IMPROVEMENT OF DETECTION OF WEAK APERIODIC BINARY SIGNALS IN A BISTABLE OPTICAL SYSTEM BY VIBRATIONAL RESONANCE

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Abstract

The experimental and numerical evidence of improvement of detection of subthreshold aperiodic binary signals by the use of phenomenon of vibrational resonance in a bistable vertical cavity surface emitting laser is presented. It is shown that an additional periodic modulation with a period much shorter than the bit duration in the aperiodic input signal allows one to increase significantly a cross-correlation coefficient between the input and the output of a bistable system as well as substantially to decrease the bit error rate. An experimental observation of a time lag between the input and the output of a bistable laser due the high-frequency modulation is demonstrated. The effect of asymmetry of a bistable quasipotential on the signal detection is also studied. The numerical results are in a qualitative agreement with experimental ones.

Key words

VCSEL, bistability, vibrational resonance, cross-correlation, bit-error-rate.

1 Introduction

Dynamics of an overdamped bistable oscillator driven simultaneously by two periodic signals with strongly differing frequencies became recently a subject of broad interest both theoretically and experimentally (Landa and McClintock, 2000; Baltanas *et al.*, 2003; Chizhevsky *et al.*, 2003; Blekhman and Landa, 2004; Casado-Pascual and Baltanas, 2004; Chizhevsky and Giacomelli, 2004; Chizhevsky and Giacomelli, 2005; Chizhevsky and Giacomelli, 2006; Casado-Pascual *et al.*, 2007). In such conditions the response of a bistable system to the weak low-frequency excitation passes through a maximum depending on the amplitude or frequency of a high-frequency signal. The phenomenon has been named a vibrational resonance (VR) (Landa and McClintock, 2000). The mechanism underlying VR is a parametric amplification of signals and fluctuations near a bifurcation corresponding to transition from bistability to monostability controlled by the high-frequency signal (Baltanas *et al.*, 2003; Blekhman and Landa, 2004; Chizhevsky and Giacomelli, 2004; Chizhevsky and Giacomelli, 2005). Experimentally, the phenomenon of VR has been revealed in the analog electric circuits (Baltanas *et al.*, 2003) and in polarization dynamics of the bistable semiconductor surface-emitting vertical-cavity laser (VCSEL) (Chizhevsky *et al.*, 2003). It was shown that the gain factor and the signal-to-noise ratio (SNR) in the detection of periodic signals in a noisy environment due to use of VR are always higher than those which can be obtained in the same conditions with use of the phenomenon of stochastic resonance (SR) (Chizhevsky and Giacomelli, 2004; Chizhevsky and Giacomelli, 2005). Moreover it was experimentally and theoretically demonstrated for subthreshold square-wave periodic signals that the output SNR greater than the input SNR can be obtained in a wide range of a noise intensity in a bistable system (Chizhevsky and Giacomelli, 2005). Recently, similar results for a subthreshold, sinusoidal signals were also obtained theoretically (Casado-Pascual *et al.*, 2007). Recent theoretical investigations have revealed an occurrence of VR in a system of two coupled overdamped oscillators as well (Gandhimathi *et al.*, 2006). It should be noted that all these studies were performed when input signals were periodic. Therefore an extension of such an approach for an improvement of detecting and recovering aperiodic signals represents significant scientific and practical interest.

It is necessary to note, that the possibility of improvement of detection of noisy aperiodic signals in bistable systems making use for these purposes the phenomenon of stochastic resonance was studied in a number of theoretical and experimental investigations (Collins *et al.*, 1995; Collins *et al.*, 1996; Barbay *et al.*, 2000; Bulsara and Zador, 1996; Misono *et* *al.*, 1998; Chapeau-Blondeau, 1997; Gang *et al.*, 1992; Barbay *et al.*, 2001). Such an aspect in studies of stochastic phenomena was named aperiodic stochastic resonance (ASR) (Collins *et al.*, 1995). Unlike the usual phenomenon of SR for periodic signals where a measure of a detection quality is a signal-to-noise ratio, in ASR for an estimation of degree of a detection quality, a cross-correlation coefficient between an input signal and an output of bistable system (Collins *et al.*, 1995; Collins *et al.*, 1996; Barbay *et al.*, 2000), the mutual information (Bulsara and Zador, 1996; Misono *et al.*, 1998), the information capacity of the channel (Chapeau-Blondeau, 1997) and the bit error rate (Gang *et al.*, 1992; Barbay *et al.*, 2001) are used as a rule.

In this paper the experimental and numerical evidence of occurrence of VR for aperiodic binary signals in VCSEL, operating in the regime of the polarization bistability in both the nearly-symmetric and asymmetric configurations of a double-well quasipotential is presented. For an estimation of the quality of the detection such measures as a cross-correlation coefficient and the bit error rate (BER) are used.

2 Experimental setup

The experimental setup was almost the same as used earlier for investigations of VR in VCSEL (Chizhevsky *et al.*, 2003). In the experiments we used VCSEL lasing at 850nm. The laser was thermally stabilized with accuracy of a few mK. The injection current was chosen so that the laser operates in the regime of polarization bistability where only very rare switchings induced by internal noise could be observed. We studied response in the laser intensity after polarization selection when the mixture of the aperiodic binary signal and the periodic signal were applied to the injection current. We used in the experiment pseudorandom binary signal with a bit rate 10 kB/s with different subthreshold amplitudes A_L, and the square-wave HF control signal with a frequency f_H =200 kHz and the amplitude A_H. The amplitude A_H is here a control parameter. The aperiodic binary signal represents a stream of randomly distributed 1300 pulses of $T_b = 100$ μsec. duration each, where $+A_L$ corresponds, for instance, to '1', and $-A_L$ to '0'. In our experiment we did not add noise to the injection current, since we use intrinsic spontaneous emission noise of VCSEL. Therefore, all measurements were performed with a fixed level of noise in VCSEL. The laser responses were detected with a fast photodetector and recorded by a USB digital 50 MHz bandwidth oscilloscope coupled with a computer to store and process the data. Each point of statistical indicators depending on their type and an expected value was obtained by averaging over 10-500 signals containing 130000 points which were received with sampling period of 1μsec.

3 Model

It is known that many features of stochastic and deterministic polarization dynamics of VCSEL can be well described in the model of an overdamped oscillator with a double-well quasipotential as shown in a number of studies (Giacomelli and Marin, 1998; Willemsen *et al.*, 1999; Frank *et al.*, 2005). Therefore, in parallel with experiments a numerical simulation of some experimental observations was performed in the framework of the equation

$$\frac{dx}{dt} = \alpha x - \beta x^3 + \Delta + A_L R(t) + A_H sgn(\sin \Omega_H t) + \zeta(t), \quad (1)$$

where $R(t)$ is the same aperiodic binary signals that used in experiments, Δ is a level of asymmetry, $sgn(x)$ is a signum function $sgn(x) = -1$, for $x < 0$, and $sgn(x) = 1$, for $x > 0$, $\zeta(t)$ is white, gaussian noise with $< \zeta(t)\zeta(t') >= 2D\delta(t - t')$ and a zero mean $\langle \zeta(t) \rangle = 0$. For distinctness, the values $\alpha = \beta = 4$ were taken. The variable x represents the polarized laser intensity. A forward Euler algorithm with a fixed step of $0.001T_H$ was used ($T_H = 2\pi/\Omega_H$).

For a quantitative characterization of aperiodic vibrational resonance, the normalized cross correlation coefficient between the input and output signals were evaluated from time series for each amplitude of the high-frequency signal

$$C_{IO}(\tau) = \frac{\langle R(t)I(t+\tau)\rangle - \langle R(t)\rangle \langle I(t+\tau)\rangle}{[(\langle R^2(t)\rangle - \langle R(t)\rangle^2)(\langle I^2(t)\rangle - \langle I(t)\rangle^2)]^{1/2}} \quad (2)$$

where $R(t)$ represents an aperiodic binary signal, $I(t)$ is a response of a bistable system, an operation $\langle ... \rangle$ means the time average. In fact, the maximal value of $C_{IO}(\tau)$ [expr. (2)] gives a degree of similarity between the input signal and the output of the system. Besides, this expression allows one to estimate a time lag between signals. The expression (2) was used for both experimental and numerical time series.

4 Response of VCSEL to the periodic modulation

First of all, we have investigated the switching thresholds between polarization states in a bistable VCSEL for rectangular periodic signals with different frequencies. In Fig 1 we show the response of the laser, obtained from a Fourier spectrum of time series, as a function of the amplitude of the applied periodic modulation to the laser at two frequencies: $f_m = 10$ kHz (curve 1) and $f_m = 200$ kHz (curve 2). The frequency $f_m = 10$ kHz corresponds to the highest base frequency in the aperiodic binary signal. From these curves we found approximately switching thresholds $\mu_L \approx 3.1$ mV (at $f_m = 10$ kHz) and $\mu_H \approx 8.5$ mV (at $f_m = 200$ kHz). There is a some uncertainty for both threshold values, since the finding of the true (noiseless) switching threshold in a bistable system with noise is not a simple task. Nevertheless, for com-

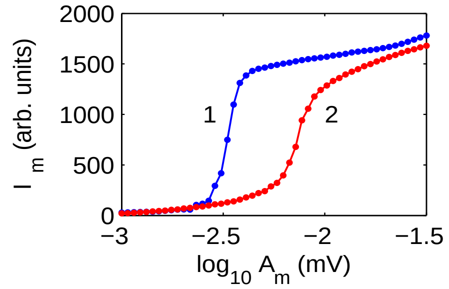

Figure 1. Experiment. Polarization-resolved response of VCSEL I_m to a squarewave periodic modulation as a function of the amplitude A_m shown for two different modulation frequencies $f_m = 10$ kHz (1) and 200 kHz (2).

parison purposes, the value μ_L was used for normalization of the amplitude of the binary signal. We have also experimentally studied the influence of the asymmetry of the quasipotential on the switching threshold in VCSEL for the same modulation frequencies. There exists the minimal thresholds for both frequencies, that corresponds to nearly-symmetric configuration of the bistable quasipotential. Therefore, in order to ensure a nearly-symmetrical configuration in experimental investigations the injection current was tuned to the value corresponding to the minimal switching threshold .

5 Improvement of a cross-correlation by VR

In Fig 2(a) we show by a thin (red) line a part of the aperiodic binary signal used in the experimental (and numerical) investigations and the response of VCSEL in the absence of the additional HF modulation [thick (blue) line]. In this case we used the amplitude of the binary signal $A_L = 0.5$ mV (or normalized to the switching threshold $\varepsilon = A_L/\mu_L \approx 0.16$). When we add a HF signal with the optimal amplitude to the injection current of the laser, one can see perfect recovering the aperiodic binary signal with a significant increase of its amplitude [Fig 2(b)]. For a quantitative characterization of a quality of the detection we performed a set of measurements of the cross-correlation coefficient depending on the amplitude of the HF signal for different amplitudes of the aperiodic binary signal. The experimental results are shown in Fig 3(a) where maximal values of the cross-correlation $C_{IO} = \max\{C_{IO}(\tau)\}$ as a function of the HF amplitude are represented. For comparison purposes, in Fig 3(b) we also present results of the numerical simulation. In the simulation some level of noise intensity ($D = 0.082$) was introduced in Eq. 1 in order to obtain a qualitative agreement between experimental and numerical results. Since we do not know exactly the level of internal noise of the laser and the noiseless switching

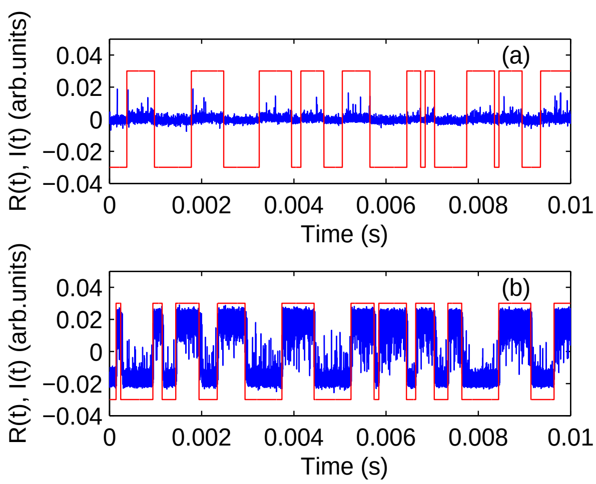

Figure 2. Experiment. Temporal behavior of the polarization-resolved intensity of VCSEL [thick (blue) line]: (a) in the absence and (b) in the presence of the additional HF modulation with the optimal amplitude. Thin (red) line is an input aperiodic binary signal with subthreshold amplitude $\varepsilon \approx 0.16$.

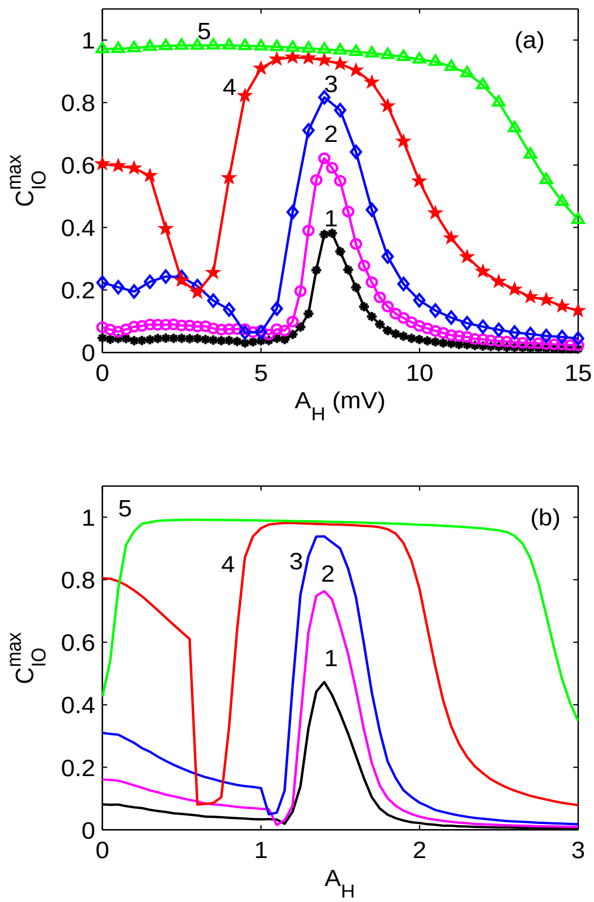

Figure 3. The coefficient of cross-correlation C_{IO} as a function of the amplitude of the HF modulation A_H shown for different values of the amplitude of binary signal (normalized to the switching threshold). (a) Experiment. $\varepsilon \approx 0.02$ (1), 0.04 (2), 0.08 (3), 0.25 (4), 0.67 (5); (b) Numerical simulation. Δ=0, $\varepsilon \approx 0.02$ (1), 0.04 (2), 0.08 (3), 0.32 (4), 0.8 (5).

threshold, therefore a quantitative agreement is rather difficult to obtain. Nevertheless one can note a good qualitative agreement between experimental results and results of the numerical simulation. First of all, one can note that C_{IO} passes through the maximum as the amplitude of the HF signal increases. Such a behavior is a manifestation of VR for the aperiodic binary signal and is observed for all dependencies presented in the figure. It should be noted the phenomenon is clearly seen even for a very weak input binary signal [$\varepsilon \approx 0.02$, curve 1 in Fig 3(a)]. One can note also the broadening of curves and the shift of the optimal value of A_H as the amplitude of the binary signal increases. All this features found experimentally are in a good agreement with the numerical results shown in Fig. 3(b). The location of the maximum of the curve is in the vicinity of the switching threshold μ_H, but slightly shifted to the lower values of A_H due to the presence of internal noise in the laser. Such a shift strongly depends on the noise intensity and can be large enough to be significant as we found in the simulation of the effect of noise on the C_{IO}^{max}. Besides, the numerical analysis shows that for the given amplitude of the binary signal, noise significantly diminishes C_{IO}^{max} , worsening a quality of the recovering of the signal. In this case a diminution of C_{IO}^{max} obeys a scaling law similar to that was found for noise-induced gain degradation for periodic signals in VR (Chizhevsky and Giacomelli, 2004): $C_{IO}^{max} \sim D^{-\gamma}$, where γ lies between 0 and 0.5 and tends to 0.5 as A_L goes to 0. Figure 4 illustrates the dependence of C_{IO}^{max} on the noise intensity D shown for different values of the aperiodic binary signal. One can

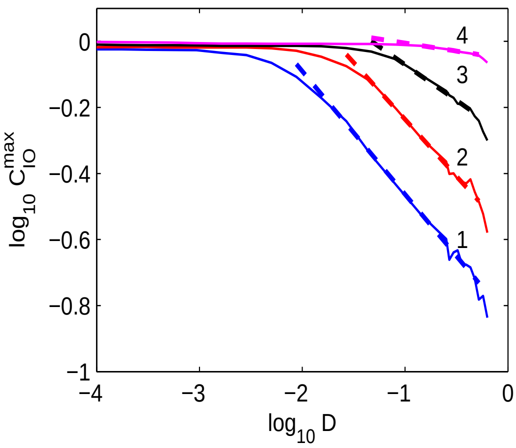

Figure 4. Numerical simulation. C_{IO}^{max} as a function of the noise intensity D shown for different values of the amplitude of binary signal (normalized to the switching threshold). Δ=0, $\varepsilon \approx 0.033$ (1), 0.065 (2), 0.13 (3), 0.325 (4).

distinguish two regions in the figure. First, in a certain range of D, the value C_{IO}^{max} practically does not depend on D. This range decreases whith lowering ε. In this range of D for all amplitude of the aperiodic signal ε, the value of C_{IO}^{max} is close to the unity (a limiting value as D decreases). For a strong enough level of noise, the decreasing of C_{IO}^{max} in the log-log scale linearly depends on D (shown by inclined dashed lines). A fitting of the numerical data gives $\gamma \approx 0.4$, 0.34, 0.2, 0.05 for $\varepsilon \approx 0.033$, 0.065, 0.13, 0.325, respectively.

In Fig. 5(a) and (b) we show experimental and numerical dependencies of $C_{IO}^{\max} = \max\{C_{IO}\}$ (curve 1 on both figures), respectively, as a function of the normalized amplitude of the aperiodic binary signal. For comparison, the coefficient of cross-correlation C_{in} in the absence of HF driving is shown (curve 2 in both figures). These results characterize a great improvement of the cross-correlation coefficient between input-output with respect to the initial value C_{in} , especially, for a weak aperiodic signal as well as a qualitative agreement between experimental and numerical results.

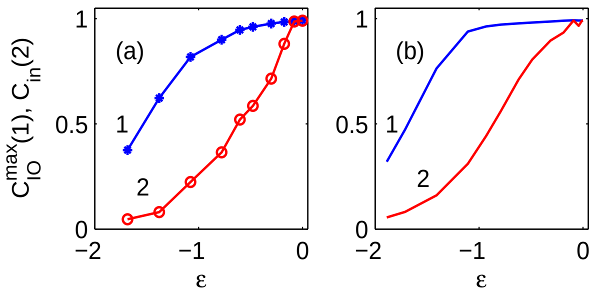

Figure 5. The $C_{IO}^{\max}$ (1) and l C_{in} (2) as a function of the amplitude of aperiodic binary signal ε. (a) Experiment. (b) Simulation. Δ=0.

6 A time lag between the input and the output in VR

One of the features of VR in bistable systems is a time lag between input and output signals as shown numerically in Ref. (Landa and McClintock, 2000). Until now such an effect was not observed experimentally. An experimental evidence of the appearance of a time lag between the input aperiodic binary signal and the laser response is presented in Fig. 6 (a). Here, a normalized time lag δT defined as $\delta T = \tau_m / T_b$, where τ_m is a value of τ corresponding to the maximum of $C_{IO}(\tau)$ for the given HF amplitude, is used. It should be noted that the phase shift between input and output periodic signals were found earlier in stochastic resonance (Dykman *et al.*, 1992). Linear response theory and analogue experiments demonstrated that such shift passes through a maximum depending on the applied noise intensity. In the case of vibrational resonance, the non-monotonic behavior is observed versus the amplitude of the HF modulation and is characterized by the appearance of the discontinuity in the initial part of the

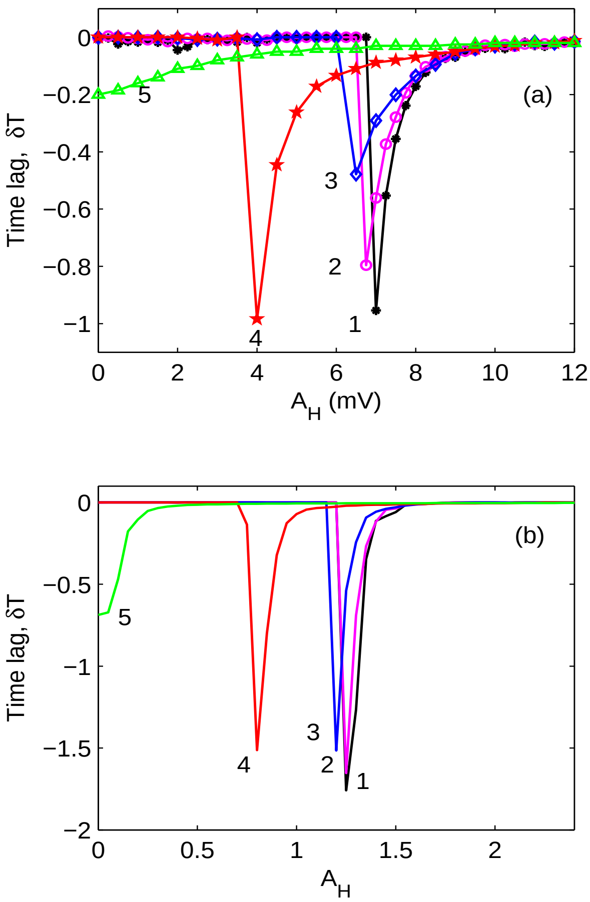

Figure 6. A time lag δT between the input aperiodic signal and the response of a bistable system as a function of the amplitude of the HF modulation A_H shown for different values of the amplitude of the aperiodic binary signal (normalized to the switching threshold). (a) Experiment. $\varepsilon \approx$ 0.02 (1), 0.04 (2), 0.08 (3), 0.25 (4), 0.67 (5); (b) Numerical simulation. Δ=0, $\varepsilon \approx$ 0.02 (1), 0.04 (2), 0.08 (3), 0.32 (4), 0.8 (5).

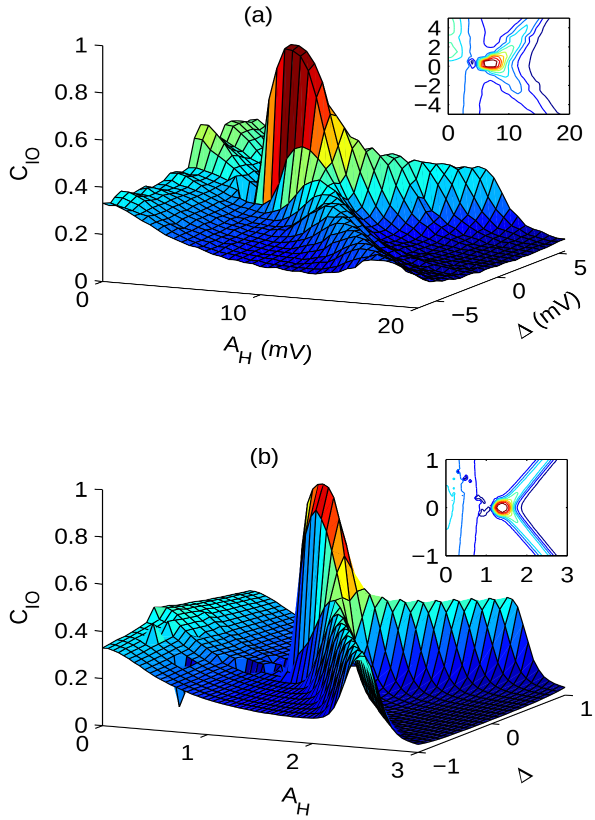

Figure 7. The coefficient of cross-correlation C_{IO} for aperiodic binary signal in the presence of noise as a function of the amplitude of the HF modulation A_H and the level of asymmetry Δ of a bistable quasipotential. (a) Experiment (ε ≈0.16); (b) Numerical simulation (ε =0.16). Inset in both figures: the contour plot for C_{IO} as a function A_H and Δ).

dependence [see Fig. 6] in contrast with a smooth behavior of the phase shift in SR.

7 Effect of asymmetry on a cross-correlation in VR

Most of studies on VR were concentrated on the use of a symmetrical configuration of a bistable potential, except Ref. (Chizhevsky and Giacomelli, 2006) where the effect of asymmetry was investigated in detail both experimentally and theoretically. Figures 7 and 8 present experimental and numerical results of the effect of asymmetry on the detection of the aperiodic binary signal. These results are important from a standpoint of influence of an instability of system parameters in the detection and recovering of aperiodic signals. Fig. 7(a) shows 3D plot for C_{IO} as a function of the HF amplitude A_H and the level of asymmetry Δ. In these experiments the injection current was changed in series with a step of 0.5 mV and, correspondingly, by this way, some level of asymmetry to the quasipotential was introduced (Giacomelli and Marin, 1998). From the figure it is seen that maximal C_{IO} for a weak input signal can be obtained only in narrow ranges of both A_H and Δ. These experimental results are well supported by the numerical results shown in Fig. 7(b). We performed series of similar measurements with different amplitudes of the aperiodic signal, which are summarized in Fig. 8(a). In this figure dependencies of $C_{IO}^{\max}$ are shown as a function of the level of asymmetry for different values of the the amplitude of the aperiodic binary signal. It follows from these results that asymmetry leads to a strong diminution of $C_{IO}^{\max}$ when the amplitude of the aperiodic signal $A_L < \Delta$ (level of asymmetry). In the opposite case one can neglect by asymmetry of quasipotential. Such a rather strong decrease of the cross-correlation coefficient with increasing the level of the asymmetry can be explained by the different temporal behavior of a bistable system when the quasipotential changes from symmetrical to strongly-asymmetrical. In the former case, with an optimal amplitude of the HF modulation, the bistable system output (to periodic or aperiodic modulations)

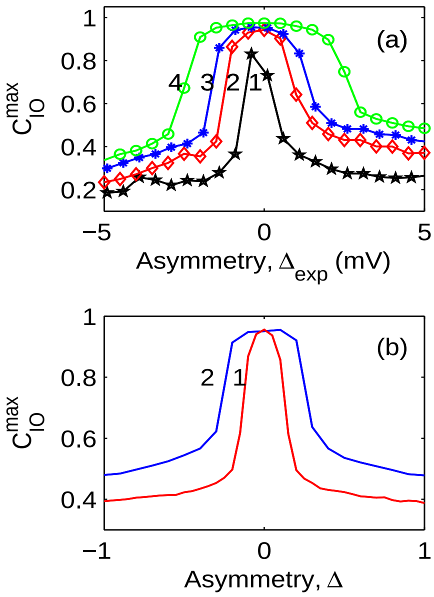

Figure 8. The coefficients of cross-correlation $C_{IO}^{\max}$ as a function of the level asymmetry Δ. (a) Experiment. ε= 0.1 (1), 0.25 (2), 0.33 (3), 0.5 (4); (b) Numerical simulation. ε= 0.1 (1) , 0.2 (2).

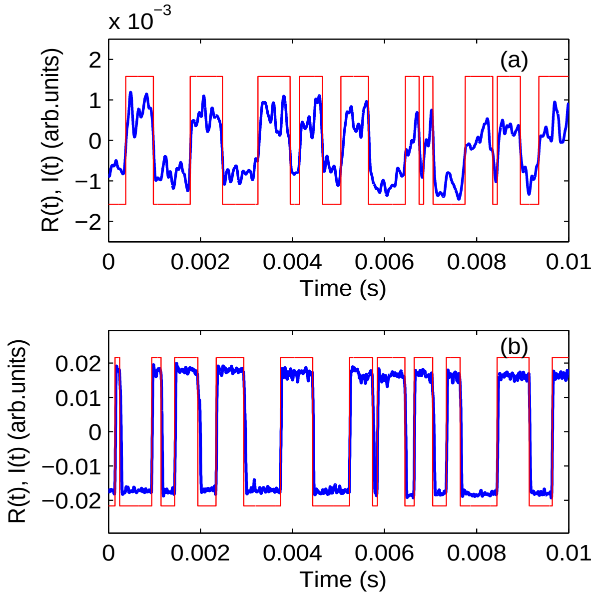

Figure 9. Experiment. A filtered polarization-resolved laser response [thick (blue) line]: (a) in the absence and (b) in the presence of the additional HF modulation with the optimal amplitude. Thin (red) line is an input aperiodic binary signal with subthreshold amplitude $\varepsilon \approx 0.16$.

almost reproduces the shape of the input signal with some minor variations of the period or of the bit duration due to the presence noise. In the latter case, the response is filled out by HF modulation pulses of one sign only. As a result, both the gain factor for the periodic signal and the cross-correlation coefficient for the aperiodic signals decrease (pictures of the temporal behavior are shown in Ref. (Chizhevsky *et al.*, 2003)).

8 Improvement of correlation by low-pass filtering

All the experimental results presented above were obtained without additional processing of time series. The response of the bistable VCSEL can be roughly separated into three parts, namely, aperiodic, HF components, and noise. Since the time-scales of deterministic signals are very different, one can expect that a simple low-pass filtering can improve further the correlation between the input and the output of VCSEL. For this purpose, we used a software-implemented acausal low-pass filter with the transfer function $H(\omega) = 1/(1 + p\omega^4)$, where the parameter p was chosen in order to maximize the input-output cross-correlation.

In Fig. 9 the filtered responses of VCSEL in the absence HF modulation (Fig. 9(a)) and in the presence of the HF modulation with the optimal amplitude (Fig. 9(b)) are shown. Comparing the two signals, one can note a (nearly) perfect recovering of the input signal with a substantial increase of its amplitude.

After the filtering process, one can characterize the gain of the amplitude induced by the HF modulation. For this purpose we introduce quantity g_c defined as $g_c = \langle \tilde{I} > 0\rangle - \langle \tilde{I} < 0\rangle$, where $\tilde{I}$ is the filtered response after subtracting the mean value $\langle I(t)\rangle$. The quantity g_c gives a signal amplitude averaged over the whole time series. The normalized value $G = g_c/g_0$ defines the amplification of the amplitude with respect to the absence of the HF signal (g_0 is the amplitude of the laser response in the absence of the HF modulation). In Fig. 10(a) we show the improved C_{IO}^{max} (open circles) and C_{IO} obtained without filtering (diamonds). Low-pass filtering increases C_{IO}^{max} by about 7

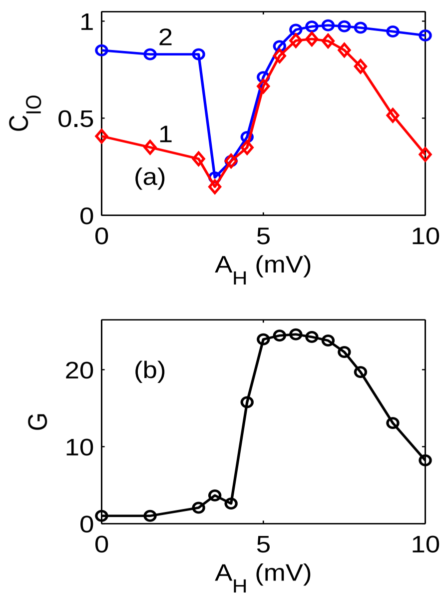

Figure 10. Experiment. (a) C_{IO}^{max} as a function of A_H: 1 - raw data, 2 - filtered data; (b). G as a function of A_H. ($\varepsilon \approx 0.16$).

percents in the range of A_H corresponding to VR. After filtering, C_{IO}^{max} reaches the value of ≈0.98 whereas with raw data (without filtering) $C_{IO}^{max} \approx 0.91$ at the optimal amplitude of the HF modulation. At the same time, the amplitude of the laser response increases by about 25 times [Fig. 10 (b)] with respect to the initial value (in the absence of the HF modulation).

9 Improvement of the bit error rate by VR

Finally, experimental results showing a diminution of the bit error rate by the use of phenomenon of VR are presented in Fig. 11. The BER is the most commonly used measure for an evaluation of the quality of the data transmission through communication channels and is defined as the percentage of wrong received bits with respect to the total number of transmitted bits. Depending on the applications, a BER from 10^{-2} to less than 10^{-8} is required. In processing of our experimental data, the BER was evaluated by averaging the output signal over the input bit length in a similar fashion as in Ref. (Barbay *et al.*, 2001). One can note a significant decrease of BER at the optimal value of the HF amplitude. For instance, for $\varepsilon = 0.25$ (a quarter of the switching threshold) the minimal BER is about 3×10^{-5}, the value which can not be obtained for this amplitude of the aperiodic binary signal by the use of the phenomenon of stochastic resonance.

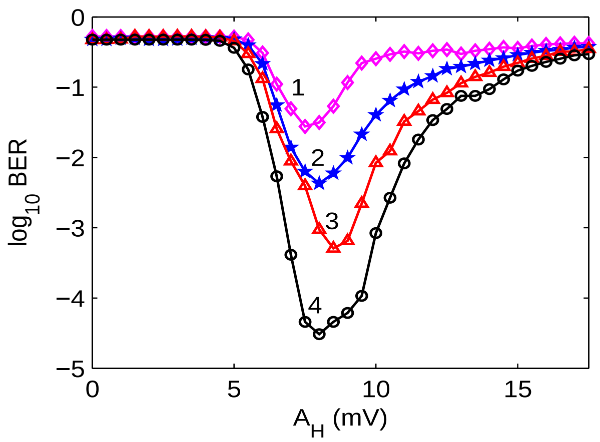

Figure 11. Experiment. Bit error rate (BER) as a function of the amplitude A_H for different values of the normalized amplitude of the aperiodic binary signal ($\varepsilon = 0.1$ (1), 0,15 (2), 0.2 (3), 0.25 (4)).

We also studied the effect of asymmetry of quasipotential on the BER and found the same regularities as for a cross-correlation coefficient presented above, as shown in the following. The BER is plotted in Fig. 12 as a function of the level of asymmetry for three different values of the the amplitude of the aperiodic binary signal. One can note that an increase of the asymmetry leads to a strong increase of the BER (thus, to a deteriorated quality of the fidelity of the transmission) and its effect shows up again when the amplitude of the aperiodic signal $A_L < \Delta$.

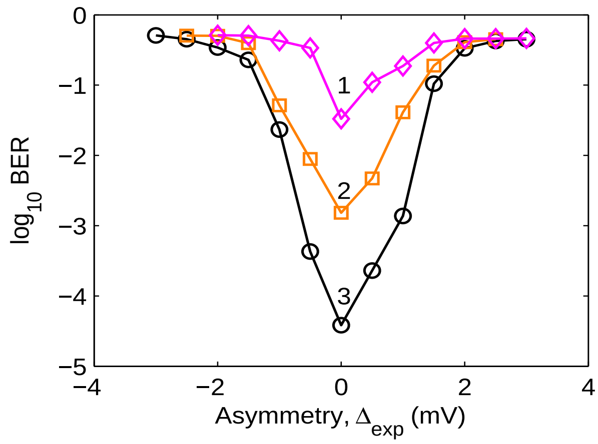

Figure 12. Experiment. Bit error rate (BER) as a function of the level of asymmetry Δ_{exp}, for different values of the normalized amplitude of the aperiodic bynary signal. ($\varepsilon \approx 0.1$ (1), 0.18 (2), 0.25 (3))

10 Conclusions

To conclude, we have shown that vibrational resonance is an efficient method for improvement of detection and recovering of weak subthreshold aperiodic binary signals in stochastic bistable systems. We have demonstrated that such an approach allows one to greatly increase the coefficient of cross-correlation between the input and the output signals and reduce the bit error rate. This method has an advantage over the recently proposed methods of tuning system parameters or parameter-induced aperiodic stochastic resonance (Xu *et al.*, 2004; Jian-Long, 2007) since it does not require to change internal parameters of bistable systems what is not always possible in practice. As vibrational resonance is based on the use of the input and the control signal with very different time-scales, then the high-frequency component can be efficiently removed by an additional low-pass filter what gives a further improvement in detection. We believe that the results we reported will be useful in applications relating to the detection and the recovering of weak aperiodic binary signals, especially, in the field of pattern recognition where a coefficient of cross-correlation is used as a degree of similarity between input and reference patterns and in the transmission of digital and analog data in noisy communication channels.

11 Acknowledgement

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