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NUMERICAL EVALUATION OF CONTROLLED SYNCHRONIZATION FOR CHAOTIC CHUA SYSTEMS OVER THE LIMITED-BAND DATA ERASURE CHANNEL

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Abstract
In the paper limit possibilities of controlled synchronization systems under information constraints are evaluated. The case of data erasure channel is studied. By the example of controlled master-slave synchronization of two chaotic Chua systems, dependence of systems synchronization on the channel information capacity and the erasure probability is numerically evaluated.

Key words
Synchronization, Control of chaos, Networked systems, Information constraints, Erasure channel

1 Introduction

At the present time, the distributed control of complex physical systems becomes increasingly important. The relevance of the intensive research in designing multi-agent systems with decentralized control over the communication network is widely demonstrated in scientific literature, see (Ishii and Francis, 2002; Goodwin *et al.*, 2004; Olfati-Saber *et al.*, 2007; De Persis and Nešić, 2005; Matveev and Savkin, 2009; Evans *et al.*, 2005; Zheng *et al.*, 2013; Pasqualetti *et al.*, 2014; Xiaofeng and Hovakimyan, 2013; Antonelli, 2013; DeLellis *et al.*, 2013) for mentioning a few. The *Networked Control System* is a real-time system in which the sensor data and control signals are transmitted over the common communication network. Design of such a kind of systems requires a simultaneous consideration of control, computation and information aspects. The networked control includes the cooperative control of a group of moving agents, such as transport robots, aircrafts, water vehicles, etc. In the control literature there is a strong interest in control of oscillations, particularly in controlled synchronization problems (Nijmeijer, 1997; Nijmeijer, 2001; Lu and Chen, 2005; Chopra and Spong, 2006; Li *et al.*, 2006; Han, 2007; Loría and Zavala-Río, 2007; Lu and Hill, 2007; Li *et al.*, 2010).

The first results on synchronization under information constraints were presented in (Fradkov *et al.*, 2006), where the so called *observer-based synchronization scheme* (Pecora and Carroll, 1990; Fradkov *et al.*, 2000) was considered. The synchronization scheme of (Fradkov *et al.*, 2006) leads to the limit synchronization error inversely proportional to the *transmission rate* (the *channel capacity*). An output feedback controlled synchronization scheme for nonlinear systems, where the control signal is computed based on a signal, transmitted over a communication channel, is studied in (Fradkov *et al.*, 2008; Fradkov *et al.*, 2009; Fradkov *et al.*, 2015). In these works, the output feedback synchronization laws based on the *passification method* of (Fradkov, 1974; Fradkov *et al.*, 1999) are proposed and theoretical analysis for master-slave synchronization of nonlinear systems is provided.

(Fradkov *et al.*, 2008) assumed that the *master system output* is transmitted to the slave system controller over the limited capacity communication channel. It is demonstrated that the approach to *observer-based* synchronization of nonlinear systems, proposed in (Fradkov *et al.*, 2006), is also suitable for *controlled* master-slave synchronization over the limited capacity communication channel. (Fradkov *et al.*, 2008) showed that upper bound on the limit synchronization error is proportional to the maximum rate of the coupling signal and inversely proportional to the information transmission rate.

The case of transmitting the *synchronizaton error* is studied in (Fradkov *et al.*, 2009; Fradkov *et al.*, 2015). It is shown that in the absence of any errors, if the data transmission rate exceeds some threshold, the proposed controlled synchronization strategy ensures asymptotically vanishing on time synchronization error.

In (Nair and Evans, 2004; Nair *et al.*, 2004; Fradkov *et al.*, 2006; Fradkov *et al.*, 2009) it is assumed that the communication channel has limited capacity unlike the ideal. The cases of packet erasure channel and 'blinking' channel are widely appear in different real-world applications and are intensively studied in Information theory, Computer and Physical sciences and Control theoretic literature, see, e.g. (Cover and Thomas, 1991; Rizzo, 1997; Tatikonda and Mitter, 2004*a*; Matveev and Savkin, 2009; Shokrollahi, 2006; Köetter and Kschischang, 2008; Patterson *et al.*, 2010; Diwadkar and Vaidya, 2011; Wang and Yan, 2014). In the present paper, the controlled synchronization schemes of (Fradkov *et al.*, 2008; Fradkov *et al.*, 2009) are examined for the case of a data erasure communication channel of limited capacity. A synchronization accuracy is numerically evaluated by the example of the unidirectionally coupled chaotic Chua systems, and admissible bounds for channel capacity R and erasure probability p are found by simulations. In the framework of the controlled synchronization scheme of (Fradkov *et al.*, 2009) the *adaptive coding procedure* (Goodman and Gersho, 1974; Andrievsky and Fradkov, 2010; Gomez-Estern *et al.*, 2011; Goodwin *et al.*, 2012; Andrievsky and Fradkov, 2014) is employed.

The rest of the paper is organized as follows. The coding procedures are briefly described in Section 2. The controlled synchronization schemes of (Fradkov *et al.*, 2008; Fradkov *et al.*, 2009) are recalled in Sec. 3. The model of a data erasure channel is given in Section 4. The simulation results for chaotic Chua systems synchronization are presented in Sec. 5. Final remarks are given in Conclusion.

2 Coding procedures

Let us briefly recall the binary coding procedures of (Fradkov *et al.*, 2006; Fradkov *et al.*, 2009; Andrievsky and Fradkov, 2010; Andrievsky and Fradkov, 2014).

2.1 Binary time-varying quantizer

Let $\vartheta[k] = \vartheta(t_k)$ be a scalar signal for quantization and transmission over the digital communication channel at discrete instants $t_k = kT$, where $k = 0, 1, \dots$ is a sequence of natural numbers, $T > 0$ is the *sampling interval*. The binary static quantizer is described as

$$q(\vartheta, M) = M[k]\,\mathrm{sign}(\vartheta), \qquad (1)$$

where $\mathrm{sign}(\vartheta) = \begin{cases} 1, & \text{if } \vartheta \geq 0, \\ 0 & \text{otherwise} \end{cases}$ is the *signum* function, parameter M is referred to as a *quantizer range*. Range $M[k]$ is depends on time k in accordance with a certain *zooming procedure*, see Sections 2.3, 2.4.

Quantizer output

$$\bar\vartheta[k] = q(\vartheta[k], M[k]) \qquad (2)$$

is represented by a binary codeword $s = \mathrm{sign}\,\bar\vartheta$ and transmitted over the communication channel to the decoder. Note that the considered coding scheme corresponds to the channel data rate of $R = T^{-1}$ bits per second.

2.2 First-order coder with a memory

Let $y(t)$ be a scalar information signal, which is generated (measured) at the side of the coder and should be reproduced by the decoder based on available (quantized) data, transmitted over a digital communication channel. In the *first-order coder with memory*, the transmitted signal $\vartheta[k]$ in (2) is a deviation between $y(t_k)$, $t_k = kT$, and some real number $c[k]$, which is refereed to as a '*centroid*' (Tatikonda, 2000; Tatikonda and Mitter, 2004*b*; Fradkov *et al.*, 2006) and defined recursively by the following algorithm:

$$c[k+1] = c[k] + \bar\vartheta[k], \quad c[0] = 0. \qquad (3)$$

Equations (1), (3) describe the coder algorithm. A similar algorithm is implemented by the decoder: the decoder output $\bar y[k]$ is defined as

$$\bar y[k] = \bar c[k] + \bar\vartheta[k], \qquad (4)$$

where centroid $\bar c[k]$ is found in the decoder in accordance with (3):

$$\bar c[k+1] = \bar c[k] + \bar\vartheta[k], \quad \bar c[0] = 0. \qquad (5)$$

In the absence of any errors this leads to identity of sequences $\bar c[k]$ and $c[k]$. In between transmission times, $\bar y(t)$ is found as $\bar y(t) = \bar y[k]$, $t \in [t_k, t_{k+1})$.

2.3 Time-based zooming procedure

At the initial stage of the system evolution, the error $|\vartheta - \bar\vartheta|$ may be large because the initial value $\vartheta(0)$ is not known. The *zooming strategy*, cf. (Tatikonda, 2000; Tatikonda and Mitter, 2004*b*; Fradkov *et al.*, 2006; Brockett and Liberzon, 2000; Liberzon, 2003; Malyavej and Savkin, 2005), is efficient at this stage.

The values of $M[k]$ may be precomputed (the *time-based zooming*), described below in this Section, or, alternatively, current quantized measurements may be used at each step to update $M[k]$ (the *event-based zooming*). A particular form of the event-based zooming is the *adaptive zooming*, described in Sec. 2.4.

In the case of time-based zooming (see, e.g. (Nair and Evans, 2003; Brockett and Liberzon, 2000; Fradkov *et* *al.*, 2009)), range $M[k]$ is defined as the following time sequence:

$$M[k] = (M_0 - M_\infty)\rho^k + M_\infty, \;\; k = 0, 1, \ldots, \quad (6)$$

where $0 < \rho \leq 1$ is the *decay parameter*, M_∞ stands for the limit value of M. Initial value M_0 should be large enough to capture all the region of possible initial values of ϑ. For avoiding computations of powers of ρ, $M[k]$ can be calculated in the following recursive form:

$$M[k+1] = \rho M[k] + m, \quad (7)$$

where $m = (1-\rho)M_\infty$, $M[0] = M_0$.

2.4 Adaptive coding

Range M is updated with time by the following event-based *adaptive zooming* strategy:

$$\begin{aligned} &\lambda[k] = (s[k] + s[k-1])/2, \\ &M[k+1] = m + \begin{cases} \rho M[k], & \text{as } |\lambda[k]| \leq 0.5, \\ M[k]/\rho, & \text{otherwise,} \end{cases} \\ &\lambda[0] = 0, \quad M[0] = M_0, \end{aligned} \quad (8)$$

where M_0 stands for initial value of $M[k]$. m denotes the minimal possible value of M. ρ, M_0, m are design parameters of the algorithm.

3 Description of the controlled synchronization schemes

Following (Fradkov *et al.*, 2008; Fradkov *et al.*, 2009), consider two identical dynamical systems modeled in the Lur'e form (Yakubovich *et al.*, 2004). Let the first ('master') system be an autonoumus one, whereas the second ('slave') system be controlled by a scalar action $u(t)$. The problem is to ensure the systems synchronization by applying an appropriate control $u(t)$ to the slave system. The control action should be generated based on the available data, transmitted over the communication channel. The systems dynamics are described by the following equations:

$$\dot{x}(t) = Ax(t) + B\varphi(y_1), \; y_1(t) = Cx(t), \quad (9)$$

$$\dot{z}(t) = Az(t) + B\varphi(y_2) + Bu, \; y_2(t) = Cz(t), \quad (10)$$

where $x(t)$, $z(t) \in \mathbb{R}^n$ are vectors of state variables; $y_1(t)$, $y_2(t)$ are scalar outputs variables; A, B, C are constant matrices of appropriate dimensions; $\varphi(y)$ is a continuous nonlinearity; vectors $\dot{x}$, $\dot{z}$ stand for time-derivatives of $x(t)$, $z(t)$ respectively. Master system is represented by Eq. (9), whereas the controlled slave system is described by Eq. (10).

3.1 Transmitting the master system output

It is assumed that the controller on the receiver side uses signal $\bar{y}_1(t)$, transmitted over the channel, instead of $y_1(t)$. The control law is taken in the form of a static linear feedback

$$u(t) = K\big(y_1(t) - \bar{y}_2(t)\big), \quad (11)$$

where K is the scalar controller gain.

Assuming that the growth rate of $y_1(t)$ is uniformly bounded, (Fradkov *et al.*, 2009) shown that the limit synchronization error can be made arbitrarily small for sufficiently large transmission rate R. This result is analytically obtained in (Fradkov *et al.*, 2009) under the assumption that the transfer function $W(\lambda) = C(\lambda I - A)^{-1}B$ of the linear part of (10) is hyper-minimum phase (HMP).

3.2 Transmitting the synchronization error

In (Fradkov *et al.*, 2009) it is supposed that a *synchronization error* is measured and transmitted over the channel. Particularly both master and slave systems may have the ability to estimate the dynamics of another system so that the output error signal can be directly computed. The other example is presented in (Malyavej *et al.*, 2006) for command guidance system, where the tracking center is used for motion control.

The control function in this case is taken in the form of a static linear feedback, depending on the quantized synchronization error $\varepsilon(t) = y_1(t) - y(t)$:

$$u(t) = K\bar{\varepsilon}(t), \quad (12)$$

where K is a scalar controller gain. It is assumed that $\bar{\varepsilon}(t)$ is generated by a decoder at the slave system side. Output synchronization error $\varepsilon(t)$ is measured at the coder side. Coded symbol $\bar{\varepsilon}[k] = \bar{\varepsilon}(t_k)$ is transmitted over a digital communication channel with a finite capacity. It is supposed that the *zero-order extrapolation* is used to convert the digital sequence $\bar{\varepsilon}[k]$ to the continuous-time input $\bar{\varepsilon}(t)$ of the controller: in between transmission times it is defined that $\bar{\varepsilon}(t) = \bar{\varepsilon}[k]$.

In (Fradkov *et al.*, 2009) it is assumed that the linear part of (10) is HMP. For coding-decoding procedure (2)–(6) (for $M_\infty = 0$) and control (11) it is shown that the there exist coder parameters ρ, T s.t the synchronization error decays exponentially on time. It is also shown that if channel capacity $R = T^{-1}$ is not sufficiently large, the synchronization process fails.

This result is extended to the case of bounded exogenous disturbances and bounded measurement errors in (Fradkov *et al.*, 2015). In (Fradkov *et al.*, 2006; Fradkov *et al.*, 2009; Fradkov *et al.*, 2015), the coder and transmission channel distortions are neglected. This paper is focused on studying the control synchronization schemes for the case of erasure communication channel.

4 Erasure channel description

In this study a *binary erasure channel* is considered. It is assumed, that the receiver either receives the bit ($s \in \{-1, 1\}$ in our convention) or it receives a message that the bit was not received (was '*erased*') with a certain *erasure probability* p, see (Cover and Thomas, 1991; Wang and Yan, 2014). It is also supposed that there exists a feedback from decoder to the encoder for acknowledgment whether the bit was erased or not. Therefore the encoder knows what information has been delivered to the decoder, i.e. the so-called *equi-memory condition* (Tatikonda and Mitter, 2004*b*) is fulfilled. Let the acknowledgment signal at time k which is sent by the decoder and received by the encoder be represented by $\sigma[k] \in \{0, 1\}$ as follows:

$$\sigma[k] = \begin{cases} 0, & \text{no erasure at instant } k, \\ 1, & \text{otherwise.} \end{cases} \tag{13}$$

Random variables $\sigma[k]$, $k = 0, 1, \dots$ are assumed to be independent and identically distributed with common distribution: $P_r(\sigma[k] = 0) = 1 - p$ and $P_r(\sigma[k] = 1) = p$.

It is further assumed that if the message of erasing bit appears, then a signal value which was received in the previous step is kept. Therefore in this model a probability p_f of receiving erroneous bits is less than erasure probability p because there is a non-zero probability that the true value coincides with the 'guessed' one (obtained in the previous cycle of transmission).

5 Numerical evaluation of chaotic Chua systems synchronization

Let us numerically study the described control synchronization schemes by the example of synchronization for chaotic Chua systems coupled via erasure communication channel.

5.1 Master and slave systems models

Master system. Let the master system (9) be represented by the following *Chua system*:

$$\begin{cases} \dot{x}_1 = r(-x_1 + \varphi(y_1) + x_2), \quad t \geq 0, \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -q x_2, \end{cases} \\ y_1(t) = x_1(t), \tag{14}$$

where $y_1(t)$ is the master system output, $x = [x_1, x_2, x_3]^{\mathrm{T}} \in \mathbb{R}^3$ is the state vector; $\varphi(\xi) = m_0 \xi + m_1(|\xi + 1| - |\xi - 1|)$; r, q, m_0, m_1 are system parameters.

Slave system. Correspondingly, the slave system equations (10) read as

$$\begin{cases} \dot{z}_1 = r\big(-z_1 + \varphi(y_2) + z_2 + u(t)\big), \\ \dot{z}_2 = z_1 - z_2 + z_3 \\ \dot{z}_3 = -q x_2, \end{cases} \\ y_2(t) = z_1(t), \tag{15}$$

where $y_2(t)$ is the slave system output, $z = [z_1, z_2, z_3]^{\mathrm{T}} \in \mathbb{R}^3$ is the state vector, $\varphi(y_2)$ is defined in (14).

One can easily check that the linear parts of (14), (15) satisfy the HMP condition, see (Fradkov *et al.*, 2008; Fradkov *et al.*, 2009) for details.

5.2 Case of output signal transmission

In this case, controller has a form (12). The first-order coding-decoding scheme with time-based zooming (2)–(6) is applied for transmission $y_1(t)$ to controller (11) input.

The following parameter values were used for the simulations (Fradkov *et al.*, 2008) [1]:

- Chua system parameters: $r = 10$, $q = 15.6$, $m_0 = 0.33$, $m_1 = 0.945$;
- sampling time T was varied in the interval $T \in [0.025, 0.1]$ s for different simulation runs (a corresponding interval for the transmission rate R is $R \in [10, 400]$ bit/s);
- erasure probability p was varied in the interval $p \in [0, 0.4]$;
- controller gain was taken as $K = 10$;
- coder parameter $M_0 = 5$ is chosen to cover the region of the initial values of y_0. This region is found based on the maximum value of $|y_0|$ over Chua system attractor;
- coder decay parameter ρ is taken for each sampling interval T as $\rho = \exp(-\eta T)$, where parameter $\eta = 0.1$;
- initial conditions for the master and slave systems were taken as $x = [0.3, 0.3, 0.3]^{\mathrm{T}}$, $z = 0$;
- simulation final time $t_{\text{fin}} = 1500$ s.

Synchronization error index $Q = Q(R)$ was found as a relative upper value of state synchronization error $e(t) = x(t) - z(t)$:

$$Q = \frac{\max\limits_{0.75 t_{\text{fin}} \leq t \leq t_{\text{fin}}} \|e(t)\|}{\max\limits_{0 \leq t \leq t_{\text{fin}}} \|x(t)\|}. \tag{16}$$

The results for various p are plotted in Figs. 1–4. For smoothing random variations of function $Q(R)$ (in percents), its approximation as an inversely-proportional

[1] For the sake of definiteness, argument '*time*' is conditionally measured in seconds.

function $\tilde{Q} = \mu_Q R^{-1}$ by a least-square method was found. The corresponding curves are also depicted in Figs. 1–3, and the values of μ_Q along with numerically calculated erroneous bit probabilities p_f are given in the figure captions. Separately, dependence of approximation parameter μ_Q on erasure probability p for $R = 100$ bit/s is plotted in Fig. 4.

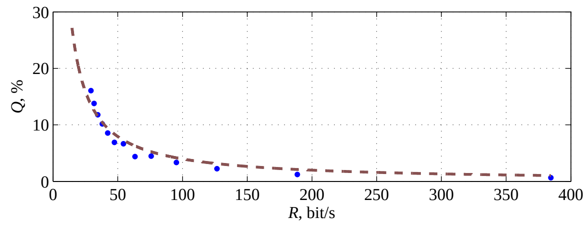

Figure 1. Synchronization error index Q [%] vs transmission rate R. Non-erasure channel, $p = p_f = 0$. $\mu_Q = 4.0$.

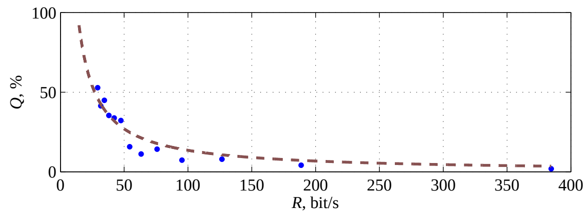

Figure 2. Synchronization error index Q [%] vs transmission rate R. Erasure channel, $p = 0.10, p_f = 0.084$. $\mu_Q = 13.5$.

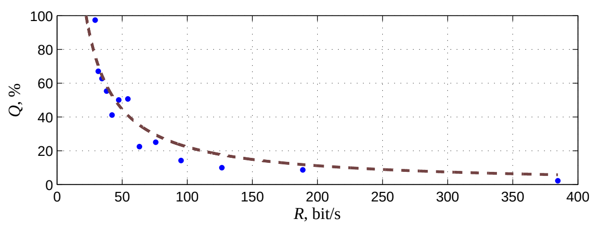

Figure 3. Synchronization error index Q [%] vs transmission rate R. Erasure channel, $p = 0.20, p_f = 0.16$. $\mu_Q = 22.3$.

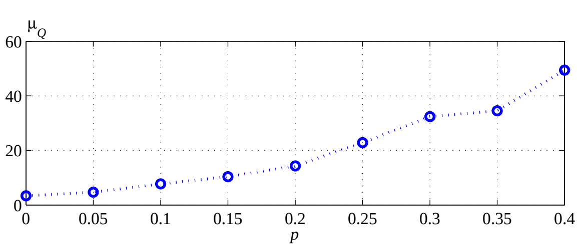

Figure 4. μ_Q vs p. $R = 100$ bit/s.

The cases of high transmission rate and erasure probability are summarized in Table 1.

Exemplary time histories of systems synchronization processes with respect to outputs $y_1(t)$, $y_2(t)$ and states $x_2(t)$, $z_2(t)$ for $R = 100$ bit/s, $p = 0$, 0.2 are depicted in Figs. 5, 6. As is seen from the plots, the synchronization transient time is about 15 seconds, which agrees with the chosen value of the coder parameter ρ.

The results obtained show that inversely proportionality of the limit synchronization error on the transmission rate, proven in (Fradkov *et al.*, 2008) for an ideal channel also valid for an erasure channel. One may notice that presence of erased bits may be treated as a certain decrease of the channel capacity. If the erasure probability tends to one, then the synchronization is lost.

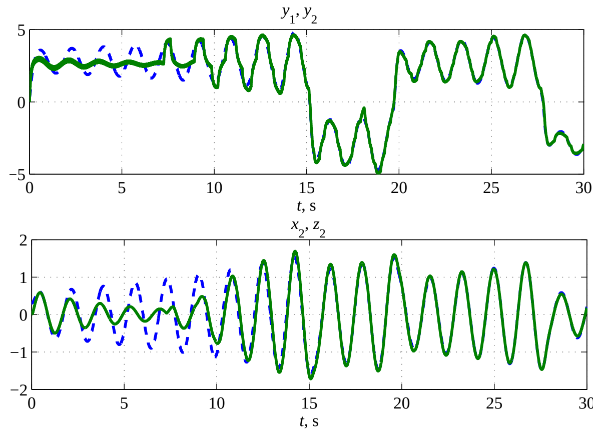

Figure 5. Time histories. Upper plot: outputs $y_1(t)$ – dashed line, $y_2(t)$ – solid line. Lower plot: states $x_2(t)$ – dashed line, $z_2(t)$ – solid line. $R = 100$ bit/s, non-erasure channel, $p = 0$.

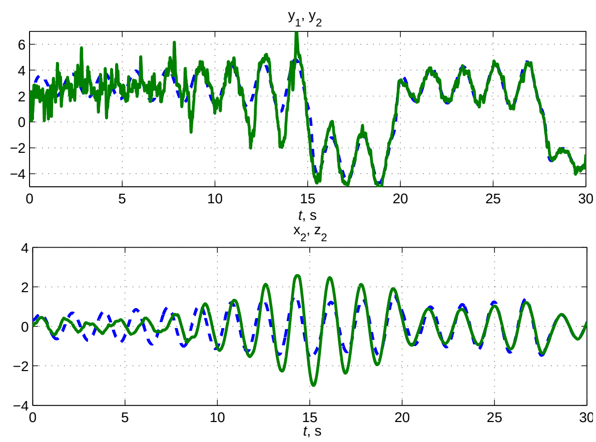

Figure 6. Time histories. Upper plot: outputs $y_1(t)$ – dashed line, $y_2(t)$ – solid line. Lower plot: states $x_2(t)$ – dashed line, $z_2(t)$ – solid line. $R = 100$ bit/s, erasure channel, $p = 0.2$. $p_f = 0.163$

5.3 Case of synchronization error transmission

In this work the adaptive coding procedure is applied and studied for the case of transmitting the synchronization error over the data erasure channel. Cod-

Table 1. Synchronization error index Q=Q(R,p) [%] for high transmission rates

R [bit/s]\ p	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
250	7.18	7.24	13.0	13.9	19.1	25.8	56.8	78.5	122	346
500	3.35	3.43	8.51	7.27	13.2	13.7	14.2	42.5	73.1	129
750	2.01	2.21	2.95	3.39	4.57	8.27	11.1	20.6	56.1	107
1000	1.61	2.18	2.3	2.74	4.28	6.32	6.8	12.7	23.4	76.5

ing procedure is defined by (1), (8). The input signal of the coder is output synchronization error $\varepsilon(t) = y_1(t) - y_2(t)$. The error signal $\bar{\varepsilon}(t)$ of the controller (12) is found by holding the value of $\bar{\varepsilon}[k]$ over the sampling interval $[kT, (k+1)T)$, $k = 0, 1, \ldots$. Initial value M_0 of the coder range and decay factor ρ in (6) are the design parameters.

For simulations, the following parameters have been taken:

- Chua system parameters: $r = 10$, $q = 15.6$, $m_0 = 0.33$, $m_1 = 0.945$;
- sampling time T was varied in the interval $T \in [0.025, 0.1]$ s for different simulation runs (a corresponding interval for the transmission rate R is $R \in [10, 400]$ bit/s);
- erasure probability p was varied in the interval $p \in [0, 0.6]$;
- controller gain was taken as $K = 10$;
- coder parameters $M_0 = 1$, $m = 0$;
- coder decay parameter ρ is taken for each sampling interval T as $\rho = \exp(-\eta T)$, where parameter $\eta = 5$;
- initial conditions for the master and slave systems were taken as $x = [3, -1, 0.3]^{\mathrm{T}}$, $z = 0$;
- simulation final time $t_{\text{fin}} = 100$ s.

Numerical evaluation results are shown in Figs. 7–17. The synchronization area on (R, p) plane is demonstrated in Figs. 7, 8. For plotting this area, the upper bound of the output synchronization error $E(R, p)$ was evaluated as the maximum error $|\varepsilon(t)|$ during the last 20 s of the system simulation. The case of $E(R, p) \geq 10^{-4}$ was considered as a synchronization failure. Zones, where synchronization takes place are shaded in Figs. 7, 8. One can see from the plots, that there exist a certain lower bound of data transmission rate, $R = R_{\min}$, and an upper bound of erasure probability, $p = p_{\max}$, where synchronization is possible. For the presented example they are $R_{\min} = 20$ bit/s, $p_{\max} = 0.55$. For pairs (R, p), close to both bounds on R and p, there exists an additional area, where synchronization fails, see details in Fig. 3.

System behavior in time domain is illustrated by Figs. 9–17. Outputs of the master and slave systems y_1, y_2, output synchronization error ε and state synchronization error $e_2 = x_2 - z_2$ for the case of $R = 20$ bit/s, $p = 0$ are plotted in Figs. 9, 10. Convergence of phase trajectories in 3D-space is demonstrated for this case by Fig. 11. One can see form the plots, that the synchronization error is negligibly small after 5 s from the process beginning. Adaptive adjustment of quantizer range $M[k]$ is shown in Fig. 12. It is seen that $M[k]$ increases when the rate of the output synchronization error is large in the magnitude, and decreases otherwise.

The similar plots for the same transmission rate $R = 20$ bit/s, and $p = 0.4$ are shown in Figs. 13–16. It is seen that the synchronization error becomes larger during the transient in compare with the previous case of $p = 0$, but it should be noticed that in this case the error also tends to zero asymptotically (with the computation accuracy). Therefore for the adopted measurement and channel model, the properties of controlled synchronization, obtained in (Fradkov *et al.*, 2009) are also valid for the data erasure channel, with the exception of possible synchronization loss for the sufficiently large erasure probability.

Figure 17 illustrates the sequences of transmitted and received codewords for $p = 0.4$. It is seen, that erroneous bits probability p_f is less than erasure probability p (for $p = 0.4$, simulation gives $p_f = 0.203$ for 3000 transmitted bits).

6 Conclusion

Limit possibilities of controlled synchronization systems under information constraints imposed by limited information capacity of the coupling channel are evaluated for the case of data erasure channel by the example of controlled master-slave synchronization of two chaotic Chua systems. It is demonstrated that the results obtained in (Fradkov *et al.*, 2008; Fradkov *et al.*, 2009) for controlled synchronization of nonlinear (chaotic) systems under channel capacity limitations may be quantitatively expanded to the data erasure channel case.

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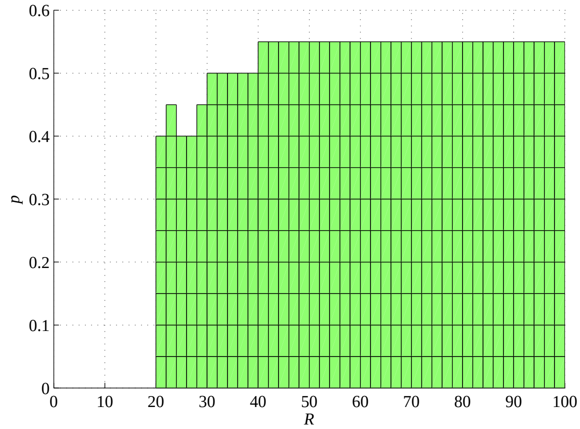

Figure 7. Synchronization area on (R, p) plane. General view.

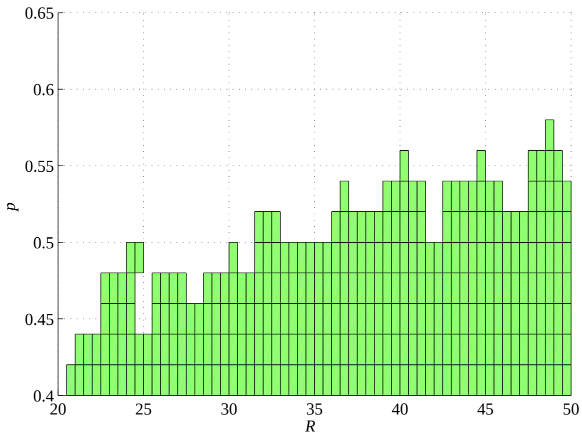

Figure 8. Synchronization area on (R, p) plane. Closer view.

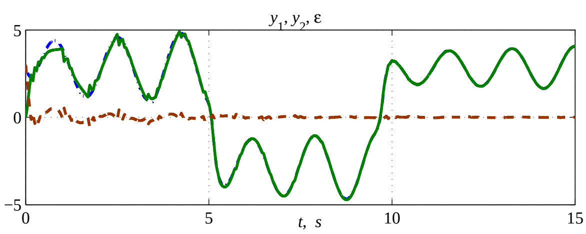

Figure 9. Outputs y_1 (dash-dot line), y_2 (solid line) synchronization. Output synchronization error ε (dashed line). $R = 20$ bit/s, $p = 0$.

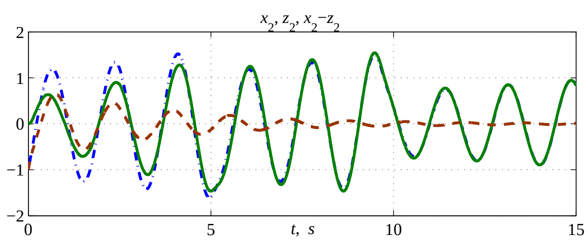

Figure 10. States x_2 (dash-dot line), z_2 (solid line) synchronization. State synchronization error $e_2 = x_2 - z_2$ (dashed line). $R = 20$ bit/s, $p = 0$.

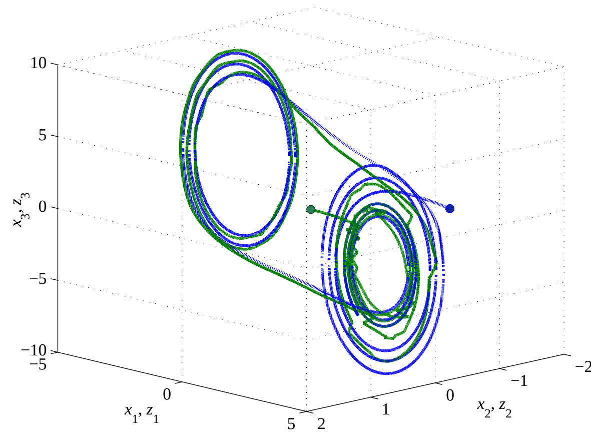

Figure 11. Phase trajectories. Master system – dash-dot line, slave system – solid line. 'o' – initial points. $R = 20$ bit/s, $p = 0$.

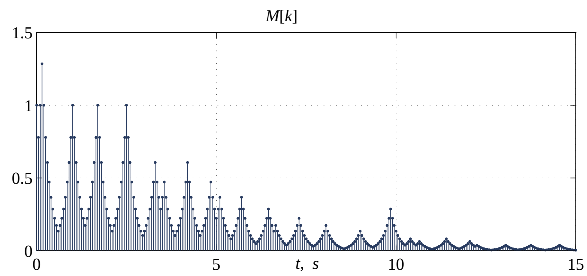

Figure 12. Adjustment of quantizer range $M[k]$. $R = 20$ bit/s, $p = 0$.

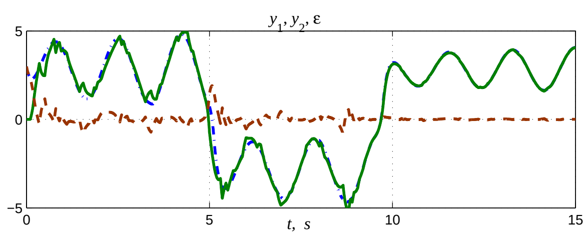

Figure 13. Outputs y_1 (dash-dot line), y_2 (solid line) synchronization. Output synchronization error ε (dashed line). $R = 20$ bit/s, $p = 0.4$.

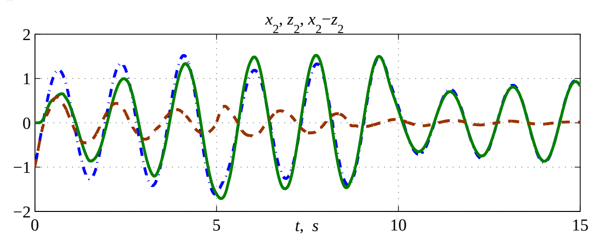

Figure 14. States x_2 (dash-dot line), z_2 (solid line) synchronization. State synchronization error $e_2 = x_2 - z_2$ (dashed line). $R = 20$ bit/s, $p = 0.4$.

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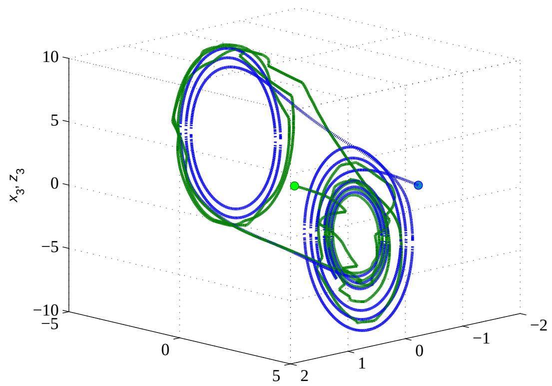

Figure 15. Phase trajectories. Master system – dash-dot line, slave system – solid line. 'o' – initial points. $R = 20$ bit/s, $p = 0.4$.

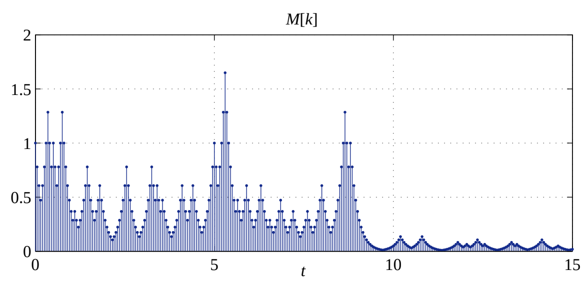

Figure 16. Adjustment of quantizer range $M[k]$. $R = 20$ bit/s, $p = 0.4$.

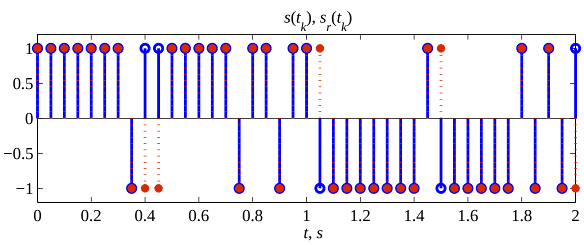

Figure 17. Sequences of transmitted (solid line, 'o') and received (dotted line, '$*$') codewords. $R = 20$ bit/s, $p = 0.4$, $p_f = 0.203$.

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