Stabilization of Reaction Wheel Pendulum on Movable Support with On-line Identification of Unknown Parameters

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Abstract — **In the paper an adaptive stabilization algorithm of a reaction wheel pendulum on a movable platform is proposed. Obtained hybrid control system allows to stabilize a pendulum fixed on the movable platform at the top unstable equilibrium. Task of swinging up and stabilizing of a pendulum is solved in conditions of full parametric uncertainty of a plant and constrained control. In contrast to other approaches, this algorithm has adaptive adjusting of controller's parameters in "on-line" mode without any preliminary identification procedure. The second goal of this work is development of applications used in educational process because investigation of control theory problems with realistic plants very important for students and their experience.**

I. Introduction

The pendulum is one of the most interest objects to research complex dynamics of mechanical system since Galileo's time. Now problems of pendulum system control don't lose actuality and cause increasing interest from both Russian and foreign researchers. Interest is caused because pendulum systems represent analogs of lot technological and physical processes. For example, one of practical tasks where proposed algorithms can be applied is rocket stabilization at the start, when the rocket can be approximately represent as inverted pendulum.

At present time mechatronics researching complexes are widely adopted for science investigation in mechatronics and automatic control systems area allowing demonstrating capability of modern algorithms application for many various tasks. Using such complex, researcher can check efficiency of obtained algorithms not only by mathematical modeling, but also at realistic physical plant. Complex of Mechatronics system Inc., disposable of Cybernetics and Control System Laboratory of Saint Petersburg State University of Information Technologies Mechanics and Optics has been actively used to research by authors (see Fig. 1).

In this paper control task of the reaction wheel pendulum is solved. Mechanical part of mechatronics researching complex represent a single-link pendulum fixed at the pivot pin with the reaction wheel. Such mechanical system is described in a number of publications [1] – [9]. Pendulum must be brought in vertical unstable equilibrium and stabilized there in conditions of full parametric uncertainty of plant and constrained control.

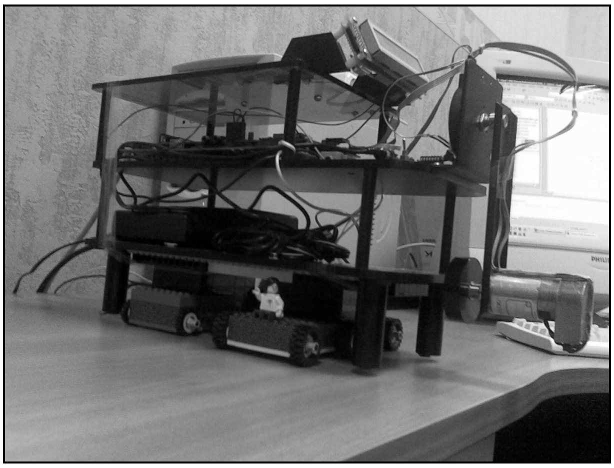

Fig. 1. Mechatronics Control Kit

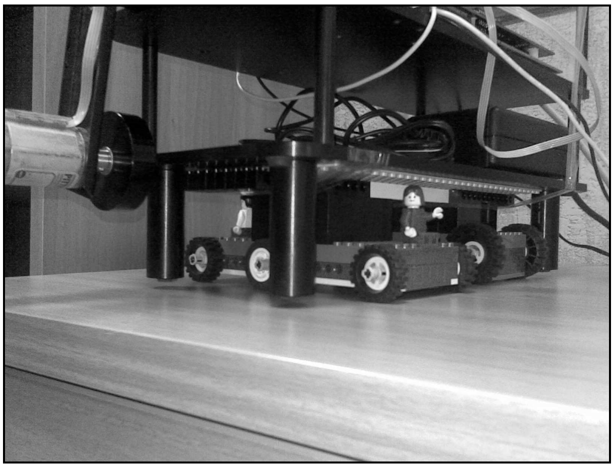

Fig. 2. Movable platform built from LEGO

The platform on which the pendulum established is movable (see Fig. 2) for advancing of approach proposed in [1]. Such platform can be easy built from constructor LEGO by hand because of simple structure. Such equipment very useful for researching of various mechanical control tasks and it is relatively cheap. This application should be used in educational process because investigation of control theory problems using realistic plants very important during studying. Students would gain

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a wide experience in control in such tasks as, for example, stabilization and disturbance compensation.

In [2] similar mechanical system is used to test stabilization algorithms of unstable equilibrium position for small initial deviation. In [3] problem of swinging-up and stabilizing in inverted equilibrium of reaction wheel pendulum with use of speed-gradient method and energy approach is solved. In [4] for swinging up a pendulum energy approach is used also, but unlike [3] at this work pacification with choice as function of stock of full energy of the system is used In [5] two approaches for swing-up and stabilization of pendulum are considered: feedback linearization and pacification.

In a spite of evidently availability of adaptive approaches application, attention is a not enough focused at the similar control task. So in [4] and [5] parameters of pendulum are well known. In [3] this task is solved in condition of parametric uncertainty, however, proposed approach need in preliminary identification procedure.

The purpose of this work is advancement of control approaches offered in [1] – [9], but in a contrast of known analogs adaptive scheme of pendulum control is obtained allowing to solve mentioned problem in condition of full parametric uncertainty without preliminary identification procedure.

II. Problem Statement

The mechanical part of the plant represents a single-link pendulum fixed at the pivot pin with the reaction wheel situated at the opposite end of pendulum. The platform of the pendulum is movable (see Fig. 3).

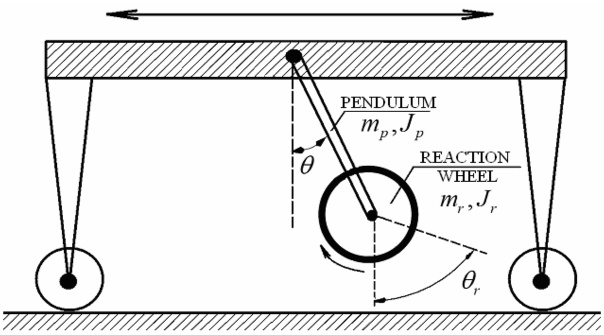

Fig. 3. Schematic diagram of pendulum system

Moving of the pendulum is provided by changing a direction and rate of turn of the reaction wheel. Wheel rotation is controlled by regulating an input voltage in DC-motor fixed together with the wheel. It is necessary to note, that the platform of system possesses high inertia in compassion with a pendulum itself.

Mathematical model describes physical character of the mechanical part of plant without movement of platform and it looks like the following system of differential equations:

$$\begin{cases} \ddot{\theta}(t) + a\sin\theta = -b_p(\bar{u} - f), \\ \ddot{\theta}_r = b_r(\bar{u} - f), \end{cases} \tag{1}$$

where θ – an angle of the pendulum, θ_r – a wheel angle, $a = \frac{mgl}{J}$, $b_p = \frac{k_u}{J}$, $b_r = \frac{k_u}{J_r}$ – unknown complex parameters of the pendulum, m – combined mass of a rotor and the pendulum, l – a distance from pivot to the center of mass of the pendulum system, J – combined moment of inertia of the pendulum system, J_r – moment of inertia of the wheel and the rotor of the electric motor about its center of mass, k_u – transfer constant of the DC-motor, g – acceleration of gravity, f – a torque of friction forces in pendulum, $|\bar{u}| \leqslant 10$ – the control signal. Variables θ and θ_r are measured.

The main goal of task is the inverted pendulum stabilization at the unstable equilibrium point in condition of parametric uncertainty and small control.

III. Main Result

In conditions of full parametric uncertainty and constrained control to stabilize a pendulum in upper equilibrium point once from any initial state is extremely difficult, and general task is divided to particular tasks: swinging-up the pendulum and stabilizing.

Friction is also must be considered. For this purpose torque f is changed on a dead zone unit serial connected in input to plant. Size of this zone can be straightforward measured. The approach proposed in paper [10] is used and following control law is obtained:

$$\bar{u}(t) = u(t) + Dsign(u(t)),$$

where $u(t)$ – value of calculated control for the plant without dead. So, the negative influence of the friction will be rejected.

A. Swing-up of pendulum

In this paper unlike [1] problem of swinging-up of a pendulum is solved in condition when the platform of a system is movable. Such attempt is caused to expand a category of plants, where offered control algorithms can be successfully applied.

To synthesize a swing-up algorithm, the speed gradient method is used [11], [12] with the objective energetic function.

In paper [3] it is shown that partial energy of the pendulum characterized pendulum moving without considering a wheel rotation is appropriate instead of full Hamiltonian of system. Also it is required to consider mobility of the platform. In this case some energy provided by a drive for swinging-up a pendulum is used for moving the platform. Therefore a desirable level of energy should be adjusted.

Remark 1. It is necessary to notice that the task of swinging-up a pendulum can be solved, only if platform of system possesses high inertia. Otherwise in conditions of constrained control pendulum cannot be brought in desirable position in general.

Equation for objective function looks like:

$$G(\theta,\dot{\theta}) = \frac{1}{2}\Delta H^2(t),$$

where function $\Delta H(t) = H(\theta,\dot{\theta}) - H^*(t)$, $H = mgl(1-\cos\theta) + \frac{1}{2}J\dot{\theta}^2$ – current partial energy of system, $H^*(t)$ – desired energy level.

Equation determining change of a desirable level of energy has a view:

$$H^*(t) = H_0^*(t)(1+\Delta H^*(t)), \qquad (2)$$

where $H_0^* = 2mgl$ – initial entry of desirable energy level, when platform is stationery, $\Delta H^*(t)$ – positive function, increasing desired energy level for compensating energy selected on platform moving. Function $\Delta H^*(t)$ can be adjusted using following adaptive scheme:

$$\Delta H^*(t) = \int_0^t \mu \, d\tau, \qquad (3)$$

where μ – positive number chosen by developer.

Correlation (2) shows that it is proposed to increase desired energy level smoothly during swinging-up of pendulum until this level will not be enough for bringing of the pendulum to top equilibrium vicinity.

Control law obtained from end formula of speed gradient approach has following form:

$$u = \gamma \Delta H(t)\sigma,$$

where γ – positive coefficient, $\sigma = \frac{\partial \dot{H}(\theta,\dot{\theta})}{\partial u}$.

In this case control law looks like:

$$u(t) = \gamma\left(\frac{1}{2}J\dot{\theta}^2 - mgl(1+\cos\theta+2\Delta H^*)\right) b_p J\dot{\theta}. \qquad (4)$$

Transform expression (4):

$$u(t) = \gamma\left(\frac{1}{2}\dot{\theta}^2 - \frac{mgl}{J}(1+\cos\theta+2\Delta H^*)\right) b_p J^2\dot{\theta} =$$
$$= K b_p\left(\frac{1}{2}\dot{\theta}^2 - a(1+\cos\theta+2\Delta H^*)\right)\dot{\theta}, \qquad (5)$$

where $K = \gamma J^2$ – an arbitrary positive constant.

Since parameters a and b_p are unknown and variable $\dot{\theta}$ is unmeasured, control law (5) is not realizable.

Realizable control law is designed basing on (5):

$$u(t) = K\hat{b}_p\left(\frac{1}{2}\hat{\dot{\theta}}^2 - \hat{a}(1+\cos\theta+2\Delta H^*)\right)\hat{\dot{\theta}}, \qquad (6)$$

where $\hat{a}$ – estimation of parameter a, $\hat{b}_p$ – estimation of parameter b, $\hat{\dot{\theta}}$ – estimation of rate of turn of pendulum, obtained from estimator. Estimation scheme of rate of turn $\hat{\dot{\theta}}$ is represented in [4].

B. Stabilizing of pendulum at the unstable equilibrium

In the pendulum stabilization task at the top equilibrium the main control purpose is expressed by limitation equation:

$$\lim_{t\to\infty}(\theta(t) - \theta^*) = 0,$$

where $\theta(t)$ – current pendulum angle, $\theta^* = \pi$ – desired pendulum angle corresponding to unstable equilibrium.

For small values θ function $\sin\theta$ can be approximately represented by argument θ. So, close to top equilibrium following substitute can be used:

$$\sin\theta \approx (\pi - \theta) \mod 2\pi.$$

We obtain linearized model of pendulum close to top position in the following form:

$$\begin{cases} \ddot{\theta} = a\bar{\theta} - b_p u, \\ \ddot{\theta}_r = b_r u, \end{cases}$$

where $\bar{\theta} = (\theta - \pi) \mod 2\pi$ – deviation of pendulum angle from desired angle.

If parameters of model (1) are known, it is possible to use an extended stabilization algorithm, proposed in paper [4], based on method of standard polynomials:

$$u(t) = -k_1(t)\bar{\theta}(t) - k_2\hat{\dot{\theta}}(t) - k_3\theta_r(t) - k_4\hat{\dot{\theta}}_r(t),$$

where $k_1 = -\dfrac{a(\omega_1^2 + 4\xi_1\xi_2\omega_1\omega_2 + \omega_2^2) + \omega_1^2\omega_2^2 + a^2}{ab_p}$, $k_2 = -\dfrac{2a(\xi_1\omega_1+\xi_2\omega_2) + 2(\xi_1\omega_2+\xi_2\omega_1)\omega_1\omega_2}{ab_p}$, $k_3 = -\dfrac{\omega_1^2\omega_2^2}{ab_r}$, $k_4 = -\dfrac{2(\xi_1\omega_2+\xi_2\omega_1)\omega_1^2\omega_2^2}{ab_r}$ – controller's parameters, $\xi_1,\ \xi_2,\ \omega_1,\ \omega_2 > 0$ – positive values influenced on regulation quality, $\hat{\dot{\theta}},\ \hat{\dot{\theta}}_r$ – estimations of rates of turn of pendulum and rotor correspondingly, obtained from estimator [4].

In the case of parametric uncertainty realizable control law is constructed in the following form:

$$u(t) = -K_1(t)\bar{\theta}(t) - K_2\hat{\dot{\theta}}(t) - K_3\theta_r(t) - K_4\hat{\dot{\theta}}_r(t), \quad (7)$$

where controller's parameters are calculated using estimations of unknown parameters $\hat{a}, \hat{b}_p, \hat{b}_r$:

$$K_1 = -\frac{\hat{a}(\omega_1^2 + 4\xi_1\xi_2\omega_1\omega_2 + \omega_2^2) + \omega_1^2\omega_2^2 + \hat{a}^2}{\hat{a}\hat{b}_p},$$
$$K_2 = -\frac{2\hat{a}(\xi_1\omega_1 + \xi_2\omega_2) + 2(\xi_1\omega_2 + \xi_2\omega_1)\omega_1\omega_2}{\hat{a}\hat{b}_p},$$
$$K_3 = -\frac{\omega_1^2\omega_2^2}{\hat{a}\hat{b}_r}, K_4 = -\frac{2(\xi_1\omega_2 + \xi_2\omega_1)\omega_1^2\omega_2^2}{\hat{a}\hat{b}_r}.$$

C. *Identification of unknown pendulum parameters*

It is necessary to make a parameterization procedure about the model (1) to obtain a regression model. Enter to consideration function and auxiliary second order filters:

$$\xi_1(t) = \frac{\lambda^2}{p^2+2\lambda p+\lambda^2}\theta(t), \xi_2(t) = \frac{\lambda^2}{p^2+2\lambda p+\lambda^2}\theta_r(t),$$
$$\xi_3(t) = \frac{\lambda^2}{p^2+2\lambda p+\lambda^2}\vartheta(t), \xi_4(t) = \frac{\lambda^2}{p^2+2\lambda p+\lambda^2}u(t),$$

where $\xi_i, i = \overline{1,4}$ – output signals for filters, $p = d/dt$ – differential operator and positive value $\lambda > 0$.

It is straightforward to show that after the parameterization model (1) can be written in the following form:

$$\begin{cases} z_1(t) = \varphi_1(t)\psi_1, \\ z_2(t) = \varphi_2(t)\psi_2, \end{cases}$$

where $z_1(t) = \ddot{\xi}_1(t)$ and $z_2(t) = \ddot{\xi}_2(t)$ – known functions, $\varphi_1^T = [\xi_3(t) \quad \xi_4(t)]$ and $\varphi_2^T = [\xi_4(t)]$ – vectors which can be calculated, $\psi_1 = [-a \quad -b]$ and $\psi_2 = [b_r]$ – vectors of unknown pendulum parameters.

Let the consequence of input signals excite the plant $\{u(1)\ u(2)\ \ldots\ u(N)\}$, providing consequences of measured outputs $\{\theta(1)\ \theta(2)\ \ldots\ \theta(N)\}$ and $\{\theta_r(1)\ \theta_r(2)\ \ldots\ \theta_r(N)\}$. For parameterized model it may be written data arrays Z_1, Z_2, Ω_1 and Ω_2:

$$Z_1^T = \{z_1(1)\ z_1(2)\ \ldots\ z_1(N)\},$$
$$Z_2^T = \{z_2(1)\ z_2(2)\ \ldots\ z_2(N)\},$$
$$\Omega_1^T = \{\varphi_1(1)\ \varphi_1(2)\ \ldots\ \varphi_1(N)\},$$
$$\Omega_2^T = \{\varphi_2(1)\ \varphi_2(2)\ \ldots\ \varphi_2(N)\},$$

where N – quantity of measured values. Values are taken consequently on the each iteration of program.

Then an identification scheme can be represented:

$$\begin{cases} \hat{\psi}_1(t) = (\Omega_1\Omega_1^T)^{-1}\Omega_1 Z_1, \\ \hat{\psi}_2(t) = (\Omega_2\Omega_2^T)^{-1}\Omega_2 Z_2, \end{cases} \quad (8)$$

where $\hat{\psi}_1$ and $\hat{\psi}_2$ – vectors of parameters estimation.

Remark 2. The algorithm of identification with increasing quantity of measurements N provides convergence of all estimation to real values, if matrixes $\Omega_1\Omega_1^T$ and $\Omega_2\Omega_2^T$ are not singular. This condition is valid if input signal $u(t)$ has enough harmonics.

Enter in consideration functions:

$$V_1(N) = \sum_{i=1}^N \xi_3(i)z_1(i), V_2(N) = \sum_{i=1}^N \xi_4(i)z_1(i),$$
$$V_3(N) = \sum_{i=1}^N \xi_4(i)z_2(i), W_1(N) = \sum_{i=1}^N \xi_3^2(i),$$
$$W_2(N) = \sum_{i=1}^N \xi_3(i)\xi_4(i), W_3(N) = \sum_{i=1}^N \xi_4^2(i).$$

Realizable identification scheme has following form:

$$\begin{cases} \hat{a}(i) = -(W_3(i)V_1(i) - W_2(i)V_2(i))/\Delta_1(i), \\ \hat{b}_p(i) = (W_2(i)V_1(i) - W_1(i)V_2(i))/\Delta_1(i), \\ \hat{b}_r(i) = V_3(i)/\Delta_2(i), \\ V_1(i) = V_1(i-1) + \xi_3(i)z_1(i),\ V_1(0) = 0, \\ V_2(i) = V_2(i-1) + \xi_4(i)z_1(i),\ V_2(0) = 0, \\ V_3(i) = V_3(i-1) + \xi_4(i)z_2(i),\ V_3(0) = 0, \\ W_1(i) = W_1(i-1) + \xi_3^2(i),\ W_1(0) = 0, \\ W_2(i) = W_2(i-1) + \xi_3(i)\xi_4(i),\ W_2(0) = 0, \\ W_3(i) = W_3(i-1) + \xi_4^2(i),\ W_3(0) = 0, \\ \Delta_1(i) = W_1(i)W_3(i) - W_2(i)W_2(i), \\ \Delta_2(i) = W_3(i). \end{cases}$$

D. *Switching between control algorithms*

Choice and ground of switching way are well described in [3, 4]. Control law for calculated control signal is presented below:

$$u(t) = \begin{cases} u_1(t), |\bar{\theta} - \pi| \leqslant \theta_s, \\ u_2(t), |\bar{\theta} - \pi| \geqslant \theta_s, \end{cases}$$

where $u_1(t)$ and $u_2(t)$ – the stabilization algorithm (7) and the swing-up algorithm (6), $\theta_s = \frac{\pi}{6}$ – the pendulum angle when switching between two algorithms is happened.

IV. Experimental results

On Fig. 4 – Fig. 10 experimental results for controller's parameters $\lambda = 5$, $K = 1,5$, $\omega_1 = 5$, $\xi_1 = 0,707$, $\omega_2 = 6$, $\xi_2 = 0,707$, $\mu = 0,015$ obtained on the Mechatronic Control Kit are shown. From charts one can see that proposed algorithm works properly.

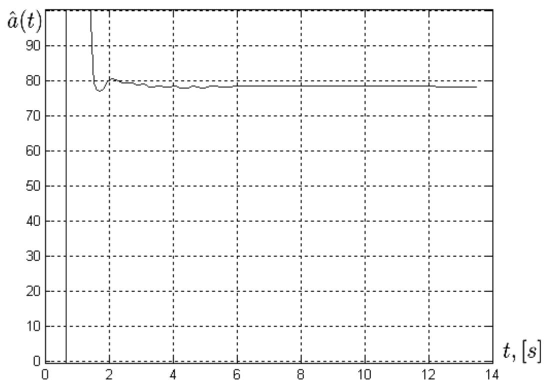

Fig. 4. Transients for $\hat{a}(t)$ in the real system

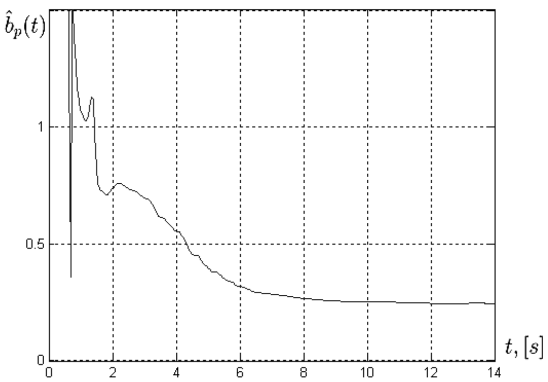

Fig. 5. Transients for $\hat{b}_p(t)$ in the real system

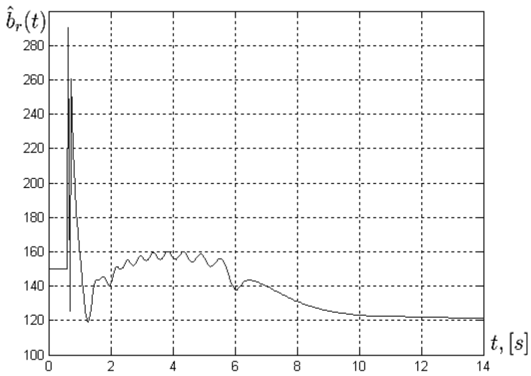

Fig. 6. Transients for $\hat{b}_r(t)$ in the real system

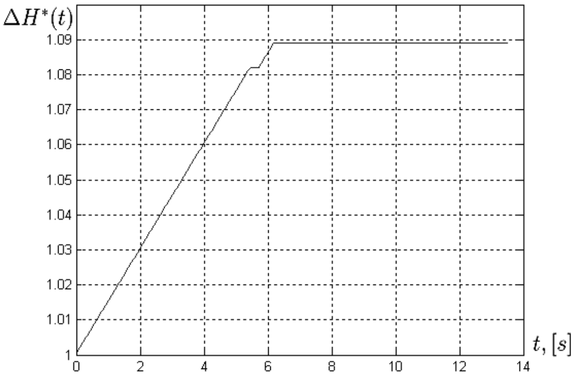

Fig. 7. Transients for $\Delta H^*(t)$ in the real system

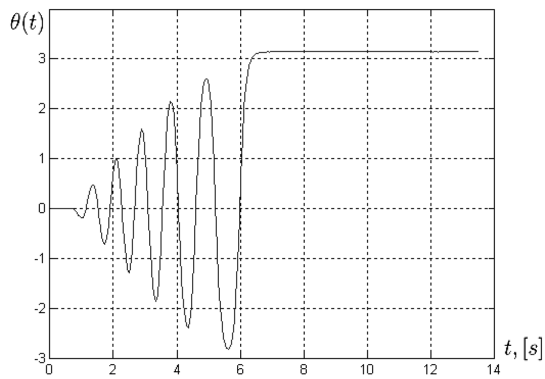

Fig. 8. Transients for $\theta(t)$ in the real system

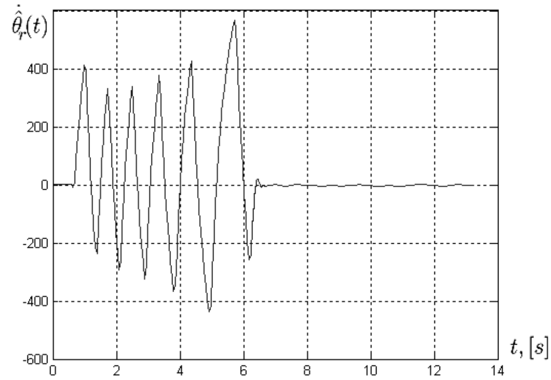

Fig. 9. Transients for $\dot{\hat{\theta}}_r(t)$ in the real system

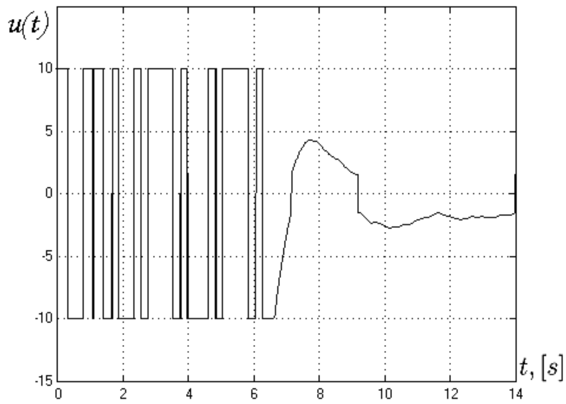

Fig. 10. Transients for $u(t)$ in the real system

V. CONCLUSION

Proposed mechatronic applications very useful in educational process because the investigation of control theory problems with realistic plants very important for students and their experience. In this paper hybrid control system on the basis of adaptive control algorithms providing setting pendulum with reaction wheel on the movable platform about top equilibrium and stabilizing there is proposed. Received algorithms are robust with respect to external disturbances, changing of parameters and initial conditions.

In contrast to other approaches this method allows to solve mentioned problem in condition of full parametric uncertainty without preliminary identification. Estimation of unknown parameters is carried out in “on-line” mode: at the same time with working algorithms of swing-up and stabilization. Also to develop the approaches presented in [1], in this paper modified sing-up algorithm based on a high-speed gradient method with dynamic objective energetic function is offered. Such approach allows to solve a stabilization problem for inverted pendulum on movable platform, therefore it is expand a category of applications were synthesized algorithms can be used.

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