CYBERNETICS AND PHYSICS, VOL. 11, NO. 2, 2022, 74–81

PARAMETERS ESTIMATION OF MULTI-SINUSOIDAL SIGNAL IN FINITE-TIME

Tung Nguyen Khac, Sergey M.Vlasov, Anton A.Pyrkin
Department of Control Systems and Robotics
ITMO University
Russia
nguyenkhactunghvhq1994@gmail.com, smvlasov@itmo.ru, a.pyrkin@gmail.com

Article history:
Received 23.05.2022, Accepted 25.09.2022

Abstract

The problems of identifying the frequency and parameters of multi-sinusoidal signals with constant parameters are considered in finite time. The signal is represented as the output of a linear generator, where the parameters of the sinusoidal signal (amplitude, phase, and frequency) are unknown. The main idea is to apply the Jordan waveform and lag to parameterize the signal and obtain a linear regression model. Unknown parameters are estimated using DREM method. The performance of algorithms considered in the article is illustrated by computer modeling. Our main contribution is to propose a new approach for parameterization of multisinusoidal signals and finite time parameter estimation.

Key words

Estimation parameters, identification algorithms, frequency, sinusoidal signal,regressor

1 Introduction

The problem of determining the unknown amplitudes, frequencies and phases of the components of a multi-sinusoidal signal represents a fundamental challenge in many engineering fields, such as active noise and vibration control, periodic disturbance rejection, power quality monitoring, etc.

The article is devoted to the method of constructing an algorithm for estimating the parameters of a multi-sinusoidal signal, including the frequency, amplitude and phase of each harmonic. Such a problem arises when solving compensating problem for a parametrically unspecified disturbance [Bobtsov, 2012], [Tsykunov, 2007], [Marino, 2007], [Marino, 2018], [Khac, 2021], which has a certain deterministic multi-sinusoidal structure. Approaches are also known when the number of harmonics in the signal spectrum is not known in advance [Marson, 2018], [Pin, 2019].

One of the main tasks in the design of automatic control systems is the leveling out of the action of parametrically undefined disturbing influences on the control object. In the theory of linear systems, there is an internal model principle for solving such problems. In the case of harmonic disturbances, the model parameters will contain unknown frequencies. The initial conditions will be set by unknown displacement, amplitudes, and phases of the disturbing signal's harmonics. In this case, it is necessary to apply adaptive internal models, which provide the possibility of parametric identification of the disturbing signal.

The most commonly used method for processing signals with constant parameters is the Fast Fourier Transform (FFT). In particular, approaches based on the FFT have been used in recognition of human emotions based on a signal from an electroencephalograph [Murugappan, 2013], in the analysis of seismic activity [Spyers-Ashby, 1998], etc. However, as is known, the accuracy of the FFT deteriorates significantly if the signal frequencies change over time [Bittanti, 2000], [Vlasov, 2018]. Moreover, the FFT works with sets of measurements of the input signal and requires quite large memory resources, especially when high accuracy of the resulting estimates is required. Despite the simplicity and efficiency of the method for signals with constant parameters, the FFT is rarely used in practical problems that require the construction of estimates of input signal parameters in real time.

A qualitatively different approach to estimate the parameters of a multi-sinusoidal signal based on an adaptive observer is presented in [Obregon-Pulido, 2002], [Xia, 2002], [Hou, 2007], [Hou, 2012], [Sharma, 2008]. The main advantage of this identification approach is the global convergence of parameter estimation errors to zero. The first algorithm of this class for a sinusoidal signal without bias, proposed in [Hsu, 1999], [Vlasov, 2019], was based on an adaptive notch filter, and the synthesized observer had a dimension equal to 3. Another algorithm [Tomei, 2002] describes an adaptive observer of order $5n$ for estimating n frequencies and an unknown offset. In publications [Obregon-Pulido, 2002], [Xia, 2002], [Hou, 2007], a $3n + 1$ dimensional estimation scheme is proposed for constructing estimates of all $3n + 1$ parameters, including frequencies, amplitudes, phases of n harmonics included in the signal, and a constant offset. In later works [Carnevale, 2011], [Pyrkin, 2015] an algorithm for identifying the minimum dynamic dimension $3n$ for a shifted polyharmonic signal. Note that the dimension of the dynamic observer directly affects the complexity of implementation and performance of the identification algorithm. Further development of this approach aimed to improve the quality of the obtained estimates in the presence of noise in the measurements [Aranovskiy, 2016], [Wang, 2017].

A common drawback of the mentioned approaches to estimating the parameters of a multi-sinusoidal signal is that harmonic frequencies are not directly estimated. As a rule, the polynomial coefficients are first estimated, the roots of which are related to the frequencies of the original signal. Thus, to construct frequency estimates, it is necessary to find the roots of a polynomial with dynamically changing coefficients, which significantly increases the computational complexity of the algorithm in the presence of a large number harmonics in the measured signal. The papers [Chen, 2014], [Wang, 2015], [Pin, 2019] propose new adaptive observers that make it possible to eliminate the aforementioned shortcoming and obtain a frequency estimate directly without any need of further processing.

In this paper, we propose methods for improving the quality of estimating parameters of multi-sinusoidal signals and converging exponential estimation errors to zero. The new method is based on the parameterization of the measured signal with a lag operator to obtain a linear regression model that depends on the measured signals and estimated parameters.

The rest of this paper is organized as follows. The problem statement is described in Section 2. Section 3 deals with the problem of parameterization of multisinusoidal signals. The algorithm for estimating unknown parameters in the regression model will be presented in Section 4. Section 5 proposes an algorithm for estimating the amplitude and phase. In section 6, the computer simulation results of the proposed algorithms are included to confirm the efficiency of the approach. The conclusion is given in Section 7.

2 Problem statement

Consider a measurable multi-sinusoidal signal

$$y(t) = \sum_{i=1}^{n} A_i \sin(\omega_i t + \varphi_i), \tag{1}$$

where $\omega_i \in R_+$ are frequencies, $\omega_i \neq \omega_j, i, j = \overline{1, n}$, $\varphi_i \in R$ are phases, $A_i \in R$ are amplitudes, n is the number of harmonics in the signal $y(t)$. The parameters A_i, ω_i, φ_i are constant and unknown. The objective is to finding the estimates $\hat{\omega}_i^{ft}(t)$ of the frequencies ω_i, $\hat{A}_i^{ft}(t)$ of the amplitudes A_i and $\hat{\varphi}_i^{ft}(t)$ of the phases φ_i, $i = \overline{1, n}$ such that

$$\tilde{\omega}_i^{ft}(t) = \omega_i - \hat{\omega}_i^{ft}(t) = 0, \tag{2}$$

$$\tilde{A}_i^{ft}(t) = A_i - \hat{A}_i^{ft}(t) = 0, \tag{3}$$

$$\tilde{\varphi}_i^{ft}(t) = \varphi_i - \hat{\varphi}_i^{ft}(t) = 0. \tag{4}$$

Our assumption is the following

Assumption . The lower $\underline{\omega}$ and upper $\overline{\omega}$ frequency boundaries for the signal (1) are known, where $0 < \underline{\omega} \leq \omega_i \leq \overline{\omega}$, $i = \overline{1, n}$.

3 Parametrization

Consider the measured signal consists of single harmonic $n = 1$

$$y(t) = A \sin(\omega t + \varphi) \tag{5}$$

The signal $y(t)$ can be represented as the outputs of linear generators [Nikiforov, 1997]

$$y(t) = H^T \xi(t) \tag{6}$$

$$\dot{\xi}(t) = \Gamma \xi(t) \tag{7}$$

where $\xi(t) \in R^q$ is the generator state vector with an initial value $\xi(0)$, $H \in R^q$ constant coefficient matrix, $\Gamma \in R^{q \times q}$ is constant vector of the corresponding dimension.

Build signal generator $y(t)$.

Choosing the signal $\xi_1 = y(t)$ as the first coordinate of the generator state vector.

Taking the time derivative ξ_1 we have

$$\xi_1 = A \sin(\omega t + \varphi),$$

$$\dot{\xi}_1 = \dot{y}(t) = A\omega \cos(\omega t + \varphi).$$

Take the derivative of the sinusoidal signal $\xi_2 = \dot{y}(t)$ as the first coordinate of the generator state vector.

Taking the time derivative ξ_2 we have

$$\dot{\xi}_2 = \ddot{y}(t) = -A\omega^2 \sin(\omega t + \varphi) = -\omega^2 \xi_1.$$

For the matrix form, expressions (6)–(7) take the form

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}, \tag{8}$$

$$\Gamma = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}, \tag{9}$$

$$H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{10}$$

Transform the equation (7) into the form

$$\dot{\xi}(t) = \Gamma \xi(t) \Rightarrow \xi(t) = e^{\Gamma t}\xi(0). \tag{11}$$

Substituting equation (11) into equation (6), we obtain

$$y(t) = H^T e^{\Gamma t}\xi(0). \tag{12}$$

Along with the measured signal $y(t)$, consider the delayed signal

$$y_1(t) = \begin{cases} y(t-d), t \geq d, \\ 0, t < d. \end{cases} \tag{13}$$

$$y(t-d) = H^T e^{\mathbf{\Gamma}(t-d)}\xi(0) = H^T e^{\Gamma t} e^{-\Gamma d}\xi(0), \tag{14}$$

where $d \in R_+$ is a constant delay.

Remark. Restriction on the selected delay value d from (14).

$$d < \frac{\pi}{2n\overline{\omega}}.$$

Based on equations (11) and (14), obtain

$$y(t-d) = H^T e^{-\Gamma d}\xi(t). \tag{15}$$

Proposition 1. The signal (15) is described by the relation

$$y(t-d) = \begin{bmatrix} \cos\omega d & -\omega^{-1}\sin\omega d \end{bmatrix} \xi(t).$$

Proof. Apply the Jordan form of the matrix for the converter $e^{\Gamma d}$ of equation (15).

First, calculate the eigenvalues of the matrix Γ

$$\det(\Gamma - \lambda I) = \begin{vmatrix} 0-\lambda & 1 \\ -\omega^2 & 0-\lambda \end{vmatrix} = 0, \Rightarrow \lambda^2 + \omega^2 = 0,$$

$$\Rightarrow \lambda_1 = i\omega, \lambda_2 = -i\omega.$$

where i is the complex numbers.

For each of the eigenvalues λ_1, λ_2, find the eigenvectors.

For a number $\lambda_1 = i\omega$

$$\begin{bmatrix} -i\omega & 1 \\ -\omega^2 & -i\omega \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0, \Rightarrow \begin{cases} -i\omega x_1 + y_1 = 0 \\ -\omega^2 x_1 - i\omega y_1 = 0 \end{cases}$$

$$\Rightarrow y_1 = i\omega x_1.$$

Assuming $x_1 = 1$, find the eigenvector $V_1 = (x_1, y_1)^T$

$$x_1 = 1, \Rightarrow y_1 = i\omega, \Rightarrow V_1 = \begin{bmatrix} 1 \\ i\omega \end{bmatrix}.$$

Similarly, we find the eigenvector $V_2 = (x_2, y_2)^T$ associated with the eigenvalue $\lambda_2 = -i\omega$

$$\begin{bmatrix} i\omega & 1 \\ -\omega^2 & i\omega \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 0, \Rightarrow \begin{cases} i\omega x_2 + y_2 = 0 \\ -\omega^2 x_2 + i\omega y_2 = 0 \end{cases}$$

$$\Rightarrow y_2 = -i\omega x_2.$$

Assuming $x_2 = 1$, find the eigenvector $V_2 = (x_2, y_2)^T$

$$x_2 = 1, \Rightarrow y_2 = i\omega, \Rightarrow V_2 = \begin{bmatrix} 1 \\ -i\omega \end{bmatrix}.$$

Compose the V matrix from the found eigenvectors V_1 and V_2

$$V = \begin{bmatrix} 1 & 1 \\ i\omega & -i\omega \end{bmatrix}.$$

Write in the Jordan formula J_Γ for a given matrix Γ, using the form [Weintraub, 2009]

$$J_\Gamma = V^{-1}\Gamma V = \begin{bmatrix} i\omega & 0 \\ 0 & -i\omega \end{bmatrix}.$$

Compose the matrix $e^{J_\Gamma d}$

$$e^{J_\Gamma d} = \begin{bmatrix} e^{i\omega d} & 0 \\ 0 & e^{-i\omega d} \end{bmatrix}.$$

Calculate the matrix exponent $e^{\Gamma d}$, using the form

$$e^{\Gamma d} = V e^{J_\Gamma d} V^{-1} = \begin{bmatrix} \frac{e^{i\omega d} + e^{-i\omega d}}{2} & \frac{1}{\omega}\frac{e^{i\omega d} - e^{-i\omega d}}{2i} \\ -\omega\frac{e^{i\omega d} - e^{-i\omega d}}{2i} & \frac{e^{i\omega d} + e^{-i\omega d}}{2} \end{bmatrix}.$$

Expand the exponential functions $e^{i\omega d}, e^{-i\omega d}$ according to the Euler formula

$$\frac{e^{i\omega d} + e^{-i\omega d}}{2} = \cos(\omega d), \frac{e^{i\omega d} - e^{-i\omega d}}{2i} = \sin(\omega d).$$

Get the following result

$$e^{-\Gamma d} = \begin{bmatrix} \cos(\omega d) & -\omega^{-1}\sin(\omega d) \\ \omega\sin(\omega d) & \cos(\omega d) \end{bmatrix}. \qquad (16)$$

From (15) and (16), we have

$$y(t-d) = H^T \begin{bmatrix} \cos(\omega d) & -\omega^{-1}\sin(\omega d) \\ \omega\sin(\omega d) & \cos(\omega d) \end{bmatrix} \xi(t).$$

$$y(t-d) = \left[\cos(\omega d) \; -\omega^{-1}\sin(\omega d)\right]\xi(t). \qquad (17)$$

which is required to prove statement 1.

Similarly for block delay $2d$.

$$y(t-2d) = \left[\cos 2(\omega d) \; -\omega^{-1}\sin 2(\omega d)\right]\xi(t). \qquad (18)$$

From (17) and (18) we have the following matrix

$$\begin{bmatrix} y(t-d) \\ y(t-2d) \end{bmatrix} = \begin{bmatrix} \cos\omega d & -\omega^{-1}\sin\omega d \\ \cos 2\omega d & -\omega^{-1}\sin 2\omega d \end{bmatrix}\xi(t).$$

Consider the following system

$$Y = \Phi\xi, \qquad (19)$$

where $Y = \begin{bmatrix} y(t-d) \\ y(t-2d) \end{bmatrix}$, $\Phi = \begin{bmatrix} \cos\omega d & -\omega^{-1}\sin\omega d \\ \cos 2\omega d & -\omega^{-1}\sin 2\omega d \end{bmatrix}$.

With n harmonics.

Consider the problem of constructing a regression model for the general case (1) with n signals.

$$y(t) = \sum_{i=1}^{n} A_i \sin(\omega_i t + \varphi_i). \qquad (20)$$

We consider discrete signal corresponging to (20)

$$y[k] = \sum_{i=1}^{n} A_i \sin(\omega_i kT + \varphi_i). \qquad (21)$$

Express the value of a discrete signal (21) at the time kT as a linear combination of $2n$ previous values of $y[(k-1)T], ..., y[(k-2n)T]$. The similar is true for a continuous signal.

For this purpose, we apply the delay operator on the measured signal (1).

Signals with multiple delays can be represented by using this delay operator, as

$$\begin{cases} y(t-d) = \Omega y(t) \\ y(t-2d) = \Omega^2 y(t) \\ \vdots \\ y(t-nd) = \Omega^n y(t) \end{cases}$$

Rewrite equation (17) as

$$\left(\Omega^2 - 2\Omega + 1\right) y(t) = 0. \qquad (22)$$

where $c = \cos(\omega d)$, Ω is the delay operators.

Proposition 2. The following relation holds for any signal $\bar{\delta}(t)$ with the sinusoids number n:

$$\left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_n + 1\right] y(t) = 0. \qquad (23)$$

where $c_i = \cos(\omega_i d), i = \overline{1,n}$.

Proof. To prove proposition 2, we use the method of mathematical induction.

The proposition is true for $n = 1$, according to (22).

Inductive step. Show that for any $k > 1$, if the equation (23) holds true $y^k(t)$ then for $y^{k+1}(t)$ it also holds true. This can be done as follows

$$\left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_k + 1\right] y^k(t) = 0. \qquad (24)$$

Expressing $y^{k+1}(t)$ with $y^k(t)$.

$$\begin{aligned} &\left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_{k+1} + 1\right] y^{k+1}(t) = \\ &= \left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_k + 1\right] y^k(t) + \\ &+ \left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_k + 1\right]\beta_{k+1}(t), \end{aligned} \qquad (25)$$

According to (24), we obtain

$$\begin{aligned} &\left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_{k+1} + 1\right] y^{k+1}(t) = \\ &\left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_k + 1\right]\beta_{k+1}(t), \end{aligned} \qquad (26)$$

where $\beta_{k+1} = A_{k+1}\sin(\omega_{k+1}t + \varphi_{k+1})$.

Note that we can consider the expression β_{k+1} as a signal (22), then

$$\left[\Omega^2 - 2\Omega c_{k+1} + 1\right]\beta_{k+1} = 0. \qquad (27)$$

Applying the operator β_{k+1} to (26) yields

$$\begin{aligned} &\left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_{k+1} + 1\right] y^{k+1} = \\ &= \left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_k + 1\right]\cdot \\ &\left[\Omega^2 - 2\Omega c_{k+1} + 1\right]\beta_{k+1} \end{aligned} \qquad (28)$$

From (27) and (28), obtain

$$\left[\Omega^2 - 2\Omega c_1 + 1\right]\cdot\ldots\cdot\left[\Omega^2 - 2\Omega c_{k+1} + 1\right] y^{k+1}(t) = 0. \qquad (29)$$

Since both the base case and the inductive step have been performed, by mathematical induction the statement holds for all natural numbers n.

This completes the proof.

Now we are constructing from (23) the regression model for the general case as

$$\Xi(t) = \psi^T(t)\,\Theta_i, \qquad (30)$$

where $\Xi \in \mathbb{R}^1$ is a dependent function, $\psi = \left[\psi_1\ \psi_2\ ...\ \psi_n\right]^T \in \mathbb{R}^n$ is regressor, $\theta = \left[\theta_1\ \theta_2\ ...\ \theta_n\right] \in \mathbb{R}^n$ is vector of unknown parameters, or more specifically

$$\left[\Omega^2+1\right]^n y\left(t\right) = \Theta_1\psi_1\left(t\right)+\Theta_2\psi_2\left(t\right)+...+\Theta_n\psi_n\left(t\right). \quad (31)$$

The $\Xi\left(t\right)$ component is obtained using the Newton binomial

$$\Xi\left(t\right) = \left[\Omega^2+1\right]^n y(t), \quad (32)$$

The components of the vector of unknown parameters Θ_i are related to $c_i, i = \overline{1,n}$ by Vieta's formulas

$$\begin{cases} \Theta_1 = c_1 + c_2 + \ldots + c_n, \\ \Theta_2 = -c_1c_2 - c_1c_3 - \ldots - c_{n-1}c_n, \\ \vdots \\ \Theta_n = (-1)^{n+1}c_1c_2\cdot\ldots\cdot c_n. \end{cases}$$

The components of the $\psi_i\left(t\right)$ regressor are as follows

$$\begin{cases} \psi_1 = 2\Omega\left[\Omega^2+1\right]^{n-1}y\left(t\right), \\ \psi_2 = 2^2\Omega^2\left[\Omega^2+1\right]^{n-2}y\left(t\right), \\ \vdots \\ \psi_n = 2^n\Omega^n y\left(t\right). \end{cases}$$

4 Estimation algorithm

Parameters estimations of the first order regression model (30) can be obtained using method DREM [Aranovskiy, 2017].

Applying the delay block $\tau_p, p = \overline{1, n-1}$ for the known elements of the regression model (30), then for (30) we get

$$\Xi\left(t-\tau_p\right) = \psi^T\left(t-\tau_p\right)\Theta_i. \quad (33)$$

Denote

$$\chi_e = \varpi_e\Theta_i, \quad (34)$$

where $\chi_e = \left[\Xi\left(t\right)\ \Xi\left(t-\tau_1\right)\ ...\ \Xi\left(t-\tau_p\right)\right]^T, \varpi_e = \left[\psi^T\left(t\right)\ \psi^T\left(t-\tau_1\right)\ ...\ \psi^T\left(t-\tau_p\right)\right].$

Multiplying (34) by adj$\varpi_e\left(t\right)$, gives

$$\chi_i\left(t\right) = \Delta\left(t\right)\Theta_i, \quad (35)$$

where $\Delta\left(t\right) = \text{det}\varpi_e\left(t\right) \in \mathbb{R}^1, \chi_i\left(t\right) = \text{adj}\varpi_e\chi_e\left(t\right) \in \mathbb{R}^n$.

Algorithm for estimating parameters Θ_i can be presented as

$$\dot{\hat{\Theta}}_i\left(t\right) = \kappa_i\Delta\left(t\right)\left(\chi_i\left(t\right) - \Delta\left(t\right)\hat{\Theta}_i\right), \quad (36)$$

where κ_i is any positive number.

To obtain an estimate in finite time, we replace the estimation error $\tilde{\Theta}_i\left(t\right)$ by definition with $\Theta_i - \hat{\Theta}_i\left(t\right)$

$$\Theta_i - \hat{\Theta}_i\left(t\right) = \Theta_i W\left(t\right) - \hat{\Theta}_i\left(0\right)W\left(t\right), \quad (37)$$

where $\dot{W}\left(t\right) = -\kappa_i\Delta^2\left(t\right)W\left(t\right), W\left(0\right) = 1 or W\left(t\right) = e^{-\kappa\int_0^t\Delta^2(s)ds}$.

Express the value of the parameter $\Theta_i = \hat{\Theta}_i^{ft}\left(t\right)$ explicitly from the relation (37)

$$\hat{\Theta}_i^{ft}\left(t\right) = \frac{\hat{\Theta}_i\left(t\right) - W\left(t\right)\hat{\Theta}_i\left(0\right)}{1-W\left(t\right)}. \quad (38)$$

Frequency Estimation

To estimate the frequency, use the function $\arccos\left(.\right)$ based on the parameter $\hat{c}_i^{ft}\left(t\right)$ from (38)

$$\hat{\omega}_i^{ft}\left(t\right) = \frac{1}{d}\arccos\left(\hat{c}_i^{ft}\left(t\right)\right), i = \overline{1,n}. \quad (39)$$

5 Amplitude and phase estimation

Construct a linear regression model depending on the measured signal $y(t)$ and frequency estimates $\hat{\omega}_i^{ft}\left(t\right)$),$i = \overline{1,n}$ to obtain estimates of the amplitudes A_i and phases φ_i, $i = \overline{1,n}$ of the signal (1).

Consideration of a signal $\hat{y}(t)$, similar to (1), in which instead of the frequency values ω_i, the estimates $\hat{\omega}_i^{ft}\left(t\right)$) obtained in the previous section are used

$$\hat{y}(t) = \sum_{i=1}^{n} A_i\sin\left(\hat{\omega}_i^{ft}\left(t\right)+\varphi_i\right). \quad (40)$$

Note that the measured signal (1) can be represented as $\hat{y}(t)$ for $t > t_0$

$$y\left(t\right) = \sum_{i=1}^{n} A_i\sin\left(\omega_i t + \varphi_i\right), \quad (41)$$

$$y\left(t\right) = \sum_{i=1}^{n} A_i\sin\left(\tilde{\omega}_i^{ft}\left(t\right) + \hat{\omega}_i^{ft}\left(t\right)t + \varphi_i\right),$$

$$y\left(t\right) = \sum_{i=1}^{n} A_i\sin\tilde{\omega}_i^{ft}\left(t\right)\cos\left(\hat{\omega}_i^{ft}\left(t\right)t+\varphi_i\right) + \\ + A_i\cos\tilde{\omega}_i^{ft}\left(t\right)\sin\left(\hat{\omega}_i^{ft}\left(t\right)t+\varphi_i\right),$$

$$y\left(t\right) = \sum_{i=1}^{n} A_i\sin\left(\hat{\omega}_i^{ft}\left(t\right)t+\varphi_i\right) = \hat{y}\left(t\right), \quad (42)$$

where $\tilde{\omega}_i^{ft}\left(t\right) = \omega_i - \hat{\omega}_i^{ft}\left(t\right)$ frequency estimation error ω_i in finite time.

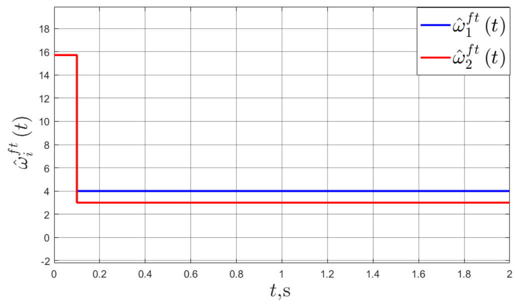

Figure 1. Frequency estimation plot $\hat{\omega}_i^{ft}(t)$

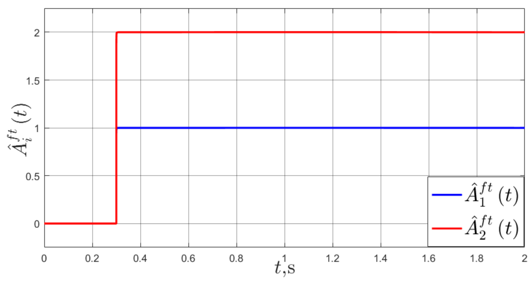

Figure 2. Amplitude estimation plot $\hat{A}_i^{ft}(t)$

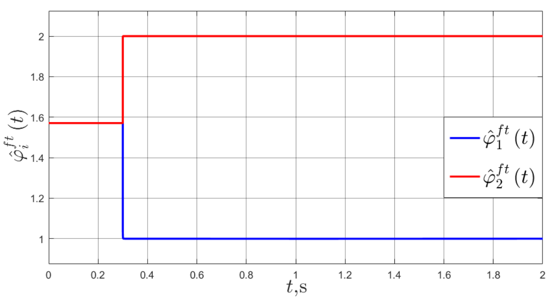

Figure 3. Phase estimation plot $\hat{\varphi}_i^{ft}(t)$

Then, proceeding from (40) and (42), we obtain

$$y(t) = \sum_{i=1}^{n} A_i \sin\left(\hat{\omega}_i^{ft}(t)\,t\right)\cos\varphi_i + \\ + A_i \sin\varphi_i \cos\left(\hat{\omega}_i^{ft}(t)\,t\right) \tag{43}$$

Rewrite expression (43) as a linear regression model

$$\upsilon(t) = \rho^T(t)\,\eta, \tag{44}$$

where $\upsilon(t) \in R$ is the measured function, $\rho(t) \in R^{2n}$ is the regressor, $\eta \in \mathbb{R}^{2n}$ - vector of unknown parameters, which are defined by the following expressions:

$$\upsilon(t) = y(t), \rho(t) = \begin{bmatrix} \sin(\hat{\omega}_1(t)\,t) \\ \cos(\hat{\omega}_1(t)\,t) \\ \sin(\hat{\omega}_2(t)\,t) \\ \cos(\hat{\omega}_2(t)\,t) \\ \dots \\ \sin(\hat{\omega}_n(t)\,t) \\ \cos(\hat{\omega}_n(t)\,t) \end{bmatrix}, \eta = \begin{bmatrix} A_1\cos\varphi_1 \\ A_1\sin\varphi_1 \\ A_2\cos\varphi_2 \\ A_2\sin\varphi_2 \\ \dots \\ A_n\cos\varphi_n \\ A_n\sin\varphi_n \end{bmatrix}. \tag{45}$$

Now let us construct estimates for the amplitudes A_i and phases φ_i, $i = \overline{1,n}$ of the signal (1) from the linear regression model (44), whose unknown parameters are the components of the constant vector η of the original regression model (44). As a result, applying the method DREM, for the first order regression models, we obtain the estimates $\hat{\eta}_j^{ft}(t)$, $j = \overline{1,2n}$ components of the unknown vector η from model (44), and the estimation errors $\tilde{\eta}_j^{ft}(t)$, $j = \overline{1,2n}$ converge to zero in a finite time.

Then the estimates $\hat{A}_i^{ft}(t)$ for the amplitudes A_i and $\hat{\varphi}_i^{ft}(t)$ for the phases φ_i, $i = \overline{1,n}$ can be obtained from $\hat{\eta}_j^{ft}(t)$ as follows

$$\hat{A}_i^{ft}(t) = \sqrt{\left[\hat{\eta}_{2i-1}^{ft}(t)\right]^2 + \left[\hat{\eta}_{2i}^{ft}(t)\right]^2}, \tag{46}$$

$$\hat{\varphi}_i^{ft}(t) = \begin{cases} \arccos\dfrac{\hat{\eta}_{2i-1}^{ft}(t)}{\hat{A}_i^{ft}(t)}, \hat{A}_i^{ft}(t) > 0, \\ 0, \hat{A}_i^{ft}(t) = 0. \end{cases} \tag{47}$$

6 Simulation

In this section, we present simulation results that illustrate the efficiency of proposed estimation algorithm. All simulations have been performed in MATLAB Simulink.

Example 1

Consider a signal $y(t)$ consisting of two sinusoids $n = 2$

$$y(t) = \sin(4t+1) + 2\sin(3t+2). \tag{48}$$

Parameter values of the proposed method:

Delay values $d = 0.1, \tau = 0.1$,

DREM method parameter $\kappa = 10^4$.

On Fig. 1–3 shows the results of estimating frequencies $\hat{\omega}_i^{ft}(t)$, amplitudes $\hat{A}_i^{ft}(t)$, phases $\hat{\varphi}_i^{ft}(t)$ signal (48) in a finite time.

Example 2

Consider a signal $y(t)$ consisting of two sinusoids $n = 3$

$$y(t) = \sin(2t+1) + 2\sin(3t+2) + 3\sin(4t+3). \tag{49}$$

Parameter values of the proposed method:

Delay values $d = 0.1, \tau_1 = 0.1, \tau_2 = 0.2$,

DREM method parameter $\kappa = 10^6$.

On Fig. 4–6 shows the results of estimating frequencies $\hat{\omega}_i^{ft}(t)$, amplitudes $\hat{A}_i^{ft}(t)$, phases $\hat{\varphi}_i^{ft}(t)$ signal (49) in a finite time.

In Fig. 1–6 show the results of estimating the frequency ω_i, amplitude A_i and phase φ_i of the signal (48), (49) in finite time. As can be seen from the graphs, the proposed estimation algorithm provides exponential convergence to the true values of the estimation of the signals parameters $y(t)$.

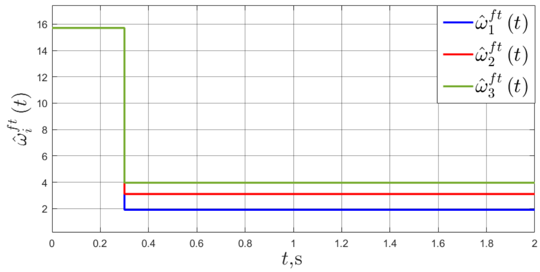

Figure 4. Frequency estimation plot $\hat{\omega}_i^{ft}(t)$

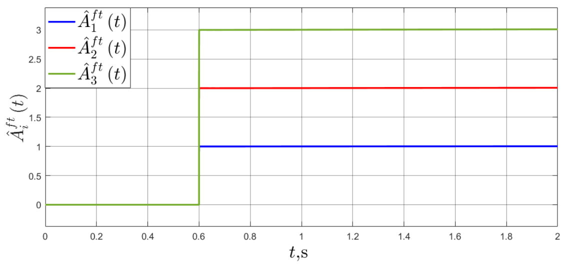

Figure 5. Amplitude estimation plot $\hat{A}_i^{ft}(t)$

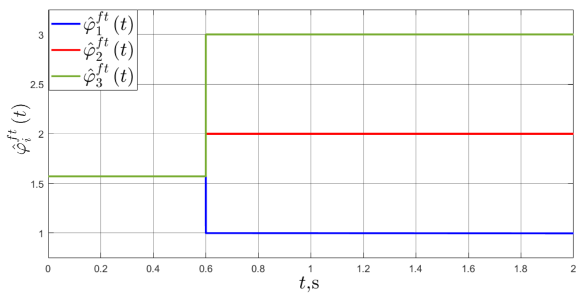

Figure 6. Phase estimation plot $\hat{\varphi}_i^{ft}(t)$

7 Conclusions

The article considers the problem of estimating the parameters of a multisinusoidal signal in finite time. A new parameterization method based on delay operator application on a measurable signal is applied to construct a linear regression model. At the first stage, the frequency estimates were obtained, and in the second stage, the amplitudes and phases of the measured signal were estimated. In each of two stages, linear regression models were built depending on the unknown parameters of original harmonic signal. The parameter vector of the regression models was estimated using the DREM method. Methods for producing estimates of the frequency of a multisinusoidal signal are presented, making it possible to obtain estimates of the parameters at a predetermined time. A computer simulation has been carried out to illustrate the performance, demonstrating the parametric convergence of algorithm variable to the correct value.

Acknowledgements

The work was supported by the President of the Russian Federation, grant No. MD-3574.2022.4.

References

Aranovskiy S., Bobtsov A., Ortega R., and Pyrkin A. (2016). Parameters estimation via dynamic regressor extension and mixing. *American Control Conference (ACC)* 2016: 6971–6976.

Aranovskiy S., Bobtsov A., Ortega R., and Pyrkin A. (2017). Performance Enhancement of Parameter Estimators via Dynamic Regressor Extension and Mixing. *IEEE Transactions on Automatic Control*; 62(7): 3546-3550. doi: 10.1109/TAC.2016.2614889

Bittanti S., and Savaresi S. (2000). On the parametrization and design of an extended Kalman filter frequency tracker. *IEEE Transactions on Automatic Control*, 45(9): 1718-1724. doi: 10.1109/9.880631.

Bobtsov A. A. (2012). Cancelation of unknown multiharmonic disturbance for nonlinear plant with input delay. *IEEE Transactions on Automatic Control*, 50: 302-315. doi: 10.1002/acs.1283.

Carnevale D, and Astolfi A. (2011). A hybrid observer for frequency estimation of saturated multi-frequency signals. *50th IEEE Conference on Decision and Control and European Control Conference*, 2577–2582.

Chen B., Pin G., and Parisini T. (2014). An adaptive observer-based estimator for multi-sinusoidal signals. *American Control Conference* 3450–3455.

Hou M. (2007). Estimation of Sinusoidal Frequencies and Amplitudes Using Adaptive Identifier and Observer. *IEEE Transactions on Automatic Control* 52(3): 493-499. doi: 10.1109/TAC.2006.890389.

Hou M. (2012). Parameter Identification of Sinusoids. *IEEE Transactions on Automatic Control* 57(2): 467-472. doi: 10.1109/TAC.2011.2164736.

Hsu L., Ortega R., and Damm G. (1999). A globally convergent frequency estimator. *IEEE Transactions on Automatic Control*, 44(4): 698–713. doi: 10.1109/9.754808.

Khac T., Vlasov S.M., and Iureva R. (2021). Estimating the Frequency of the Sinusoidal Signal using the Parameterization based on the Delay Operators//ICINCO 2021 - Proceedings of the 18th International Conference on Informatics in Control, Automation and Robotics, 2021, pp. 656-660.

Nikiforov V.O. (1997). Adaptive servomechanism controller with an implicit reference model. *International Journal of Control* 68(2): 277-286. doi: 10.1080/002071797223604.

Marino R., Santosuosso G.L., and Tomei P. (2007) Adaptive stabilization of linear systems with outputs affected by unknown sinusoidal disturbances. *European Control Conference (ECC)*: 129-134.

Marino R., Santosuosso G.L., and Tomei P. (2008). Regulation of Linear Systems with Unknown Additive Sinusoidal Sensor Disturbances. *IFAC Proceedings Volumes*, 41(2): 4102-4107. 17th IFAC World

Congressdoi: https://doi.org/10.3182/20080706-5-KR-1001.00690.

Murugappan M., and Murugappan S. (2013). Human emotion recognition through short time Electroencephalogram (EEG) signals using Fast Fourier Transform (FFT). *IEEE 9th International Colloquium on Signal Processing and its Applications*, 289-294.

Pin G., Marson E., Posa P., and Schiava L.D. (2018). Direct Estimation of Line-Current Harmonics from DC-link measurements. *IEEE Conference on Control Technology and Applications (CCTA)*, 1179-1184.

Obregon-Pulido G., Castillo-Toledo B., and Loukianov A. (2002). A globally convergent estimator for n-frequencies. *IEEE Transactions on Automatic Control*, 47(5): 857-863. doi: 10.1109/TAC.2002.1000286.

Pin G., Wang Y., Chen B., and Parisini T. (2015). Semi-global direct estimation of multiple frequencies with an adaptive observer having minimal parameterization. *54th IEEE Conference on Decision and Control (CDC)* 3693–3698.

Pin G, Wang Y, Chen B, and Parisini T. (2019). Identification of multi-sinusoidal signals with direct frequency estimation: An adaptive observer approach. *Automatica*, 99: 338-345. doi: https://doi.org/10.1016/j.automatica.2018.10.026.

Pyrkin A. (2015). Estimation of polyharmonic signal parameters. *Automation and Remote Control*, 76: 1400–1416. doi: 10.1134/S0005117915080068.

Sharma BB, Kar IN. (2008). Design of Asymptotically Convergent Frequency Estimator Using Contraction Theory. *IEEE Transactions on Automatic Control* 53(8): 1932-1937. doi: 10.1109/TAC.2008.927682.

Spyers-Ashby J.M,, Bain P.G., and Roberts S.J. (1999). A comparison of fast fourier transform (FFT) and autoregressive (AR) spectral estimation techniques for the analysis of tremor data. *Journal of Neuroscience Methods*, 83: 35-43.

Marino R., and Tomei P. (2002). Global estimation of n unknown frequencies. *IEEE Transactions on Automatic Control*, 47(8): 1324-1328. doi: 10.1109/TAC.2002.800761.

Tsykunov A.M. (2007). Robust control algorithms with compensation of bounded perturbations. *Automation and Remote Control*; 68: 1213-1224. doi: 10.1134/S0005117907070090.

Vlasov S.M., Kirsanova A.S., Dobriborsci D., Borisov O.I., Gromov V.S., Pyrkin A.A., Maltsev M.V., and Semenev A.N. (2018). Output Adaptive Controller Design for Robotic Vessel with Parametric and Functional Uncertainties//26th Mediterranean Conference on Control and Automation, MED 2018, 2018, pp. 547-552.

Vlasov S.M., Margun A.A., Kirsanova A.S., and Vakhvianova P.D. (2019). Adaptive controller for uncertain multi-agent system under disturbances//ICINCO 2019 - 16th International Conference on Informatics in Control, Automation and Robotics, 2019, **2**, pp. 198–205.

Wang J., Gritsenko P.A., Aranovskiy S., Bobtsov A.A., and Pyrkin A.A. (2017). A method for increasing the rate of parametric convergence in the problem of identification of the sinusoidal signal parameters. *Automation and Remote Control / Avtomatika i Telemekhanika* 78(3): 389–396. doi: 10.1134/S0005117917030018.

Weintraub S. (2009). Jordan Canonical Form: Theory and Practice. *Morgan and claypool publishers*.

Xia X. (2002). Global frequency estimation using adaptive identifiers. *IEEE Transactions on Automatic Control*, 47(7): 1188- 1193. doi: 10.1109/TAC.2002.800670.