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DESIGN OF DYNAMIC CONTROLLER FOR REJECTION OF PERSISTENT DISTURBANCES

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Abstract
We design the dynamic controller for optimal rejection of persistent exogenous disturbances in linear control systems. The solution technique based on the invariant ellipsoids concept is developed. Robust version of the problem is addressed as well. The approach is exemplified through a control problems for a gyroplatform.

Key words
Linear systems, invariant ellipsoids, linear matrix inequalities, dynamic controller.

1 Introduction

The problem of rejection of nonrandom bounded exogenous disturbances (also known as peak-to-peak gain minimization) has the long history. It is the subject of l_1-optimization theory, see [Barabanov and Granichin, 1984; Dahleh and Pearson, 1987]. However, l_1-optimization technique often leads to high-dimensional controllers and is hard to implement in the continuous-time case.

A natural way to overcome these difficulties is to appeal to the invariant sets ideology, see [Blanchini and Miani, 2008] in order to reduce complexity and attain the control objectives. Among various possible "shapes" of invariant sets utilized in the research areas above, *ellipsoids* should be distinguished because of their simple structure and direct connection to the quadratic Lyapunov functions approach. Moreover, the ellipsoidal description allows to exploit the powerful machinery of linear matrix inequalities (LMI) and semidefinite programming (SDP), see [Boyd, El Ghaoui, Feron, and Balakrishnan, 1994] as a technical solution tool. Among the first papers in this direction is [Abedor, Nagpal, and Poolla, 1996], also see [Polyak, Nazin, Topunov, and Nazin, 2006]. Under this framework the static state feedback controller was proposed in [Polyak, Nazin, Topunov, and Nazin, 2006]; the linear output controller was designed in [Polyak and Topunov, 2008] based on the Luenberger observer.

In the present paper we address the above mentioned problem by use of the full-order output linear dynamic controller. It is worth noting that as was shown in [Diaz-Bobillo and Dahleh, 1992], l_1-optimal regulator should be dynamic.

Apparently, the idea of stabilizing by output dynamic controller appeared in [Francis, 1977; Francis and Wonham, 1976] under the name of *internal model principle*. The case of L_2-bounded disturbances has been considered in [Balandin and Kogan, 2008]. Up to the authors' knowledge, the design of general dynamic controller for rejection of L_∞-bounded disturbances remained an open problem.

The structure of the the paper is as follows. In the next section we provide the basics of the invariant ellipsoid technique. Then we present the problem formulation and propose the main result. Thereafter we consider an example for gyroplatform stabilization. Finally, we treat the robust case of the problem.

2 The invariant ellipsoids technique

In this section we remind the idea of the invariant ellipsoids technique for analysis of linear systems. We consider the continuous-time system given by

$$\begin{aligned} \dot{x} &= Ax + Dw, \\ z &= Cx, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$ are fixed known matrices, $x(t) \in \mathbb{R}^n$ is the state vector, $z(t) \in \mathbb{R}^l$ is the system output, $w(t) \in \mathbb{R}^m$ is the exogenous disturbance satisfying the Euclidean norm constraint

$$w^{\mathrm{T}}(t)w(t) \le 1 \quad \forall t \ge 0. \tag{2}$$

It is assumed that system (1) is stable (matrix A is Hurwitz), pair (A, D) is controllable, and C is a full-rank matrix.

The ellipsoid

$$\mathcal{E}_x = \left\{x \in \mathbb{R}^n : \quad x^{\mathrm{T}} P^{-1} x \le 1\right\}, \quad P \succ 0, \quad (3)$$

centered at the origin and specified by matrix P is said to be *state-invariant* (or simply *invariant*) for system (1) if condition $x(0) \in \mathcal{E}_x$ implies $x(t) \in \mathcal{E}_x$ for all $t \ge 0$. In other words, starting at any point in $\mathcal{E}_x$, the trajectory of the system is guaranteed to remain inside $\mathcal{E}_x$ for all admissible disturbances (2).

As shown in [Khlebnikov, 2010], if the state of system (1) starts from a point outside $\mathcal{E}_x$, it will tend to $\mathcal{E}_x$ as time increases, thereby any invariant ellipsoid is the attractable one.

Associated with the state-invariant ellipsoid (3) is the *bounding* ellipsoid for the output variable z specified by

$$\mathcal{E}_z = \left\{z \in \mathbb{R}^m : \quad z^{\mathrm{T}} \left(CPC^{\mathrm{T}}\right)^{-1} z \le 1\right\}, \quad (4)$$

where P is the matrix of the state-invariant ellipsoid. There exist various criteria of minimality for bounding ellipsoid (4); here we adopt the following trace criterion: $f(P) = \mathrm{tr}[CPC^{\mathrm{T}}]$, which characterizes the “size” (the sum of squared semiaxes) of the corresponding ellipsoid. The key point is that the condition for invariance of ellipsoids can be formulated as LMI, see [Boyd, El Ghaoui, Feron, and Balakrishnan, 1994; Polyak, Nazin, Topunov, and Nazin, 2006].

3 Problem formulation

Now we proceed to the linear control system given by

$$\begin{aligned} \dot{x} &= Ax + B_1 u + D_1 w, \quad x(0) = x_0,\\ y &= C_1 x + D_2 w,\\ z &= C_2 x, \end{aligned} \quad (5)$$

where $A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times p}$, $D_1 \in \mathbb{R}^{n\times m}$, $D_2 \in \mathbb{R}^{l\times m}$, $C_1 \in \mathbb{R}^{l\times n}$, $C_2 \in \mathbb{R}^{r\times n}$ are fixed known matrices, $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^l$ is the observed output, $z(t) \in \mathbb{R}^r$ is the controlled output, $u(t) \in \mathbb{R}^p$ is the control, $w(t) \in \mathbb{R}^m$ is the exogenous disturbance satisfying the Euclidean norm constraint (2). Pair (A, B_1) is controllable, pair (A, C_1) is observable.

Suppose state vector x of system (5) is not available and the information on the system is given by its output y. The problem is to design a controller which stabilizes the system and provides the minimal (in the certain sense) bounding ellipsoid, which contains the controlled output z. The controller will be designed as linear dynamic full-order one:

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r y, \quad x_r(0) = 0,\\ u &= C_r x_r + D_r y, \end{aligned} \quad (6)$$

where $A_r \in \mathbb{R}^{n\times n}$, $B_r \in \mathbb{R}^{n\times l}$, $C_r \in \mathbb{R}^{p\times n}$, $D_r \in \mathbb{R}^{p\times l}$ are parameters of the controller, $x_r \in \mathbb{R}^n$ is its state vector.

So, we need to determine the parameters of controller (6) in order to obtain the minimal bounding ellipsoid for output z of corresponding closed-loop system (5).

4 The main result

Introduce the vector $g = \begin{pmatrix} x \\ x_r \end{pmatrix} \in \mathbb{R}^{2n}$. By using (5) and (6), we obtain the closed-loop system

$$\begin{aligned} \dot{g} &= A_c g + D_c w, \quad g(0) = \begin{pmatrix} x(0) \\ 0 \end{pmatrix},\\ z &= Cg, \end{aligned} \quad (7)$$

where

$$A_c = \begin{pmatrix} A + B_1 D_r C_1 & B_1 C_r \\ B_r C_1 & A_r \end{pmatrix}, \quad (8)$$

$$D_c = \begin{pmatrix} B_1 D_r D_2 + D_1 \\ B_r D_2 \end{pmatrix}, \quad C = \begin{pmatrix} C_2 & 0 \end{pmatrix}. \quad (9)$$

We restrict state vector g of system (7) in the invariant ellipsoid $\mathcal{E}_g$ with the matrix $P \in \mathbb{R}^{2n\times 2n}$, $P \succ 0$, and minimize the bounding ellipsoid for the output vector z with the matrix CPC^{T}.

The solution is given by the following theorem.

Theorem 1. *Let $\widehat{P}_{11}, \widehat{Q}_{11}, \widehat{\alpha}$ be the solution of the minimization problem*

$$\mathrm{tr}\, C_2 P_{11} C_2^{\mathrm{T}} \longrightarrow \min \quad (10)$$

subject to the constraints

$$\begin{pmatrix} AP_{11} + P_{11}A^{\mathrm{T}} + \alpha P_{11} - \mu_1 B_1 B_1^{\mathrm{T}} & D_1 \\ D_1^{\mathrm{T}} & -\alpha I \end{pmatrix} \preceq 0, \quad (11)$$

$$\begin{pmatrix} \Psi & Q_{11} D_1 - \mu_2 C_1^{\mathrm{T}} D_2 \\ D_1^{\mathrm{T}} Q_{11} - \mu_2 D_2^{\mathrm{T}} C_1 & -\alpha I - \mu_2 D_2^{\mathrm{T}} D_2 \end{pmatrix} \preceq 0, \quad (12)$$

$$\begin{pmatrix} P_{11} & I \\ I & Q_{11} \end{pmatrix} \succeq 0, \quad (13)$$

where

$$\Psi = Q_{11}A + A^{\mathrm{T}} Q_{11} + \alpha Q_{11} - \mu_2 C_1^{\mathrm{T}} C_1, \quad (14)$$

with respect to matrix variables $P_{11} = P_{11}^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, $Q_{11} = Q_{11}^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, scalar variables μ_1, μ_2 and scalar parameter α.

Then, $C_2\widehat{P}_{11}C_2^{\mathrm{T}}$ is the matrix of the optimal bounding ellipsoid for the controlled output z of system (5), (6) *provided that $x_0 = 0$.*

Thus, the parameters $\Delta_r = \begin{pmatrix} A_r & B_r \\ C_r & D_r \end{pmatrix}$ of dynamic controller (6) *satisfy the LMI*

$$\begin{pmatrix} \widetilde{A}\widehat{P} + \widehat{P}\widetilde{A}^{\mathrm{T}} + \widehat{\alpha}\widehat{P} & \widetilde{D} \\ \widetilde{D}^{\mathrm{T}} & -\widehat{\alpha}I \end{pmatrix} + M\Delta_r N \begin{pmatrix} \widehat{P} & 0 \\ 0 & I \end{pmatrix} + \begin{pmatrix} \widehat{P} & 0 \\ 0 & I \end{pmatrix} (M\Delta_r N)^{\mathrm{T}} \preceq 0, \quad (15)$$

where

$$\widehat{P} = \begin{pmatrix} \widehat{P}_{11} & V \\ V & V \end{pmatrix}, \quad V = \widehat{P}_{11} - \widehat{Q}_{11}^{-1}, \quad \widetilde{A} = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}, \quad (16)$$

$$\widetilde{D} = \begin{pmatrix} D_1 \\ 0 \end{pmatrix}, \; M = \begin{pmatrix} 0 & B_1 \\ I & 0 \\ 0 & 0 \end{pmatrix}, \; N = \begin{pmatrix} 0 & I & 0 \\ C_1 & 0 & D_2 \end{pmatrix}. \quad (17)$$

To prove Theorem 1, we need the following lemmas.

Lemma 1. *([Iwasaki and Skelton, 1994; Wang, Duan, Yang, and Huang, 2009]) Let matrices $G = G^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, $M \in \mathbb{R}^{n\times k}$ and $N \in \mathbb{R}^{l\times n}$ be given such that range M and range N are linearly independent. Then LMI*

$$G + M\Delta N + (M\Delta N)^{\mathrm{T}} \preceq 0 \quad (18)$$

is feasible for a matrix $\Delta \in \mathbb{R}^{k\times l}$ iff there exist scalars μ_1, μ_2 such that

$$G \preceq \mu_1 MM^{\mathrm{T}}, \quad G \preceq \mu_2 N^{\mathrm{T}}N. \quad (19)$$

Lemma 2. *([Balandin and Kogan, 2008]) Let matrices $X_{11} = X_{11}^{\mathrm{T}} \in \mathbb{R}^{n\times n}$ and $Y_{11} = Y_{11}^{\mathrm{T}} \in \mathbb{R}^{n\times n}$ be given. Then there exist $(2n \times 2n)$-matrices*

$$X = \begin{pmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{pmatrix} \succ 0, \quad Y = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{12}^{\mathrm{T}} & Y_{22} \end{pmatrix} \succ 0, \quad (20)$$

such that $XY = I$, iff

$$\begin{pmatrix} X_{11} & I \\ I & Y_{11} \end{pmatrix} \succeq 0. \quad (21)$$

Moreover, if $V = X_{11} - Y_{11}^{-1} \succ 0$ then the matrix X can be recovered as follows:

$$X = \begin{pmatrix} X_{11} & V \\ V & V \end{pmatrix}. \quad (22)$$

Now we are in position to prove Theorem 1.

Proof. Introduce the quadratic Lyapunov function

$$V(g) = g^{\mathrm{T}}Qg, \quad Q \in \mathbb{R}^{2n\times 2n}, \quad Q \succ 0, \quad (23)$$

considered on the solutions of system (7). As shown in [Khlebnikov, 2010], the trajectories $g(t)$ of the system (7) remain in the ellipsoid $\mathcal{E}_g = \{g \in \mathbb{R}^{2n} \colon \; V(g) \le 1\}$, iff there exists $\alpha > 0$ such that

$$\begin{pmatrix} PA_c^{\mathrm{T}} + A_cP + \alpha P & D_c \\ D_c^{\mathrm{T}} & -\alpha I \end{pmatrix} \preceq 0, \quad P = Q^{-1}. \quad (24)$$

We rewrite matrix inequality (24) in the form

$$\begin{pmatrix} \widetilde{A}P + P\widetilde{A}^{\mathrm{T}} + \alpha P & \widetilde{D} \\ \widetilde{D}^{\mathrm{T}} & -\alpha I \end{pmatrix} + M\Delta_r N \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix} + \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix} (M\Delta_r N)^{\mathrm{T}} \preceq 0. \quad (25)$$

By Lemma 1, the obtained inequality is feasible for a matrix Δ_r iff there exist scalars μ_1, μ_2 such that

$$\begin{pmatrix} \widetilde{A}P + P\widetilde{A}^{\mathrm{T}} + \alpha P & \widetilde{D} \\ \widetilde{D}^{\mathrm{T}} & -\alpha I \end{pmatrix} - \mu_1 MM^{\mathrm{T}} \preceq 0, \quad (26)$$

and

$$\begin{pmatrix} \widetilde{A}P + P\widetilde{A}^{\mathrm{T}} + \alpha P & \widetilde{D} \\ \widetilde{D}^{\mathrm{T}} & -\alpha I \end{pmatrix} - \mu_2 \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix} N^{\mathrm{T}}N \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix} \preceq 0. \quad (27)$$

After multiplication of the last inequality from the both sides on the matrix $\begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix}$, we obtain

$$\begin{pmatrix} \widetilde{A}^{\mathrm{T}}Q + Q\widetilde{A} + \alpha Q & Q\widetilde{D} \\ \widetilde{D}^{\mathrm{T}}Q & -\alpha I \end{pmatrix} - \mu_2 N^{\mathrm{T}}N \preceq 0. \quad (28)$$

Let us represent the matrices P and Q as

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{12}^{\mathrm{T}} & P_{22} \end{pmatrix}, \quad Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{pmatrix}, \quad (29)$$

and rewrite relations (26), (28) in the form

$$\begin{pmatrix} AP_{11} + P_{11}A^{\mathrm{T}} + \alpha P_{11} - \mu_1 B_1 B_1^{\mathrm{T}} & * & D_1 \\ * & * & * \\ D_1^{\mathrm{T}} & * & -\alpha I \end{pmatrix} \preceq 0, \quad (30)$$

$$\begin{pmatrix} \Psi & * & Q_{11}D_1 - \mu_2 C_1^{\mathrm{T}} D_2 \\ * & * & * \\ D_1^{\mathrm{T}} Q_{11} - \mu_2 D_2^{\mathrm{T}} C_1 & * & -\alpha I - \mu_2 D_2^{\mathrm{T}} D_2 \end{pmatrix} \preceq 0, \tag{31}$$

where

$$\Psi = Q_{11}A + A^{\mathrm{T}}Q_{11} + \alpha Q_{11} - \mu_2 C_1^{\mathrm{T}} C_1. \tag{32}$$

The obtained matrix inequalities are equivalent to the inequalities (11) and (12), where the matrices P_{11} and Q_{11} are corresponding blocks of matrices P and Q respectively, see [Balandin and Kogan, 2008].

By Lemma 2, there exist matrices P and Q with corresponding blocks P_{11} and Q_{11} iff condition (13) holds. Moreover, matrix P could be written as follows:

$$P = \begin{pmatrix} P_{11} & V \\ V & V \end{pmatrix}, \quad V = P_{11} - Q_{11}^{-1} \succ 0. \tag{33}$$

This concludes the proof.

It is worth noting that for every fixed $\alpha > 0$, trace criterion (10) and constraints (11)–(13) are linear in $P_{11}, Q_{11}, \mu_1, \mu_2$. Hence, for α fixed, the minimization of (10) under the LMI constraints above is a semidefinite program. For computations one can use any of the numerous toolboxes that are presently available for SDP solving, e.g., MATLAB-based packages SeDuMi and Yalmip.

There exists certain freedom in the selection of the particular solution Δ_r of the LMI (15). As simulation confirms, the satisfactory result is given by the minimization of $\|A_r\|$ over (15). For this purpose we demand $\lambda \longrightarrow \min$ under the additional constraint

$$\begin{pmatrix} \lambda I & A_r \\ A_r^{\mathrm{T}} & I \end{pmatrix} \succeq 0. \tag{34}$$

In Theorem 1 we required $x(0) = 0$. The extension for nonzero initial state is straightforward.

Lemma 3. *Let $x(0) = x_0 \neq 0$. In order to guarantee the uniform estimate for the controlled output of the system* (5) *it suffices to add LMI*

$$x_0^{\mathrm{T}} Q_{11} x_0 \leq 1, \tag{35}$$

into constraints of Theorem 1.

Similarly the following corollary covers possible uncertainty in the initial state.

Corollary 1. *Let*

$$x(0) \in \mathcal{E}_0 = \left\{x \in \mathbb{R}^n : \quad x^{\mathrm{T}} P_0^{-1} x \leq 1\right\}, \quad P_0 \succ 0. \tag{36}$$

Then it suffices to incorporate the LMI $Q_{11} \preceq P_0^{-1}$, into constraints of Theorem 1.

5 Example: the gyroplatform

Let us consider the control problem for the gyroplatform described by the equations (see [Alexandrov and Chestnov, 1998]):

$$\begin{aligned} \ddot{q}_1 + 400\dot{q}_1 + 0.342\dot{q}_3 + 0.94\dot{q}_4 - 940q_3 + 342q_4 &= 0, \\ \ddot{q}_2 + 400\dot{q}_2 + 0.866\dot{q}_3 + 0.5\dot{q}_4 - 500q_3 + 866q_4 &= 0, \\ 803\dot{q}_1 + 154\dot{q}_2 + 100\dot{q}_3 + 754q_3 + 1130q_4 &= u_1 + w_1, \\ -718\dot{q}_1 - 1070\dot{q}_2 + 200\dot{q}_4 - 867q_3 - 754q_4 &= u_2 + w_2, \end{aligned} \tag{37}$$

where q_1, q_2 are the precession angles (measured rotation angles) of gyroscopes, q_3, q_4 are projections of absolute angular velocities of the platform on its axes, u_1, u_2 are torques of stabilization (control) motors, w_1, w_2 are external disturbances. Here the observed and controlled outputs coincide: $y = z = \begin{pmatrix} q_1 & q_2 \end{pmatrix}^{\mathrm{T}}$.

Introducing the auxiliary variables $q_5 = \dot{q}_1, \ q_6 = \dot{q}_2$, we obtain the system in form (5) with the matrices

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -7.5400 & -11.3000 & -8.0300 & -1.5400 \\ 0 & 0 & 4.3350 & 3.7700 & 3.5900 & 5.3500 \\ 0 & 0 & 938.5038 & -341.6792 & -400.6283 & -4.5023 \\ 0 & 0 & 504.3621 & -858.0992 & 5.1590 & -401.3414 \end{pmatrix},$$

$$B_1 = D_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.0100 & 0 \\ 0 & 0.0050 \\ -0.0034 & -0.0047 \\ -0.0087 & -0.0025 \end{pmatrix},$$

$$C_1 = C_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad D_2 = 0.$$

The constraints on the disturbing moments are given in the interval form:

$$|w_1(t)| \leq 1000, \quad |w_2(t)| \leq 1000 \quad \forall t \geq 0. \tag{38}$$

The corresponding suboptimal analogue of Theorem 1 could be obtained by the replacement $-\alpha I$ with $-\operatorname{diag}\{\beta_1 \ldots \beta_m\}$ in the constraints (11) and (12) with addition of the constraint $\sum\limits_{i=1}^{m} \beta_i \leq \alpha$, where $\beta_1, \ldots, \beta_m$ are scalar variables; and by the replacement $-\widehat{\alpha} I$ with $-\operatorname{diag}\{\widehat{\beta}_1 \ldots \widehat{\beta}_m\}$ in relation (15).

Application of this modification of Theorem 1 yields the matrix

$$\begin{pmatrix} 0.6320 & -0.0830 \\ -0.0830 & 0.4988 \end{pmatrix} \cdot 10^{-5} \tag{39}$$

of bounding ellipse by the controlled output of the system under consideration.

The closed-loop system is stable with $\max_i \operatorname{Re} \lambda_i(A_c) \approx -313.4384$.

For the harmonic exogenous disturbances

$$w_1(t) = 410 \sin 5t + 565 \cos 7t, \tag{40}$$

$$w_2(t) = 565 \sin 5t + 410 \sin 7t, \tag{41}$$

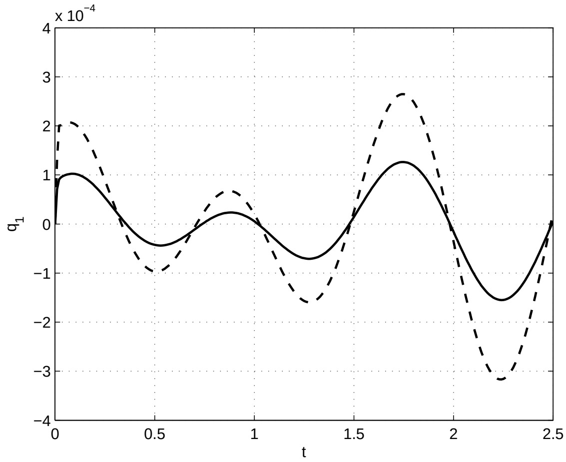

Figure 1. Gyroplatform (output $q_1(t)$, harmonic disturbances).

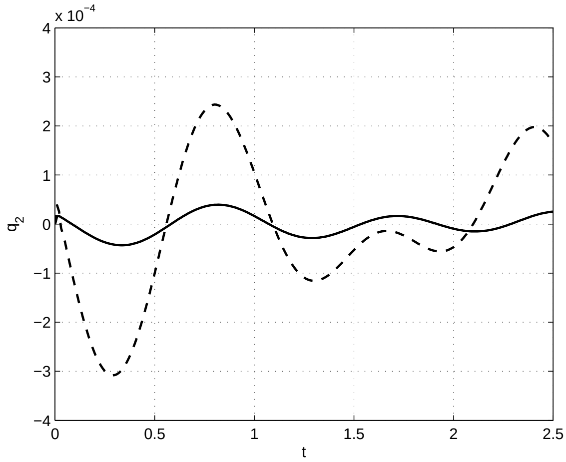

Figure 2. Gyroplatform (output $q_2(t)$, harmonic disturbances).

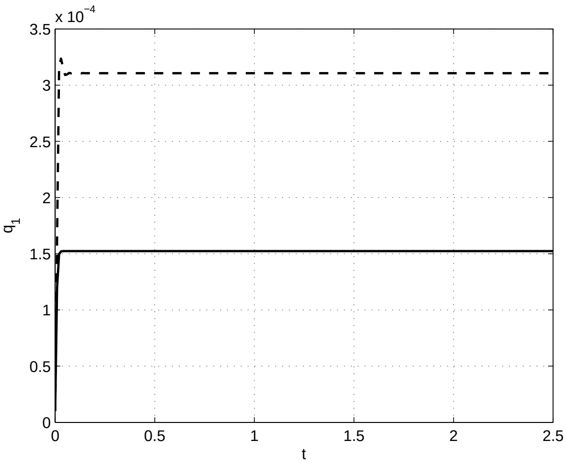

Figure 3. Gyroplatform (output $q_1(t)$, step disturbances).

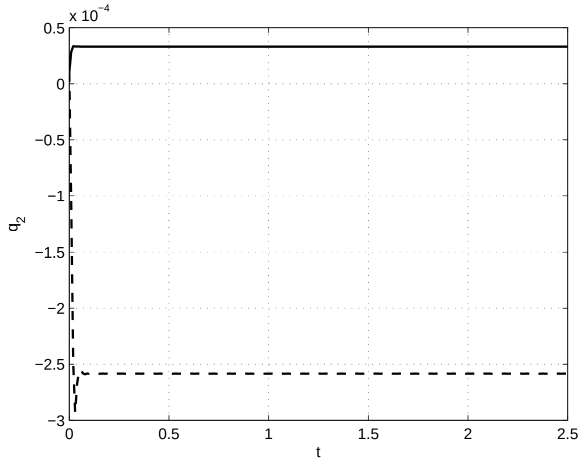

Figure 4. Gyroplatform (output $q_2(t)$, step disturbances).

the output trajectories for the obtained dynamic controller (solid line) and for the dynamic controller from [Alexandrov and Chestnov, 1998] (dashed line) is shown in Fig. 1 and Fig. 2. It is easy to see that our results are preferable.

The difference in results more essential for $w_1(t) = w_2(t) \equiv 1000$ as shown in Fig. 3 and Fig. 4.

6 The robust case

Suppose system (5) contains uncertainty in matrix A:

$$A = A_0 + \Delta A \tag{42}$$

specified in the form $\Delta A = F_A \Delta_A H_A$, where A_0 is the nominal value of matrix A, and F_A, H_A are known "wrapping" matrices of appropriate dimensions, and matrix uncertainty Δ_A satisfies the condition $\|\Delta_A\| \leq 1$.

The corresponding analogue of Theorem 1 is presented below.

Theorem 2. *Let $\widehat{P}_{11}, \widehat{Q}_{11}, \widehat{\alpha}$ be the solution of the minimization problem*

$$\operatorname{tr} C_2 P_{11} C_2^{\mathrm{T}} \longrightarrow \min \tag{43}$$

subject to the constraints

$$\begin{pmatrix} \Phi & D_1 & P_{11} H_A^{\mathrm{T}} \\ * & -\alpha I & 0 \\ * & * & -\varepsilon_1 I \end{pmatrix} \preceq 0, \quad \begin{pmatrix} P_{11} & I \\ I & Q_{11} \end{pmatrix} \succeq 0, \tag{44}$$

$$\begin{pmatrix} \Psi & Q_{11} D_1 - \mu_2 C_1^{\mathrm{T}} D_2 & Q_{11} F_A \\ * & -\alpha I - \mu_2 D_2^{\mathrm{T}} D_2 & 0 \\ * & * & -\varepsilon_2 I \end{pmatrix} \preceq 0, \tag{45}$$

where

$$\Phi = A_0 P_{11} + P_{11} A_0^{\mathrm{T}} + \alpha P_{11} - \mu_1 B_1 B_1^{\mathrm{T}} + \varepsilon_1 F_A F_A^{\mathrm{T}}, \tag{46}$$

$$\Psi = Q_{11}A_0 + A_0^{\mathrm{T}}Q_{11} + \alpha Q_{11} - \mu_2 C_1^{\mathrm{T}}C_1 + \varepsilon_2 H_A^{\mathrm{T}}H_A, \quad (47)$$

with respect to the matrix variables $P_{11} = P_{11}^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, $Q_{11} = Q_{11}^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, *scalar variables* μ_1, μ_2, ε_1, ε_2, *and the scalar parameter* α.

Then, $C_2\widehat{P}_{11}C_2^{\mathrm{T}}$ *is the matrix of the bounding ellipsoid for the controlled output of system* (5), (42), (6).

Moreover, the parameters $\Delta_r = \begin{pmatrix} A_r & B_r \\ C_r & D_r \end{pmatrix}$ *of dynamic controller* (6) *satisfy LMI*

$$\begin{pmatrix} \Omega & \begin{pmatrix} \widehat{P}\begin{pmatrix} H_A^{\mathrm{T}} \\ 0 \end{pmatrix} \\ 0 \end{pmatrix} \\ \left(\begin{pmatrix} H_A & 0 \end{pmatrix} \widehat{P} \;\; 0 \right) & -\varepsilon I \end{pmatrix} \preceq 0, \quad (48)$$

where ε *is a scalar variable, and*

$$\Omega = \begin{pmatrix} \widetilde{A}_0\widehat{P} + \widehat{P}\widetilde{A}_0^{\mathrm{T}} + \widehat{\alpha}\widehat{P} + \begin{pmatrix} \varepsilon F_A F_A^{\mathrm{T}} & 0 \\ 0 & 0 \end{pmatrix} & \widetilde{D} \\ \widetilde{D}^{\mathrm{T}} & -\widehat{\alpha} I \end{pmatrix} + M\Delta_r N \begin{pmatrix} \widehat{P} & 0 \\ 0 & I \end{pmatrix} + \begin{pmatrix} \widehat{P} & 0 \\ 0 & I \end{pmatrix} (M\Delta_r N)^{\mathrm{T}}, \quad (49)$$

$$\widehat{P} = \begin{pmatrix} \widehat{P}_{11} & V \\ V & V \end{pmatrix}, \; V = \widehat{P}_{11} - \widehat{Q}_{11}^{-1}, \; \widetilde{A}_0 = \begin{pmatrix} A_0 & 0 \\ 0 & 0 \end{pmatrix}, \quad (50)$$

$$\widetilde{D} = \begin{pmatrix} D_1 \\ 0 \end{pmatrix}, \; M = \begin{pmatrix} 0 & B_1 \\ I & 0 \\ 0 & 0 \end{pmatrix}, \; N = \begin{pmatrix} 0 & I & 0 \\ C_1 & 0 & D_2 \end{pmatrix}. \quad (51)$$

The proof of Theorem 2 follows the lines of the proof of Theorem 1 combined with application of Petersen's lemma (see [Petersen, 1987]).

7 Conclusion

We present simple and effective method of design the linear full-order dynamic controller for linear systems with nonrandom bounded exogenous disturbances. The approach is based on invariant ellipsoids technique; its use makes possible to reduce the problem to LMIs, while finding the parameters of the dynamic controller can be performed by using SDP and one-dimensional optimization.

The proposed approach is also applicable to discrete-time systems and to another robust problem formulations. These results are not presented here and will be addressed in the journal version of the paper. Also it is possible to construct the nonfragile dynamic controller.

The efficiency of the approach is illustrated on real-life control problem for the gyroplatform.

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