Application of Robust Decentralized Control to the Group Flight of Airplanes [†]

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Abstract: The problem of a robust control system design for interconnected systems with structural and parametrical uncertainty was solved for the case where derivatives of input and output parameters cannot be measured. The order of the mathematical model may change over time. Operability of the designed control systems in the case of non-measurable and bounded disturbances acting on the controlled plant was demonstrated. Only the measurable variables of the local subsystems are used to generate the control actions, that is, control is completely decentralized.

Keywords: robust control, decentralized law, compensation

1. INTRODUCTION

The problem of control with scalar input and output has become one of the classical problems of modern control theory and plenty of methods for robust control design have been developed. The key developments in robust control theory, as well as a comprehensive bibliography, can be found in (Polyak et al., 2002; Nikiforov, 2003). In monograph (Bukov, 2006) the classification of disturbances of various types and methods of their compensation are given.

In Bukov (2006), Nikiforov (2004a,b) an internal model of disturbances is used to solve the problem whereas (Nikiforov, 1997; Miroshnik et al., 2000) use the methods of the theory of robust and adaptive systems. The approach to the synthesis of static, robust controllers for linear systems that is based on the linear-quadratic problem that is in turn based on the parameterization of Lurie–Riccati equations is presented in (Bukov et al., 2007). Robust systems with compensation of disturbances that use these methods are studied in Bobtsov, (2003a,b)

A simple robust control algorithm that remains the same for various types of plants is proposed in Tsykunov (2008). It is shown that the algorithm compensates for parametric and external disturbances with a given accuracy. A closed system works here as an implicitly given nominal model whose parameters are used in control.

It is important to note that almost all the suggested methods are based on an assumption that the structure of a plant is known i.e. the order of a system of differential equations is known and parametric and external disturbances are unknown. There are various studies devoted to the problems of control with an unknown order (Tao et al., 1993; Hoang et al., 2007; Furtat et al., 2008). Sources (Hoang et al., 2007) consider control problems of linear, stationary systems with an unknown and constant order of numerator and denominator for their transfer functions. Source (Furtat et al., 2008) considers a wider class of systems with disturbances that are able to influence both the parameters of the system as well as its order.

This paper considers the problem of robust control for interconnected systems with unknown parameters which are subject to the uncontrolled external and parametric disturbances. These disturbances may change the order of a system in unpredictable ways. This means that the order of a system is unknown and scalar input and output signals can only be measured. To solve the problem, a simple robust control algorithm is proposed that compensates for this class of uncertainties with a given accuracy and a finite time. Only the measurable variables of the local subsystems are used for the control i.e. control is completely decentralized.

Decentralized control can be used for a wide rage of large-scale complex systems including satellite networks, group flights, electric power systems, robots etc. Decentralized control is also very efficient when there is a need to design the control algorithms relying on local information. Modern computer networks provide an efficient infrastructure for a real implementation of such algorithms.

2. PROBLEM STATEMENT

Let us consider an interconnected system whose local subsystems' dynamic processes are described by the following equations

$$Q_i(P)y_i(t) = k_i R_i(P)u_i(t) + f_i(t) + \sum_{j=1}^{M} S_{ij}(P)y_j(t), \quad i \neq j, \quad i = \overline{1, M}, \qquad (1)$$

where $P = d/dt$ – differential operator; $Q_i(P)$, $R_i(P)$, $S_{ij}(P)$ are the linear differential operators with unknown constants

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parameters; $u_i(t)$ is a scalar control action; $y_i(t)$ is a scalar controlled variable in the *i*-subsystem which can be measured; $f_i(t)$ is an uncontrolled disturbance.

Decentralized control for such a system is defined as the problem of finding M local control blocks, each of which only can access current information about a system. Required quality of transition processes in a subsystem is defined by equations of the local nominal models

$$Q_{mi}(P)y_{mi}(t) = k_{mi}r_i(t), \qquad i = \overline{1, M}. \qquad (2)$$

Here $Q_{mi}(P)$ are linear differential operators; $k_{mi} > 0$; $r_i(t)$ are the scalar bounded control actions.

It is necessary to design a control system for which the following condition will be satisfied:

$$\overline{\lim_{t\to\infty}}|e_i(t)| = \overline{\lim_{t\to\infty}}|y_i(t) - y_{mi}(t)| < \delta \text{ if } t \geq T. \qquad (3)$$

Here δ is the accuracy of the dynamic error $e_i(t)$; T is the time beyond of which the dynamic error should not exceed the value δ. It is forbidden to use measurable parameters of one subsystem in other local subsystems.

Assumptions:

i) $R_i(\lambda), Q_{mi}(\lambda), R_{mi}(\lambda)$ are Hurwitz polynomials (λ is a complex variable in Laplace transformation);

ii) the orders of $\deg Q_i = n_i$; $\deg R_i = m_i$; $\deg S_{ij} = n_{ij}$; $n_{ij} < n_i - 1$ are unknown and relative degree of the local system $\gamma_i = n_i - m_i > 1$;

iii) the upper bound $\gamma_{ui} \geq \gamma_i$ of the relative order γ_i as well the upper bound of the operator Q_i are defined, i.e. $n_i \leq \bar{n}_i$;

iv) the orders of the polynomials Q_{mi} are equal to γ_{ui};

v) we know the coefficients' sings k_i and assume that $k_i > 0$;

vi) the coefficients of the operators $R_i(\lambda), Q_i(\lambda)$ depend on the vector of unknown parameters $\xi \in \Xi$, where Ξ is a bounded set;

vii) control actions $r_i(t)$ are bounded functions;

viii) we cannot use the derivatives $y_i(t),\ u_i(t),\ r_i(t)$ of the signals.

ix) the signal of local nominal model $y_{mi}(t)$ and its derivatives γ_{ui} are bounded functions;

x) the external disturbance $f_i(t)$ is a bounded function of time with an unknown changes range;

3. Method of solution

Let us first write the $Q_i(P),\ R_i(P)$ in the following form:

$$Q_i(P) = Q_{0i}(P) + \Delta Q_i(P), \quad R_i(P) = R_{0i}(P) + \Delta R_i(P),$$

where $Q_{0i}(P)$ is an arbitrary linear differential operator, such that the polynomial $Q_{0i}(\lambda)$ is Hurwitz polynomial, $\deg Q_{0i} = \bar{n}_i$. Then the operator $\Delta Q_i(P)$ is $\deg \Delta Q_i \leq \bar{n}_i$, i.e. if $\deg Q_i < \deg Q_{0i}$ then $\deg \Delta Q_i = \deg Q_{0i}$, and if $\deg Q_i = \deg Q_{0i}$ then $\deg \Delta Q_i \leq \bar{n}_i - 1$. Introduce the arbitrary linear differential operator $R_{0i}(P)$, $\deg R_{0i} = \bar{n}_i - \gamma_{ui}$ such that the polynomial $R_{0i}(\lambda)$ is Hurwitz polynomial. The structure of $\Delta R_i(P)$ is such that if $m_i < \bar{n}_i - \gamma_{ui}$ then $\deg \Delta R_i = \bar{n}_i - \gamma_{ui}$, and if $m_i > \bar{n}_i - \gamma_{ui}$ then $\deg \Delta R_i = m_i$. This means that the above decomposition of the operators $Q_i(P),\ R_i(P)$ is correct because $\Delta Q_i(P)$ and $\Delta R_i(P)$ either have non-zero coefficients or an appropriate amount of their components are equal to zero. The decomposition (Furtat et al., 2008) is different from the known methods of parameterization of the equations.

Let us transform the equation of a system (1):

$$y_i(t) = \frac{k_i R_{0i}(P)}{Q_{0i}(P)}\left(u_i(t) + \frac{\Delta R_i(P)}{k_i R_{0i}(P)}u_i(t) + \right. \qquad (4)$$

$$\left. + \frac{1}{k_i R_{0i}(P)} f_i(t) - \frac{\Delta Q_i(P)}{k_i R_{0i}(P)} y_i(t) + \sum_{j=1, i\neq j}^{M} \frac{S_{ij}(P)}{k_i R_{0i}(P)} y_j(t)\right),$$

since operators $Q_{0i}(P)$ and $R_{0i}(P)$ are arbitrary, we can choose them in order that the following condition is obeyed

$$\frac{R_{0i}(\lambda)}{Q_{0i}(\lambda)} = \frac{1}{Q_{mi}(\lambda)}. \qquad (5)$$

Let us write the equation for error $e_i(t) = y_i(t) - y_{mi}(t)$, subtracting (2) from (4), and taking into consideration (5),

$$Q_{mi}(P)e_i(t) = k_i u_i(t) + \left(\frac{\Delta R_i(P)}{R_{0i}(P)}u_i(t) - \frac{\Delta Q_i(P)}{R_{0i}(P)}y_i(t) + \right.$$

$$\left. + \frac{1}{R_{0i}(P)} f_i(t) - k_{mi}r_i(t) + \sum_{j=1, i\neq j}^{M} \frac{S_{ij}(P)}{R_{0i}(P)} y_j(t)\right) \qquad (6)$$

To obtain the main result, let's use the approach (Tsykunov, 2008), which allows to compensate disturbance. Let choose a local control law in the following form

$$u_i(t) = \alpha_i \vartheta_i(t). \qquad (7)$$

where $\alpha_i > 0$; $\vartheta_i(t)$ is an additional control action. Then the following equation of error can be derived from (6)

$$Q_{mi}(P)e_i(t) = \vartheta_i(t) + \varphi_i(t), \qquad (8)$$

$$\varphi_i(t) = \frac{1}{R_{0i}(P)}\left(\Delta R_i(P)\ u_i(t)\right) - \qquad (9)$$

$$-\frac{1}{R_{0i}(P)}\left(\Delta Q_i(P) y_i(t) - \sum_{j=1, i\neq j}^{M} \frac{S_{ij}(P)}{R_{0i}(P)} y_j(t)\right) +$$

$$+ \frac{1}{R_{0i}(P)} f_i(t) - k_{mi} r_i(t) + (k_i \alpha_i - 1)\vartheta_i(t).$$

Signal $\varphi_i(t)$ contains all components action of which in the error needs to be compensated. It is necessary to extract the signal.

Let's define the additional loop

$$Q_{mi}(P)\tilde{e}_i(t) = \vartheta_i(t) \qquad (10)$$

and write the equation with the error signal $\zeta_i(t) = e_i(t) - \tilde{e}_i(t)$:

$$Q_{mi}(P)\zeta_i(t)=\varphi_i(t).$$

If the derivatives γ_{ui} of the output signal $y_i(t)$ can be measured then defining the variation law of the additional control action in the following form

$$\vartheta_i(t)=-Q_{mi}(P)\zeta_i(t)=-\varphi_i(t), \qquad (11)$$

we will get the following equation of the closed loop system using the error equation (8)

$$Q_{mi}(P)e_i(t)=0. \qquad (12)$$

Let us show that all the signals in the closed loop system are bounded. It is necessary for the efficiency of the algorithm which will be described later. Equation (12) shows that the signal $y_i(t)$ and its derivatives γ_{ui} are bounded due to assumption x). Then from conditions of the assumptions $\deg\Delta Q_i(P)=\overline{n}_i$ and because $R_{0i}(\lambda)$ is Hurwitz polynomial of $\overline{n}_i-\gamma_{ui}$ degree we can conclude that

$$\varphi_{1i}(t)=\frac{1}{R_{0i}(P)}f_i(t)-k_{mi}r_i(t)-$$
$$-\frac{1}{R_{0i}(P)}\left(\Delta Q_i(P)y_i(t)-\sum_{j=1,i\neq j}^{M}\frac{S_{ij}(P)}{R_{0i}(P)}y_j(t)\right)$$

is a bounded value. It is necessary to show that the chosen control action is bounded. For that purpose let's substitute $\varphi_i(t)$ in (11) with the statement above and resolve derived equation for $\vartheta_i(t)$:

$$\vartheta_i(t)=-\frac{1}{k_i\alpha_i}\left(\varphi_{1i}(t)+\frac{1}{R_{0i}(P)}\left(\Delta R_i(P)u_i(t)\right)\right). \qquad (13)$$

Let us substitute $\vartheta_i(t)$ in equation (9) and resolve it for $u_i(t)$, taking into consideration following parameterization $k_iR_i(P)=k_iR_{0i}(P)+\Delta R_i(P)$:

$$k_iR_i(P)u_i(t)=-\varphi_{1i}(t).$$

From condition of assumption ii) and boundedness of $\varphi_{1i}(t)$ boundedness of local control action $u_i(t)$ is followed.

Because we cannot measure the derivatives, let's formulate the local law of additional control action $\vartheta_i(t)$ in the following form

$$\vartheta_i(t)=-g_{mi}^T\overline{\zeta}_i(t), \qquad (14)$$

where $g_{mi}^T=\left[q_{m\gamma_{ui}},\ldots,q_{m1},1\right]$ – vector composed with polynomial coefficients $Q_{mi}(\lambda)=\lambda^{\gamma_{ui}}+q_{m1}\lambda^{\gamma_{ui}-1}+\ldots+q_{m\gamma_{ui}}$; $\overline{\zeta}_i(t)=col\left(\zeta_i,\overline{\zeta}_{i1},\overline{\zeta}_{i2},\ldots,\overline{\zeta}_{i\gamma_{ui}}\right)$; $\overline{\zeta}_{ik}(t)$ is estimation of derivatives $P^k\zeta_i(t)$, obtained from filters

$$\dot{z}_{ik}(t)=\frac{1}{\mu}F_iz_{ik}(t)+\frac{1}{\mu}b_iP^k\zeta_i(t),$$
$$\overline{\zeta}_{ik}=L_{0i}z_{ik},\qquad i=\overline{1,M},\quad k=\overline{1,\gamma_{ui}}. \qquad (15)$$

Where $z_{ik}\in R^{\gamma_{ui}}$; $L_{0i}=[1,0,\ldots,0]$; $b_i^T=[0,\ldots,0,1]$;

$$F_i=\begin{bmatrix}-1 & 1 & 0 & \cdots & 0\\ 0 & -1 & 1 & \ddots & \vdots\\ \vdots & \ddots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & -1 & 1\\ 0 & \cdots & 0 & 0 & -1\end{bmatrix};$$

$\mu>0$ is small number. If we use (14) and (15) in Laplace transformation we'll get the following

$$\vartheta_i(\lambda)=-\frac{Q_{mi}(\lambda)}{(\mu\lambda+1)^{\gamma_{ui}}}\zeta_i(\lambda).$$

Taking into consideration (10) and statement for error signal $\zeta_i(t)=e_i(t)-\widetilde{e}_i(t)$ we have

$$\vartheta_i(\lambda)=-\frac{Q_{mi}(\lambda)}{(\mu\lambda+1)^{\gamma_{ui}}-1}e_i(\lambda).$$

Substituting $\vartheta_i(t)$ in equation (7) with the obtained statement and using the original of Laplace transformation we'll get control algorithm. Obviously that control law now is technically feasible since it contains only known or measurable variables.

Proposition. *If assumptions i) - x) are obeyed then there are numbers $\mu_0>0$, $T_0>0$ such that under conditions $\mu\le\mu_0$, $T\ge T_0$ control algorithm*

$$\left((\mu P+1)^{\gamma_{ui}}-1\right)u_i(t)=-\alpha_iQ_{mi}(P)e_i(t) \qquad (16)$$

guarantees that target condition (3) is obeyed, where $\alpha_i>0$.

It is necessary to note that the described algorithm remains invariant if there is state delay in a system as well as in the case when a system is in a steady state with unknown parameters with known boundaries.

Proof. Let's consider vectors of the estimation error of derivatives $P^k\zeta_i(t)$

$$\eta_{ik}(t)=z_{ik}(t)+F_i^{-1}b_iP^k\zeta_i(t),\quad k=\overline{1,\gamma_{ui}},\quad i=\overline{1,M}.$$

Here the vector $F_i^{-1}b_i=h_i$ has first component equal to -1. If to prove that the value $|\eta_{ik}(t)|$ is small, then from condition $\left|\overline{\zeta}_{ik}(t)-P^k\zeta_i(t)\right|<|\eta_{ik}(t)|$ it follows that estimation $\overline{\zeta}_{ik}(t)$ is ear to $P^k\zeta_i(t)$. From (15) we'll get the equation of dynamic for vectors $\eta_{ik}(t)$:

$$\dot{\eta}_{ik}(t)=\frac{1}{\mu}F_i\eta_{ik}(t)+h_iP^{k+1}\zeta_i(t),$$
$$\Delta_{ik}(t)=L_i\eta_{ik}(t),\qquad i=\overline{1,M},\quad k=\overline{1,\gamma_{ui}}.$$

Taking into account that the additional control action is formulated as (14), we can transform the equation of error into the following form

$$Q_{mi}(P)e_i(t)=-q_{mi}^T\Delta_i(t), \qquad (17)$$

where $q_{mi}^T=\left[q_{m\gamma_{ui}-1},\ldots,q_{m1},1\right]$; $\Delta_{ik}(t)=\overline{\zeta}_{ik}(t)-P^k\zeta_i(t)$; $\Delta_i(t)=col\left(\Delta_{i1}(t),\Delta_{i2}(t),\ldots,\Delta_{i\gamma_{ui}}(t)\right)$. Let's transform equation (17) into vector-matrix form. As a result we'll get the following equations set of the closed loop system:

$$\begin{cases} \dot{\varepsilon}_i(t) = A_{mi}\varepsilon_i(t) + b_i q_{mi}^T \Delta_i(t), \quad e_i(t) = L_i \varepsilon_i(t), \\ \mu_1 \dot{\eta}_{ik}(t) = F_i \eta_{ik}(t) + \mu_2 h_i P^{k+1} \zeta_i(t), \\ \Delta_{ik}(t) = L_i \eta_{ik}(t), \qquad i = \overline{1, M}, \quad k = \overline{1, \gamma_{ui}}, \end{cases} \tag{18}$$

where $\mu_1 = \mu_2 = \mu$. We've got singularly perturbanced system as μ – small enough number. Let us use Lemma (Brusin, 1995).

Lemma (*Brusin, 1995*). *If a system is defined by the equation* $\dot{x} = f(x, \mu_1, \mu_2)$, $x \in R^{m_1}$, *where* $f(t)$ *is a continuous function that is Lipshits function with respect to x and in the case when* $\mu_2 = 0$ *it has a bounded closed region of dissipation* $\Omega_1 = \{x \mid F(x) < \widetilde{C}\}$, *where* $F(x)$ *– positive defined continuous piecewise smooth function, then there is* $\mu_0 > 0$ *such that under* $\mu_2 \le \mu_0$ *the initial system has the same dissipative region* Ω_1, *if for some numbers* $\widetilde{C}_1$ *and* $\overline{\mu}_1$ *for* $\mu_2 = 0$ *following condition is obeyed*

$$\sup_{|\mu_1| \le \overline{\mu}_1} \left(\left(\frac{\partial F(x)}{\partial x} \right)^T f(x, \mu_1, 0) \right) \le -\widetilde{C}_1, \ \text{if} \ F(x) = \widetilde{C}. \tag{19}$$

In the case of $\mu_2 = 0$ in (18) we have asymptotically stable system for variables $\varepsilon_i(t)$ and $\eta_{ik}(t)$, since A_{mi}, F_i are Hurwitz matrixes. It is the same situation which we had for measuring the derivatives i.e. $\lim_{t \to \infty} e_i(t) = 0$. It was proved that if this condition is obeyed all the signals in the system are bounded. It means that there is a certain region

$$\Omega = \{\varepsilon_i(t), \eta_{ik}(t), \zeta_i(t): \ \left|P^{k+1}\zeta_i(t)\right| \le \delta_{1k}, \left|\varepsilon_i(t)\right| < \delta_{2k},$$
$$\left|\eta_{ik}(t)\right| < \delta_{3k}, F(\varepsilon_i, \eta_{ik}) < C_1\}, \qquad k = \overline{1, \gamma_{ui}},$$

where signals $e_i(t), \eta_{ik}(t), \zeta_i(t)$ are within their boundaries for some initial conditions from Ω_0.

Let us consider two vectors $\theta_i^T(t) = \left[\ddot{\zeta}_i(t), \dots, \overset{(\gamma_{ui}+1)}{\zeta_i}(t) \right]$, $\eta_i^T(t) = \left[\eta_{i1}(t), \eta_{i2}(t), \dots, \eta_{i\gamma_{ui}}(t) \right]$, and block-diagonal matrixes with γ_{ui} diagonal blocks $F_{0i} = diag\{F_i, F_i, \dots, F_i\}$, $B_i = diag\{h_i, h_i, \dots, h_i\}$, $C_i = diag\{L_i, L_i, \dots, L_i\}$, then equations (18) will take the following form

$$\begin{cases} \dot{\varepsilon}_i(t) = A_{mi}\varepsilon_i(t) + b_i q_{mi}^T \Delta_i(t), \quad e_i(t) = L_i \varepsilon_i(t), \\ \mu_1 \dot{\eta}_i(t) = F_{0i} \eta_i(t) + \mu_2 B_i \theta_i(t), \\ \Delta_i(t) = C_i \eta_i(t), \qquad i = \overline{1, M}. \end{cases} \tag{20}$$

Evidently that condition (19) was obeyed if to take Lyapunov function for F_i

$$V(\varepsilon_i(t), \eta_i(t)) = \sum_{i=1}^{M} \left(\varepsilon_i^T(t) H_{1i} \varepsilon_i(t) + \eta_i^T(t) H_{2i} \eta_i(t) \right), \tag{21}$$

where the positive defined symmetric matrixes H_{1i}, H_{2i} are determined from equations solution

$$\begin{aligned} H_{1i} A_{mi} + A_{mi}^T H_{1i} &= -\rho_{1i} I_{\gamma_{ui}} - Q_{1i}, \\ H_{2i} F_i + F_i^T H_{2i} &= -\rho_{2i} I_{\gamma_{ui}} - Q_{2i}, \end{aligned} \tag{22}$$

where $\rho_{1i} > 0$, $\rho_{2i} > 0$, $Q_{1i} = Q_{1i}^T > 0$, $Q_{2i} = Q_{2i}^T > 0$. Thus in accordance with Lemma (Bukov et. al., 2008), there is $\mu_0 > 0$ such that if $\mu < \mu_0$ then Ω remains dissipative region of system (18).

However it is necessary to note that keeping the dissipative region doesn't guarantee that the set of attraction Ω_1 remains the same in a singularly perturbed system.

Let us calculate the full derivative of function (21) on system's trajectories (20), taking into account equation (22) and assigning $\mu_1 = \mu_2 = \mu_0$:

$$\dot{V}(\varepsilon_i(t), \eta_i(t)) = \sum_{i=1}^{M} \Big(-\rho_{1i} \left\|\varepsilon_i(t)\right\|^2 - \varepsilon_i^T(t) Q_{1i} \varepsilon_i(t) +$$
$$+ 2\varepsilon_i^T(t) H_{1i} b_i q_{mi}^T \Delta_i(t) - \frac{\rho_{2i}}{\mu_0} \left\|\eta_i(t)\right\|^2 -$$
$$- \frac{1}{\mu_0} \eta_i^T(t) Q_{2i} \eta_i(t) + 2\eta_i^T(t) H_{2i} B_i \theta_i(t) \Big). \tag{23}$$

Let us use estimations

$$2\varepsilon_i^T(t) H_{1i} b_i q_{mi}^T \Delta_i(t) \le \left\|\varepsilon_i(t)\right\|^2 + \rho_{3i} \left\|\eta_i(t)\right\|^2,$$
$$2\eta_i^T(t) H_{2i} B_i \theta_i(t) \le \frac{1}{\mu_0} \left\|\eta_i(t)\right\|^2 + \mu_0 \rho_{4i},$$
$$-\varepsilon_i^T(t) Q_{1i} \varepsilon_i(t) \le -\frac{\lambda_{\min}(Q_{1i})}{\lambda_{\max}(H_{1i})} \varepsilon_i^T(t) H_{1i} \varepsilon_i(t),$$
$$-\eta_i^T(t) Q_{2i} \eta_i(t) \le -\frac{\lambda_{\min}(Q_{2i})}{\lambda_{\max}(H_{2i})} \eta_i^T(t) H_{2i} \eta_i(t),$$

where $\rho_{3i} = \left\|H_{1i} b_i q_{mi}^T C_i\right\|^2$, $\rho_{4i} = \left\|H_{2i} B_i\right\| \sum_{i=1}^{2\gamma_{ui}} \delta_{1i}^2$; $\lambda_{\min}$, $\lambda_{\max}$ are the minimal and maximal characteristic numbers of the mentioned matrixes. Using those estimations into (23) we'll get

$$\dot{V}(\varepsilon_i(t), \eta_i(t)) \le -\sigma_0 V + \sum_{i=1}^{M} \Big(-(\rho_{1i} - 1) \left\|\varepsilon_i(t)\right\|^2 -$$
$$- \left(\frac{\rho_{2i}}{\mu_0} - \frac{1}{\mu_0} - \rho_{3i} \right) \left\|\eta_i(t)\right\|^2 + \mu_0 \rho_{4i} \Big),$$

where $\sigma_0 = \min\left\{ \frac{\lambda_{\min}(Q_{1i})}{\lambda_{\max}(H_{1i})}, \frac{\lambda_{\min}(Q_{2i})}{\lambda_{\max}(H_{2i})} \right\}$. If to choose ρ_{1i}, ρ_{2i} from conditions

$$\rho_{1i} - 1 > 0, \quad \frac{\rho_{2i}}{\mu_0} - \frac{1}{\mu_0} - \rho_{3i} > 0, \tag{24}$$

the following inequality is correct:

$$\dot{V}(\varepsilon_i(t), \eta_i(t)) \le -\sigma_0 V(\varepsilon_i(t), \eta_i(t)) + \sum_{i=1}^{M} \mu_0 \rho_{4i}.$$

If we solve the inequality

$$V(\varepsilon_i(t), \eta_i(t)) \le V(0) e^{-\sigma_0 t} + \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0},$$

we can see that if to choose μ_0 small enough we get the following region of attraction:

$$\Omega_2 = \left\{ \varepsilon_i(t), \eta_i(t): \quad V\big(\varepsilon_i(t), \eta_i(t)\big) \le \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0} \right\}.$$

Inserting the required value T_0 from the target condition (3) into the right part and taking into consideration the inequalities

$$|e_i(t)|^2 \le \|\varepsilon_i(t)\|^2 \le \frac{V(0)e^{-\sigma_0 t}}{\lambda_{\min}(H_{1i})} + \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0 \lambda_{\min}(H_{1i})}$$

we get the estimation of the value δ in the target condition (3)

$$\delta \le \sqrt{\frac{1}{\lambda_{\min}(H_{1i})}\left(V(0)e^{-\sigma_0 t} + \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0} \right)},$$

that shows that there are numbers μ_0 and T_0 guaranteeing that target condition will be obeyed. Thus for $\mu \le \mu_0$ varying ρ_{1i} in (24) and μ, we can get the required value δ in the target condition (3).

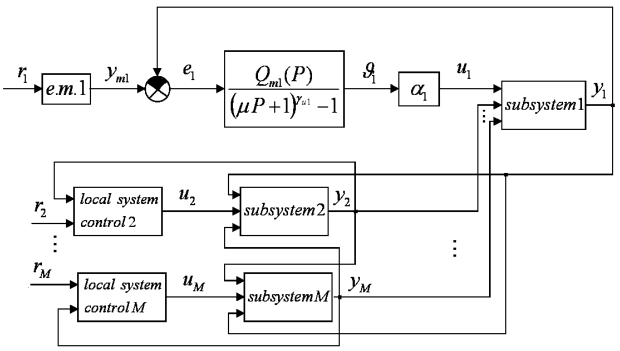

Fig. 1. The structure scheme of robust control system

4. EXAMPLE

As an example, the system can be used to solve the problem of decentralized control of the trajectory of the group of the pilotless aircrafts of different types in the horizontal plane. The aircrafts do not exchange data with each other. Trajectory control for each aircraft is performed using radio commands from a ground-based control station.

First, the robust local etalon models (2) are selected. Then, we generate the local regulators for each aircraft using (16). Using numerical analysis the group flight under wind disturbance is considered.. The obtained results demonstrate the efficiency of the suggested approach to decentralized control.

Unlike the work (Bukov et. al., 2008; Krasovsky et. al., 1986) a broader class of the systems is considered here because of taking into account the ability of the systems to adapt to external, parametrical, and structural disturbance. Let us consider for simplicity the flight of two aircrafts (the number of the aircrafts in the model can be easily increased).

The first aircraft will be the lead aircraft and the second one is a wingman aircraft. The wingman aircraft is controlled by the speed according to a predefined program. That aircraft has to

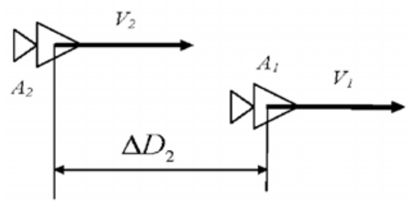

Fig. 2. Model of the aircraft group flight

keep the distance ΔD_2. Let us also assume that the wingman can measure this distance. The change of the distance is described by the equation $\Delta\dot{D} = V_1 - V_2$.

The speed V_1 is defined by the dynamics of the lead aircraft and V_2 is defined by the wingman aircraft. If we introduce the model

$$\begin{bmatrix} \Delta\dot{V}_1 \\ \Delta\dot{P}_1 \\ \Delta\dot{D}_1 \\ \Delta\dot{V}_2 \\ \Delta\dot{P}_2 \\ \Delta\dot{D}_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} -a_{x1}^{V} & -a_{x1}^{P} & 0 & 0 & 0 & 0 \\ \frac{k_{\partial 1}^{V}}{T_{\partial 1}} & -\frac{1}{T_{\partial 1}} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -a_{x2}^{V} & -a_{x2}^{P} & 0 \\ 0 & 0 & 0 & \frac{k_{\partial 2}^{V}}{T_{\partial 2}} & -\frac{1}{T_{\partial 2}} & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \end{array}\right] \left[\begin{array}{c} \Delta V_1 \\ \Delta P_1 \\ \Delta D_1 \\ \hline \Delta V_2 \\ \Delta P_2 \\ \Delta D_2 \end{array}\right] +$$

$$+ \left[\begin{array}{cc} 0 & 0 \\ \frac{k_{\partial 1}^{\partial_{py\partial}}}{T_{\partial 1}} & 0 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & \frac{k_{\partial 2}^{\partial_{py\partial}}}{T_{\partial 2}} \\ 0 & 0 \end{array}\right] \left[\begin{array}{c} \Delta\partial_{py\partial 1} \\ \hline \Delta\partial_{py\partial 2} \end{array}\right] + \left[\begin{array}{c} -\dot{U}_{x1} - a_{x1}^{\alpha}\Delta\alpha_1 - a_{x1}^{\Theta}\Delta\Theta_1 \\ 0 \\ U_{x1} \\ \hline -\dot{U}_{x2} - a_{x2}^{\alpha}\Delta\alpha_2 - a_{x2}^{\Theta}\Delta\Theta_2 \\ 0 \\ U_{x2} \end{array}\right]$$

with the distance ΔD_2 as output, we will get two interconnected systems. Let, for example, nitially $V_1 = V_2$, then $\Delta D_2 = const$. If we change the thrust lever of the lead aircraft, we change the speed of the aircraft; the equation $V_1 = V_2$ is no longer valid and the distance $\Delta\dot{D} = V_1 - V_2$ will be changed as well. If the goal is to keep the distance $\Delta D_2 = const$, the control of both aircrafts should be performed in a coordinated way. Any time when the speed of the lead aircraft is being changed, we have to change the speed of the second aircraft.

Let us assume that 1) the aircrafts do not exchange the information; 2) the distance ΔD_2 can be measured by the wingman. Under these assumptions we may treat the problem as the problem of the decentralized control (16). Here ΔV is the increment of the speed with respect to the balance value V_0 (m/c); ΔP is the increment of the thrust with respect to the balance value P_0 (n); $\dot{U}_x = dU_x/dt$ is the change of the wing speed on the axis OX, U_x is the wing speed on the axes OX; $\Delta\alpha$ is the increment of the angle of attack to the balance value; $\Delta\Theta$ is the increment of the pitch angle; $\Delta\partial_{py\partial}$ is the engine throttle; $a_x^{\Theta} = g$ (free fall acceleration); $a_x^P \approx \frac{1}{m}$, where m is the aircraft's weight; k_∂^V takes into account the change of engine throttle when the speed is been changed (can be assumed = 0); the constant T_∂ of the engine depends on the flight regime (Polyak et. al., 2002); $k_\partial^{\partial_{py\partial}}$ allows to take into account the change of the engine throttle when changing the thrust lever. We can assume for the simplicity that this dependency is linear. Then, if thrust-to-weight ratio equal to 0,7, we get

$$k_\partial^{\partial_{py\partial}} = \frac{P_{\max}}{\delta_{py\partial max}} = \frac{0{,}7m}{g\delta_{py\partial max}}.$$

Additional assumption is that the aircraft is highly maneuvering.

The inputs of the system are unknown wing and other uncontrolled dynamic disturbances. If the distance is relatively small (10 – 50 m), then we may assume the same wing $U_{x1} = U_{x2}$ for each aircraft. Otherwise we have to take into account the order in which the aircrafts meet the blasts. There are two control channels for the group flights:

- speed / distance control using the engine throttle $\Delta\partial_{py\partial}$;
- altitude control using the elevator $\Delta\delta_в$.

It is desirable to have the independent channels. The example is the following control law:

$$\Delta\delta_в = k_{\delta в}^{\omega_z}\omega_z + k_{\delta в}^{\vartheta}\Delta\vartheta + k_{\delta в}^{H}(H_0 - H),$$

H_0 and H is the targeted and current altitudes

$$(\Delta\dot{H} = (V + U_x)\sin\Theta \approx (V + U_x)\Delta\Theta).$$

This altitude control creates a disturbance in the speed control channel which has to be compensated.

5. CONCLUSION

The Paper considers the problem of decentralized control with an nominal model for interconnected system with unknown parameters and an unknown order when derivatives of input and output signals of the local subsystems cannot be measured.

Considered robust control system allows compensating parametric and external disturbances with given accuracy δ for the period of time T. Values δ and T can be small enough using the appropriate parameters of the closed loop system. It is necessary to note that the closed loop system is functioning as an implicitly defined nominal model and parameters of the model are used in control algorithm.

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