta(t). \tag{28}$$

$$L\,\frac{di_5(t)}{dt} + Ri_5(t) = n_0SB\sin\theta(t)\cdot\dot\theta(t), \tag{29}$$

$$L\,\frac{di_6(t)}{dt} + Ri_6(t) = n_0SB\cos\theta(t)\cdot\dot\theta(t). \tag{30}$$

The equation of motion of rotor with exciting winding and damping windings have the form

$$\begin{aligned} I\ddot\theta = {} & -2\beta nBll_0\sin\alpha[(i_1(t) + i_2(t))\sin\theta + \\ & + (i_3(t) + i_4(t))\cos\theta] - \beta_0n_0SB[i_5(t)\sin\theta + \\ & + i_6(t)\cos\theta] - M. \end{aligned} \tag{31}$$

Here $\beta_0$ is a coefficient of proportionality, $n_0$ is a number of winds in each damping winding.

The equilibrium states of system (25)–(31) under the condition (20) are the points $\theta = \theta_i + 2m\pi$ ($m$ is an integer number), $\dot\theta = 0$, $i_1(t) = i_2(t) = i_3(t) = i_4(t) = e/R$, $i_5(t) = i_6(t) = 0$. Here $\theta_i$ $(i = 0, 1)$ are roots of equation (21), where the function $\varphi(\theta)$ is given by relation (22).

A stationary set of system (25)–(31) is empty under the condition (23).

The characteristic polynomial of Jacobian matrix:

$$\left(\frac{R}{L} + p\right)^5 \left[p^3 + \frac{R}{L}\,p^2 + (c_0d_0 + \right.$$
$$\left. + \ 2a_\alpha b_\alpha + \varphi'(\theta_i))\,p + \frac{R}{L}\,\varphi'(\theta_i)\right],$$

is stable if $\varphi'(\theta_i) > 0$ and unstable if $\varphi'(\theta_i) < 0$.

Here

$$c_0 = \frac{n_0SB}{L}\,, \qquad d_0 = \frac{\beta_0n_0SB}{I}\,.$$

Therefore the equilibrium states $\theta = \theta_0 + 2m\pi,\ \dot{\theta} = 0,\ i_1(t) = i_2(t) = i_3(t) = i_4(t) = e/R,\ i_5(t) = i_6(t) = 0$ are stable and correspond to operating modes of synchronous motor, in which case the equilibrium states $\theta = \theta_1 + 2m\pi,\ \dot{\theta} = 0,\ i_1(t) = i_2(t) = i_3(t) = i_4(t) = e/R,\ i_5(t) = i_6(t) = 0$ are physically unrealizable.

Assuming $L = 0$ in (25)–(30), substituting (25)–(30) in (31), and replacing $\theta$ by $-\theta$, we obtain equation (18), where

$$k = \frac{\beta_0 n_0 (SB)^2}{I} + \frac{8\beta n\nu B^2 l l_0 \sin^2\alpha}{IR},$$
$$b = \frac{4\sqrt{2}\,\beta neBll_0 \sin\alpha}{IR}, \quad \gamma = \frac{M}{I}.$$

Having performed the change of variables $\theta = \theta - \frac{\pi}{4}$ we again arrive at equation (6).

**Acknowledgements**

This work was partly supported by projects of Ministry of education and science of RF (2.1.1/3889, NK-14P), Grants board of President RF (2387.2008.1), RFBR (07-01-00151) and CONACYT (000000000078890).

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