MATHEMATICAL MODELS OF CONTROL BY THE INTERCONNECTED MECHANICAL AND THERMAL PROCESSES IN NONLINEAR DYNAMIC SYSTEMS DISTURBED BY TEMPERATURE EFFECTS

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Abstract

The opportunity of control by the interconnected mechanical and thermal processes in nonlinear disturbed dynamic systems is proved, both at their irregular movements, and at parametrical temperature indignations. Conditions on parameters of systems and the control providing stability of stationary points are received. It is shown, that it is possible to provide a regularity of chaotic fluctuations and is also possible to provide a control of fluctuations when take place parametrical temperature disturbances by a choice of parameters of the connected thermal and mechanical subsystems. For control by the interconnected mechanical and thermal processes it is necessary to include into feedback of a mechanical subsystem a signal proportional to temperature difference. The quantitative estimations confirming the qualitative analysis of constructed mathematical models are received.

Use of interrelation of thermal and mechanical energy for control by movement and temperature fields in mechanical dynamic systems with sources of heat is essential and modern [1] problem. For example, this problem is important for perspective micromechanical gyroscopic sensors and sensors of other physical values, of gradiometers or for macromechanical designs of space appointment with mobile elements.

Relative mechanical motion of nonlinear dynamic system and her the thermal state is described by system of the ordinary differential equations in a matrix kind [1,2]:

$$\dot{X} = A(T,T_c)F(X) + B(X,T), \quad (1)$$

$$\dot{T} = G(X)T + D(T,T_c,X), \quad (2)$$

where X - vector of a mechanical state; T, T_c - vectors of a thermal state of system and an environment; $A(T,T_c)$ - a matrix of inertial, dissipative and elastic properties of elements of system; $G(X)$ - a matrix of thermal properties; $B(X,T)$, $D(T,T_c,X)$- vectors of controls and disturbances of mechanical and thermal subsystems.

Feature of the equations (1),(2) is that mechanical and thermal subsystems are connected through matrixes A,G and entrance disturbances and the controls ***B*,*D***.

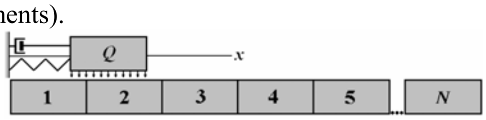
It allows to solve problems of the analysis and control of the interconnected mechanical and thermal subsystems.

1. Control of irregular behavior of nonlinear disturbed dynamic system

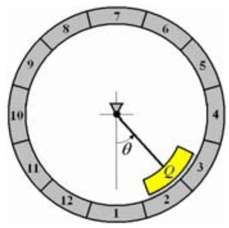
The very important feature of nonlinear disturbed dynamic systems (1), (2) is that in them, at the determined combinations of parameters, occurrence of irregular behavior [3] is possible.

We research a possibility and the conditions of a regularization of chaotic motions of a subsystem (1) and of the thermal processes of a subsystem (2) by means of the thermal controlling effect included into a feedback (1).

Examples of such systems are nonlinear disturbed by temperature the oscillator (fig.1*a*) or a mathematical pendulum (fig.1*b*), which are heat sources of power Q and which make vibratory movements above a motionless body and which we divided on the elementary volumes (elements).



a)



b)

Fig.1

Let's assume that the conductive heat transfer occurs only. Motion of a mechanical subsystem is presented by the differential equation of the second order.

Dimension of a thermal subsystem $R_T = N$, where N - quantity of motionless elementary volumes. For analytical quality estimations it is assumed N =2. For conducting numerical calculations we shall choose $N \geq 12$.

Let's consider the nonlinear equation of motion of the disturbed oscillator (fig.1*a*) with possible controlling thermal affecting:

$$\ddot{x} + 2n\dot{x} + 0.5L\omega^2 \sin(\pi x/L) = F\sin(\nu t + \delta) + \gamma(T_1 - T_2). \quad (3)$$

The equations of balances for a thermal subsystem (N =2):

$$c\dot{T}_1 + q(T_1 - T_2) + q_c(T_1 - T_c) = -\frac{Q}{L}x + \frac{Q}{2}, \quad (4)$$

$$c\dot{T}_2 + q(T_2 - T_1) + q_c(T_2 - T_c) = \frac{Q}{L}x + \frac{Q}{2}, \quad (5)$$

where $x(t)$ - coordinate of a mobile element with a heat source; $T_1(t)$, $T_2(t)$ - temperatures of motionless elements; q, q_c - thermal conductivity between elements and with a environment; c - heat capacity; L - characteristic distance between elements; $n = n_0(1 - \eta\Delta T_c)$ - dependence of a damping from temperature of environment; $\Delta T_c = T_c - T_{c0} = T_{cc}\sin\omega_c t$ - the law of change of the temperature environment; n_0 - a nominal damping; η -the parameter reflecting dependence of dissipating properties of a mechanical subsystem from temperature; ω - frequency of natural oscillations; F, ν, δ - amplitude, frequency and a phase of forcing force; Q - power of a heat source; γ - factor of controlling thermal affecting.

From (4),(5) we shall have the equation for temperature difference $\Delta T = T_1 - T_2$:

$$\Delta\dot{T} + \lambda\Delta T = -\frac{2Q}{Lc}x\,, \qquad (6)$$

where $\lambda = (2q + q_c)/c$.

For deriving the approximated estimations of a possibility of control by nonlinear dynamic system (3),(6) we suppose, that dissipative properties of an oscillator do not depend on temperature ($\eta = 0$).

Stability conditions of stationary points of system (3), (6), deduced by a procedure [2,3], look like:

$$4n\lambda > \pi\omega^2\,, \qquad (7)$$

$$\gamma_{\min} = \frac{Lc\lambda\pi\omega^2}{4Q} < \gamma < \frac{Lcn}{Q}\left(\lambda^2 + 2n\lambda - 0.5\pi\omega^2\right) = \gamma_{\max}\,. \quad (8)$$

On the basis of the equations for stationary points and relations (8) graphs (fig.2) dependences of amplitude of a driving force from coordinate x_* of stationary points are constructed at various values of factor controlling affecting $\gamma = 0$, $\gamma = \gamma_{\min}$, $\gamma = \gamma_{\max}$.

The zone of steady stationary points is shaded.

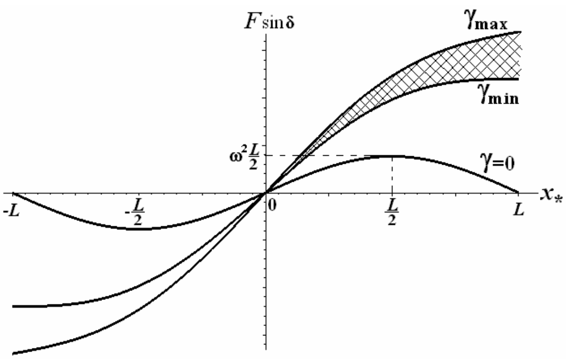


Fig.2

In case of a nonlinear disturbed mathematical pendulum (Fig.1*b*) with possible controlling thermal affecting, the equations (1) will become:

$$\ddot{\theta} + 2n\dot{\theta} + \omega^2\sin\theta = F\sin(\nu t + \delta) + \gamma(T_1 - T_2)\,, \quad (9)$$

where $\theta(t)$ - coordinate of a mobile element with a heat source

The equations of the balances (2) for a thermal subsystem with dimension $N = 2$ will accept kind:

$$c\dot{T}_1 + q(T_1 - T_2) + q_c(T_1 - T_c) = -\frac{Q}{\pi}\left(\arcsin(\sin\theta) - \frac{\pi}{2}\right), \quad (10)$$

$$c\dot{T}_2 + q(T_2 - T_1) + q_c(T_2 - T_c) = \frac{Q}{\pi}\left(\arcsin(\sin\theta) + \frac{\pi}{2}\right). \quad (11)$$

From (10),(11) we shall have the equation for temperature difference ΔT:

$$\Delta\dot{T} + \lambda\Delta T = -\frac{2Q}{\pi c}\arcsin(\sin\theta)\,. \qquad (12)$$

Occurrence of irregular motion of nonlinear disturbed dynamic systems is possible [3] under condition of existence of unstable fixed points.

Hence, if to choose parameters of system (3),(6) and factor γ, according to conditions (7),(8) so that all fixed points were steady it is possible to ensure regularization of motions of system and regularization of thermal processes.

Numerical calculations on nonlinear models of considered dynamic systems have confirmed the received analytical results.

At calculations input data have been used:
$\omega = 0{,}062\ \mathrm{s}^{-1}$; $F = 0{,}075\ \mathrm{s}^{-2}$; $\nu = 0{,}03\ \mathrm{s}^{-1}$; $n = 0{,}01\ \mathrm{s}^{-1}$; $\lambda = 0{,}42\ \mathrm{s}^{-1}$; $\eta = 0$; $T_{c0} = 20\ ^0\mathrm{C}$; Q =10 Wt and other.

The results of the numerical integration and the phase portraits of a nonlinear oscillator in a mode of irregular motion are shown on fig.3 without thermal control $\gamma = 0$ (fig.3*a*) and with thermal control with the chosen factor $\gamma_{\min} < \gamma < \gamma_{\max}$ (fig.3*b*).

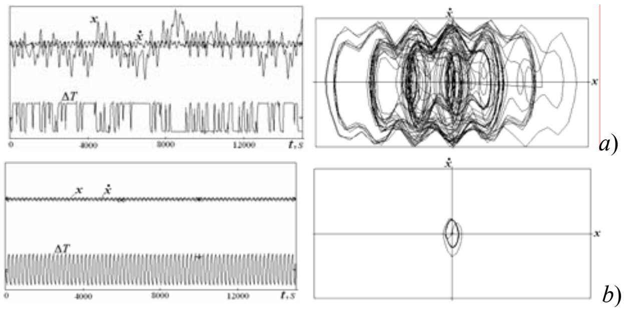


Fig.3

Apparently, without control $\gamma\Delta T$ in considered nonlinear disturbed dynamic systems possibly occurrence of irregular mechanical and thermal processes (fig.3*a*).

Inclusion into a feedback of the mechanical control subsystem proportional to temperature difference $\gamma\Delta T$ with chosen on conditions (7), (8) factor γ, provides regularization of mechanical and temperature oscillations of nonlinear dynamic system of an oscillator (fig.3*b*).

Similar results are received and for a nonlinear disturbed mathematical pendulum.

2. Control of nonlinear disturbed dynamic system (1), (2) with parametrical temperature perturbations

Let's consider a nonlinear disturbed mathematical pendulum (рис.1б), mechanical and thermal state of which is described by the equations (9) - (12).

In a mechanical subsystem the dependence of her of dissipative properties from temperature of a environment ($\eta \neq 0$) and the presence of a periodic driving force and the controlling temperature action $\gamma\Delta T$ is considered.

In a thermal subsystem we shall consider presence of a mobile heat source of a constant power.

Solving the equation (12) by method of a variation of a constant, and, substituting this solution in the equation of a pendulum (9), we shall receive the nonlinear equation of a disturbed pendulum:

$$\begin{aligned}&\ddot{\theta} + 2n_0(1 - \eta T_{cc}\sin\omega_c t)\dot{\theta} + \omega^2\sin\theta = \\ &= F\sin(\nu t + \delta) - \gamma\frac{2Q}{\pi c}\exp(-\lambda t)\int_0^t \arcsin(\sin\theta)\exp(\lambda t)dt.\end{aligned} \qquad (13)$$

For deriving analytical estimations of a possibility of control by the interconnected mechanical and thermal processes in such system, we suppose that an angle of a deviation of pendulum θ and a parameter of temperature action $\mu = \eta T_{cc}$ is small.

Then, from (13) we shall have the following equation:

$$\ddot{\theta} + 2n_0(1 - \eta T_{cc} \sin \omega_c t)\dot{\theta} + \omega^2\theta =$$
$$= F \sin(\nu t + \delta) - \gamma \frac{2Q}{\pi c} \exp(-\lambda t) \int_0^t \theta \exp(\lambda t) dt . \qquad (14)$$

Let's solve the equation (14) by method of small parameter for the important case when frequency of harmonic mechanical force coincides with frequency of harmonic environment temperature $\nu = \omega_c$. This case can be named "a thermal resonance".

An example of such case [1] is oscillations of the artificial satellite of the Earth with gyroscopic dampers.

Forced oscillations of disturbed by temperature a pendulum when $\nu = \omega_c$ will become:

$$\theta = E_1 \cos\nu t + E_2 \sin\nu t + \mu(S_1 \sin 2\nu t + S_2 \cos 2\nu t) + \mu S_0 , \qquad (15)$$

where $E_1 = \dfrac{A(a_0 - a_2\nu^2) - D(a_1\nu - \nu^3)}{(a_0 - a_2\nu^2)^2 + (a_1\nu - \nu^3)^2}$,

$$E_2 = \frac{D(a_0 - a_2\nu^2) + A(a_1\nu - \nu^3)}{(a_0 - a_2\nu^2)^2 + (a_1\nu - \nu^3)^2},$$

$$S_1 = \frac{R_1(a_0 - 4a_2\nu^2) - R_2(8\nu^3 - 2a_1\nu)}{(a_0 - 4a_2\nu^2)^2 + (2a_1\nu - 8\nu^3)^2},$$

$$S_2 = \frac{R_2(a_0 - 4a_2\nu^2) + R_1(8\nu^3 - 2a_1\nu)}{(a_0 - 4a_2\nu^2)^2 + (2a_1\nu - 8\nu^3)^2},$$

$$S_0 = -\frac{E_1 n_0 \lambda \nu}{\omega^2\lambda + \gamma \dfrac{2Q}{\pi c}},$$

$$R_1 = n_0\nu(E_2\lambda - 2E_1\nu), \quad R_2 = n_0\nu(E_1\lambda - 2E_2\nu),$$
$$R_0 = -n_0 E_1 \nu\lambda ,$$

$$a_0 = \omega^2\lambda + \gamma\frac{2Q}{\pi c}, \quad a_1 = 2n_0\lambda + \omega^2, \quad a_2 = 2n_0 + \lambda ,$$
$$A = F(\nu\cos\delta + \lambda\sin\delta), \qquad D = F(\lambda\cos\delta - \nu\sin\delta)$$

It is important to note, that, when $\nu = \omega_c$ in a solution (15), there is a constant component μS_0.

In view of labels in (15) we shall have:

$$S_0 = \frac{F n_0 \lambda \nu^2 \left(2n_0(\lambda^2 + \nu^2) - \gamma \dfrac{2Q}{\pi c} \right)}{\left(\omega^2\lambda + \gamma \dfrac{2Q}{\pi c} \right) \left[\nu^2\left(\nu^2 - \omega^2 - 2n_0\lambda\right)^2 + \left(\lambda(\omega^2 - \nu^2) - 2n_0\nu^2 + \gamma \dfrac{2Q}{\pi c} \right)^2 \right]}.$$

This constant component can be minimized, if to include controlling temperature action $\gamma\Delta T$ to mechanical subsystem, so that $n_0(\lambda^2 + \nu^2) - \gamma Q/(\pi c) \to 0$.

Numerical calculations on nonlinear models of considered dynamic systems have confirmed the received analytical results.

At calculations input data have been used:
$\omega = 0{,}005\ s^{-1}$; $F = 5 \cdot 10^{-5}\ s^{-2}$; $\nu = \omega_c = 0{,}012\ s^{-1}$; $n_0 = 0{,}01 s^{-1}$; $\lambda = 0{,}42\ s^{-1}$; $T_{cc} = 10\ ^0C$; $T_{c0} = 20\ ^0C$; Q=10 Wt and others.

Dependences $\theta(t)$, $\Delta T(t)$ and also phase portraits $(\theta, \dot{\theta})$ of a pendulum are shown on fig.4*a* when the controlling temperature action is absent $(\gamma = 0)$.

On fig.4*b* is shown, how these dependences appear at embodying $(\gamma \neq 0)$ controlling temperature action.

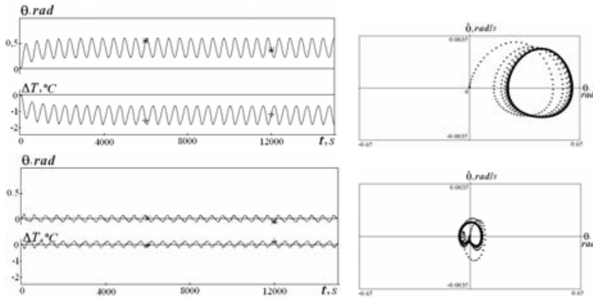
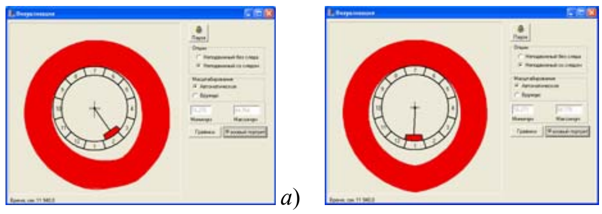


Fig.4

On fig.5 are given a fields of temperatures as functions of time and spatial coordinates of the fixed elements of a mathematical pendulum, at presence (fig.5*a*) and absence (fig.5*b*) of the controlling temperature action.


a) *b*)
Fig.5

Completely similar results are gained and at parametric perturbations of dynamic system of oscillator (fig.1*a*).

Inclusion into a feedback of a mechanical subsystem of the signal $\gamma\Delta T$ proportional to temperature difference between the fixed elements allows to control the interconnected mechanical and thermal processes.

Thus, the opportunity of control by the interconnected mechanical and thermal processes in nonlinear disturbed dynamic systems of a view (1),(2) is shown and is proved both at their irregular motions, and at parametric temperature perturbations.

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