Abstract: In the report it is developed a straight method of motion parameters calculation of an observed object based on analysis of image sequence of the object.

Key words: optical flow, velocity of moving object, object image, gradient motion estimation, functionalization method

1. INTRODUCTION

At the present time there exists a class of high performance registration image methods based on optical flow computation. Optical flow is a vector field of apparent velocity of moving object images. A brightness constancy constraint equation (BCCE) gives one of the ways to calculate optical flow [Black, M. J., et al., Lucas B.D., et al., Horn B.K.P., et al.]. The BCCE equation determines a connection between time-spatial variations of image intensity and image motion parameters, BCCE appears as

$$\frac{\partial E(x,y)}{\partial t} + v_x \frac{\partial E(x,y)}{\partial x} + v_y \frac{\partial E(x,y)}{\partial y} = 0, \quad (1)$$

where $v_x, v_y$ – components of an apparent velocity vector,

$E(x,y)$ – image intensity function.

Methods based upon the equation (1) are referred to as gradient-based motion estimation methods (GM methods).

Comprehensive surveys of the GM methods are performed in [J. Barron et al.] and [M. Irrani. et al.]. The GM methods are efficient in solving a parametric image registration problem: global translation, rotation, affine and projective motions and some others.

The GM method uses spatio-temporal derivatives of image intensity functions, but these functions are non-differentiable and noncontinuous in a general case. That makes the GM method non rigorous and as a consequence of that there arise errors in results of image motion parameters calculations. Considerable efforts were undertaken to eliminate this shortcoming of the GM method [Black, M. J. et al., Horn B.K.P. et al., Lucas B.D. et al., Schalkov R.J., J. Barron et al.]. Nevertheless implementations of all the known versions of the method require preliminary image smoothing through filtration with using of convolution. In a general case convolution is a non-linear operation in respect of velocity vector components. Nonlinearity takes place, for example, when image rotates. So, smoothing yields additional errors to results of calculations. In addition, the GM method gives estimations not of real motion parameters of an observed object but of apparent image velocity vectors.


The functionalization method is free from the main shortcomings of the GM method as it does not uses derivatives of an image intensity function. It uses a fundamental functional connection equation (FC equation), that determines a functional dependence of measurable characteristics of an observed object image on the object motion parameters. A measurable image characteristic is one that can be calculated as a particular definite integral on any image regions of non-nil square. Spatio-temporal derivatives of image intensity are not measurable image characteristics in this sense as well as an image intensity function itself. The functionalization method gives a generalization of the class of GM methods.

2. FUNCTIONALIZATION METHOD

The functionalization method differs from GM methods in that it operates not upon an image intensity functions but upon a special functionals, determined on a set of image intensity functions. That brings considerably new features of universality and accuracy to a technique of image motion parameters estimation. I this section we develop the method for a case, when an observed object is a flat underlying surface and a platform, carrying an imaging device or a video
camera, executes, a motion with six degrees of freedom relative to an underlying surface.

2.1 The model of moving underlying surface imaging

We assume that the carrying platform (CP) executes a translation and free rotation motion relative to it’s own center of mass, and that an underlying surface is flat, rigid, possess plane albedo and is uniformly illuminated with an off-site light source, distance between CP and underlying surface plane albedo and is uniformly illuminated with an off-site of mass, and that an underlying surface is flat, rigid, possess translation and free rotation motion relative to it’s own center.

We assume that the carrying platform (CP) executes a motion with six degrees of freedom (six DOF) relative to an underlying surface.

Position of coordinate system \( oxyz \) relative to coordinate system \( o'x'y'z' \) is identified with a vector \( \mathbf{r}_s = [x_s, y_s, z_s]^T \) ([] \( T \) - symbol of transposition).

Position of coordinate system \( o'x'y'z' \) relative to coordinate system \( OXYZ \) is identified with a vector \( \mathbf{R} = [X(t), Y(t), Z(t)]^T \) and with a transformation matrix (matrix of direction cosiness) \( \mathbf{A} = \mathbf{A}(t) = [a_{ij}(t)] \) \( (i, j = 1, 2, 3) \). It is well known that matrix \( \mathbf{A} \) is orthogonal and \( \mathbf{A}^T = \mathbf{A}^{-1} \).

Each point belonging to \( P \) maps it’s image at the plane \( P_k \) (for example, on Fig. 1 a point \( M \) maps it’s own image at a point \( m \)).

Position of a point \( M \in P \) in a coordinate system \( OXYZ \) is identified on Fig.1 with a vector \( \mathbf{R} = [X, Y, 0]^T \), and in a mobile coordinate system \( oxyz \) - with a vector

\[
\mathbf{r}_p = \mathbf{r}(t) = [x(t), y(t), -f]
\]

From Fig.1 follows vector equality:

\[
\mathbf{r} = \mathbf{r}_p = k(\mathbf{A}(\mathbf{R} - \mathbf{R}_0) - \mathbf{r}_3), \quad (2)
\]

where \( k = k_m = \frac{(\mathbf{r} / |\mathbf{r}|)^T}{f} \), \( \mathbf{r} = [x, y, z]^T \) - modulus of vectors \( \mathbf{r} \) and \( \mathbf{r}_3 \).

\( \mathbf{A} = [a_{ij}a_{jk}a_{lk}] \) - row-matrix that is the third row of matrix \( \mathbf{A} \); \( \mathbf{A}^T = [a_{lj}, a_{kj}, a_{ij}] \) - row-matrix that is the third row of matrix \( \mathbf{A}^T \).

2.2 Image motion equation

To obtain image motion equation of underlying surface we derive (2) in respect of time \( t \) and then get:

\[
\dot{\mathbf{r}} = -\frac{1}{f} \mathbf{r} \mathbf{A}^T + \mathbf{r} \mathbf{A} \mathbf{R}^T + k_m \left( -\frac{1}{f} \mathbf{r} \mathbf{A} + \mathbf{A} \right) \mathbf{V}_H, \quad (3)
\]

where \( \dot{\mathbf{r}} = [\mathbf{v}_1, \mathbf{v}_2, 0]^T \) - image velocity vector ;

\[
\mathbf{V}_H = \left[ \begin{array}{ccc}
0 & -\omega_3 & \omega_2 \\
\omega_3 & 0 & -\omega_1 \\
-\omega_2 & \omega_1 & 0
\end{array} \right]
\]

where elements \( \omega_1, \omega_2, \omega_3 \) of matrix \( \mathbf{V}_H \) are projections of vector \( \omega = \omega(t) = [\omega_1(t), \omega_2(t), \omega_3(t)] \) of angular velocity of CP on axes of coordinate system \( o'x'y'z' \); \( \omega = [\omega_2, \omega_1, 0] \) - row-matrix, that is the third row of the matrix \( \mathbf{V}_H \).
\( \mathbf{V}_H = \mathbf{R}_H = [V_{xH}, V_{yH}, V_{zH}]^T \) – velocity vector of transient motion of CP relative to underlying surface.

This equation (3) allows getting estimations of an apparent velocity vector of moving image. We will use this expression below to solve a problem of motion vector estimation of underlying surface.

2.3 Functional connection equation (FC equation)

Let it be allocated singly connected regular region of analysis (analyzing window) \( D \) with boundary \( B(D) \) at a plane \( P_h \). Then we will define a functional \( \Phi (\mathbf{r}, t) \) on set of image intensity functions \( \{E(\mathbf{r}, t)\} \) in the window \( D \):

\[
\Phi = \iint_D K(\mathbf{r})E(\mathbf{r}(t), t) \, ds;
\]

(4)

\( K(\mathbf{r}) \) – weight function that is continuous, uniformly bounded and differentiable one time almost everywhere in respect of all the arguments; we assume that \( K(\mathbf{r}) = 0 \), if \( \mathbf{r} \notin B(D); E(\mathbf{r}(t), t) \) – image intensity function, uniformly bounded.

Let us calculate a total derivative of functional (4) with respect to time \( t \) under the assumption that image motion equation (3) is fulfilled. Then we get:

\[
\Phi'(t) = \Phi'_1(t) + \iint_D K(\mathbf{r})E(\mathbf{r}(t), t) \frac{\partial \mathbf{r}}{\partial t} \, ds,
\]

(5)

where

\[
\Phi_1'(t) = \iint_D [K(\mathbf{r})E(\mathbf{r}(t), t)\partial \mathbf{r}/\partial t + \Omega(\mathbf{r} + k \mathbf{x}, \mathbf{r})] \, ds;
\]

With the Green formula, connecting double and curvilinear integrals, transformation of (5) yields:

\[
\Phi'(t) = \Phi'_1(t) + \Phi_1(t) + \Phi_2(t),
\]

(6)

where

\[
\Phi_1(t) = \iint_D [K(\mathbf{r})E(\mathbf{r}(t), t)\partial \mathbf{r}/\partial t + \Omega(\mathbf{r} + k \mathbf{x}, \mathbf{r})] \, ds;
\]

\[
\Phi_2(t) = -\iint_D [K(\mathbf{r})E(\mathbf{r}(t), t)\partial \mathbf{r}/\partial t + \Omega(\mathbf{r} + k \mathbf{x}, \mathbf{r})] \, ds;
\]

where \( K(\mathbf{r}) = [K(\mathbf{r})^T, K(\mathbf{r})^T, 0] \) – vector of partial derivatives of weight function \( K \) with respect to spatial variables \( x \) and \( y \) correspondingly.

In the sequel we will admit \( \Phi'_1(t) = 0 \), i.e. the off-site light source has constant intensity in time.

The equation (6) is the desired FC equation. It connects object motion parameters \( V_H \) and \( \Omega \) and measurable characteristics of the object image; the characteristics are double integrals defined on the analysing window \( D \).

An example of a suitable weight function \( K \) gives a pyramid like function presented at Fig.2, where coordinates are count off the diagonals cross point.

\[
\forall (x,y): x \in [-a-a], y \in [-kx, kx], \rightarrow f(x,y) = u(1-|x|/a);
\]

\[
\forall (x,y): y \in [-b,b], x \in [-y/k, y/k], \rightarrow f(x,y) = u(1-|y|/b),
\]

where \( u = a/b \).

**Fig. 2. Pyramid like weight function (a) and location of subdomains \( D_1 \ldots D_4 \) (b) on the analysing window**

Analysis of the FC equation (6) shows, that it can be transformed to an algebraic equation, that is linear with respect to components of the full motion vector:

\[
b_1 V_{xH} + b_2 V_{yH} + b_3 V_{zH} + b_4 \omega_1 + b_5 \omega_2 + b_6 \omega_3 = h,
\]

(7)

where \( V_{xH}, V_{yH}, V_{zH} \) – the full velocity vector components of the carrying platform;

\[
h = d\Phi/dt \quad – \text{total derivative of functional } \Phi \text{ with respect to time } t.
\]

\( b_1, b_2 \) – coefficients, given by magnitude of integrals in (6).

The set of equations needed to calculate components of the full velocity vector \( \Lambda = [V_{xH}, V_{yH}, V_{zH}, \omega_1, \omega_2, \omega_3]^T \) composes by parameterization of functional \( \Phi \). There is a vast option for the parameter. It can be time \( t \), position, number and shape of the analysing windows as well as a form of weight function \( K(\mathbf{r}) \) etc. Then we get a set of parameterized equations:

\[
\mathbf{B} \Lambda = \mathbf{H},
\]

(8)
where $\mathbf{B}=[b_{ij}]$ – matrix of coefficients $b_i$; $\mathbf{H}=[h_i]^T$ – column-vector; $i=1...N$ ($N \gg M$), $j=M$ ($M=1,...,6$) – number of freedom degrees of CP motion.

We over define the system (7) ($N \gg M$) to decrease errors in velocity estimations and applying generalized inverse method we arrive at a solution:

$$\mathbf{A}=(\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{H}. \quad (9)$$

In a simple case, when CP makes only transient motion relative to the underlying surface, and with the main axis of OS directed to nadir then FC equation (7) takes form:

$$\dot{\mathbf{r}} = \mathbf{a}(\mathbf{r}) + \mathbf{b}(\mathbf{r}) \mathbf{v}(\mathbf{r}) + \mathbf{c}(\mathbf{r}) \mathbf{v}(\mathbf{r})^2 + \mathbf{d}(\mathbf{r}) \mathbf{v}(\mathbf{r})^3$$

In this section we investigate an additive noise impact on the OS, but only its additive part $\delta \mathbf{r} = \delta x \mathbf{a}_x + \delta y \mathbf{a}_y + \delta z \mathbf{a}_z$.

$$\mathbf{A}(t_0) = \mathbf{A}_0.$$

$$\mathbf{A}(t) = -\mathbf{A}(t), \quad \mathbf{A}(t_0) = \mathbf{A}_0. \quad (11)$$

3. IMPACT OF ADDITIVE NOISE

In this section we investigate an additive noise impact on the displacement of motion vector estimates derived with the use of functionalization method. We examine a case of plane-parallel motion of CP relative to underlying surface. In the case the main functional (4) will have a form:

$$\tilde{\mathbf{r}}(t) = \int_D K(z, y, -f) \tilde{E}(z, y, -f, t) dz,$$  \quad (12)$$

where

$$\tilde{E}(z, y, -f, t) = E(z, y, -f, t) = E(z, y, -f, t) + n(z, y, -f, t)$$

- video signal, available for instrumental measurement;
- $n(z, y, -f, t)$ – noise (stochastic function). We assume that noise and image intensity function are not mutually correlated and indispensable conditions for differentiability of realizations of noise $n(z, y, -f, t)$ in respect of time are fulfilled. Derive functional (12) in respect of time $t$, then we get

$$\tilde{\mathbf{r}} = -\dot{\mathbf{r}} \tilde{\mathbf{a}} - \dot{\mathbf{r}} \tilde{\mathbf{a}}^2 + \mathbf{H}, \quad (13)$$

where $\dot{\mathbf{r}} = \mathbf{v} = \mathbf{v}_x \mathbf{a}_x + \mathbf{v}_y \mathbf{a}_y$.

As the stochastic function $\mathbf{H}$ cannot be practically measured it is impossible to use equation (13) for calculation of image velocity vector $\mathbf{v} = [\mathbf{v}_x, \mathbf{v}_y, 0]^T$. Then we will use for this aim an approximate form (14) deduced from (13) by eliminating the term $\mathbf{H}$ from right-hand side of equation (13):

$$\tilde{\mathbf{r}} = -\tilde{\mathbf{r}} \tilde{\mathbf{a}} - \tilde{\mathbf{r}} \tilde{\mathbf{a}}^2, \quad (14)$$

where $\tilde{\mathbf{r}} = \tilde{\mathbf{r}} = \tilde{\mathbf{r}}_x \mathbf{a}_x + \tilde{\mathbf{r}}_y \mathbf{a}_y$ – estimations of components of image motion velocity vector; $\Delta \mathbf{r}_x$ and $\Delta \mathbf{r}_y$ – estimation errors. All of the coefficients in right-hand side of (14) are measurable in defined above sense.

To reduce errors $\Delta \mathbf{r}_x$ and $\Delta \mathbf{r}_y$ we will over define equation (14) taking $D_i (i \in \{1...l\})$, $l \gg 2$ and thus compose the next set of equations:

$$\tilde{\mathbf{r}}_x = \mathbf{v}_x + \Delta \mathbf{v}_x \quad \text{and} \quad \tilde{\mathbf{r}}_y = \mathbf{v}_y + \Delta \mathbf{v}_y$$

- estimations of components of image motion velocity vector; $\Delta \mathbf{v}_x$ and $\Delta \mathbf{v}_y$ – estimation errors. All of the coefficients in right-hand side of (14) are measurable in defined above sense.

To reduce errors $\Delta \mathbf{r}_x$ and $\Delta \mathbf{r}_y$ we will over define equation (14) taking $D_i (i \in \{1...l\})$, $l \gg 2$ and thus compose the next set of equations:
The iterative scheme we put forward is the next. On every \(i\)-th step of the procedure we use information obtained from windows \(OA_\alpha\) and \(OA_\beta\) to calculate estimations of mutual displacement \(\{\tilde{s}_{\alpha i} = \tilde{u}_{\alpha i} \Delta t\text{ and } \tilde{s}_{\beta j} = \tilde{v}_{\beta j} \Delta t\}\) of the two windows with the use of functionalization method. Forming FC equation in that case we use the first difference of main functional (12) volumes in the windows \(OA_\alpha\) and \(OA_\beta\) instead of the first temporal derivative of the functional.

In order to form over defined set of FC equations we will use additional analyzing windows covering up the main windows \(OA_\alpha\) and \(OA_\beta\), with sets of identical windows of fewer dimensions. An example of such covering is shown at Fig.3.

At the example each of the coverings consists of five windows and consequently we get a set of five FC equations. A solution of over defined set of FC equations is arrived by applying generalized inverse method.

At the next step of the iterative scheme the analyzing window \(OA_\beta\) is warped towards \(OA_\alpha\) with the use of current estimates \((\tilde{s}_{\alpha i}, \tilde{s}_{\beta j})\) of the windows displacement and we get a new position of the window \(OA_\beta\), denoted at Fig.3 as \(OA_{\beta i+1}\). The procedure iterates till displacements \(\tilde{s}_{\alpha i}\) and \(\tilde{s}_{\beta j}\) become less than a preliminarily specified radius \(\rho\) of accessibility tube and will not leave the tube in the sequel. This procedure can be written as

\[
s(k+1) = s(k) + \Delta s(k),
\]

where \(s(s_\alpha, s_\beta), \Delta s(\Delta s_\alpha, \Delta s_\beta)\); \(s(0)=s_0\); \(k=1, 2, \ldots\) - number of an iteration. At every iteration we get increments of displacement estimations \(\Delta s(k) = (\Delta s_\alpha(k), \Delta s_\beta(k))\) by solving set of FC equations, coefficients of which are formed with the use of information obtained from windows \(OA_\alpha\) and \(OA_\beta\).

\[
\Delta s(k) = -(\Phi_1'(k))^{-1}((\Phi_1'(k))^T \Delta \Phi(k)),
\]

where \(\Delta \Phi(k) = [\hat{\Phi}_2(k) - \hat{\Phi}_1(k)]; i=1, 2, \ldots;\n\]

\(\Phi_1, \Phi_2\) – volumes of the main functionals defined in the windows \(OA_\alpha\) and \(OA_\beta\); \(\Phi_1'(k) = \frac{1}{2}[\hat{\Phi}_1'(k) + \hat{\Phi}_2'(k)].\)
with this method. A modeling system encloses three modules: a module of video signal modeling, a module of elaborating estimations of image displacement of two image frames and a module for overlapping two frames of image. A computational model of a video signal corresponded a mixture of pure video signal and additive random noise. Random noise had a uniform frequency distribution function. Amplitude of noise was specified in lower order bits (l.o.b). The main functional, used at modeling, had a pyramid-like weight function. Results of computer modeling are represented at Fig.4. Experiments were undertaken to search estimations for radius $(\rho)$ of iterations were undertaken to get reliable from 2 to 50 pixels. Sufficient number (no more the 4 – 6, as a fact) of iterations were undertaken to get reliable estimations for radius $(\rho)$ of accessibility tube.

5 CONCLUSIONS

Computational experiments show that functionalization method introduced in the report allows to eliminate the main shortcomings of gradient motion estimation method. The functionalization method does not involve calculation of spatial derivatives of image dense function. More over the iterative technique of the method eludes calculation of temporal derivatives of image dense. That brings essentially new features of universality and accuracy to the image motion estimation methods.

Fig. 4. Experimental data. Relationship of mathematical expectation $M$ of a radius $(\rho)$ of an accessibility tube and a parameter $A_n$ (an amplitude) of noise. $D_{0.95}$ - (gray bacground) confidence interval

REFERENCES


