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OPTIMAL CONTROL STRATEGIES FOR LEGGED LOCOMOTION

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Abstract
Legged locomotion belongs to the category of Nonlinear hybrid systems, which makes it particularly interesting. To adequately design a controller for such systems, requirements as ability to ensure integrity, articulation of long time and short time horizons, Scalability in time and space, Coordinated decentralization of the decision and control system, Adaptivity and Robustness arise. Optimal control results have been explored, once they can provide the required control synthesis. This led to the selection of a Model Predictive Control scheme. After presenting the theoretical background, use-cases with different degrees of complexity have been formulated and implemented.

The results show that optimal control techniques are very powerful, but also tricky and time consuming in case of higher complexity.

Key words
Legged Locomotion, Model Predictive Control, Hybrid Systems

1 Introduction

Legged locomotion is hard to duplicate, due to the great complexity of such physical system, given by the large number of degrees of freedom (DOF), intrinsic instability and others. Regardless, legs are efficient and versatile, which makes them appropriated for nature and man-made environments, since these are unstructured and soft-surface based.

Artificial bipedal systems are already a reality and significant research is been currently conducted on the topic, but they still face challenges in a variety of fields, namely power management, performance or anthropomorphism.

This paper focus is on the control system. The goal is to design a control layer that allows the bipedal system to perform well and reliably while ensuring anthropomorphic moves.

For such, a model predictive controller has chosen and put to test.

This paper is organized as follows: the system and background concepts are presented first then the considered requirements and proposed approach is carefully detailed and finally the experimental use-case and conclusions are debated.

2 The System and related Concepts

From an Engineering perspective, this is a multi-input Multi-Output (MIMO) dynamical system, with high dimension and a chain configuration.

The governing equation of this dynamical system is

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu + J^T F_c \quad (1)$$

where M is the inertia matrix, C is the centrifugal and coriolis forces' matrix, G groups the gravitical terms while J is the jacobian and F_c the terms respectant to the impact with the ground.

The modeling details were presented in [Guimaraes, 2015].

Equation 1 can be rewritten into a first order nonlinear system generalized form

$$\dot{x} = f(x, u, t) \quad (2)$$

From here, a variety of techniques can be employed to analyze the system.

The two most relevant characteristics are the existence of impacts and periodic motion. This places us in the context of hybrid systems and limit cycle convergence/stability analysis.

2.1 Hybrid Systems

A hybrid system displays the evolution in time of both continuous and discrete variables. It is composed of

– a set of discrete states Q,
– a set of continuous states $X = R^n$,
– a vector field f : Q x X $\rightarrow R^n$,
– a set of initial states Init $\subseteq$ Q x X ,
– a domain D : Q $\rightarrow PS(X)$,
– a set of edges E $\subseteq$ Q x Q,
– a guard condition G :$E \rightarrow PS(X)$,
– a reset map R : E x X $\rightarrow PS(X)$

forming the tuple $H = (Q, X, f, Init, D, E, G, R)$. Notice that PS(X) stands for the power set of X (set of all subsets) and a set of control inputs U might be included.

Modeling hybrid systems is "especially challenging" [Lygeros, 2004], due to problems in simulation when the existence and uniqueness of solution are not guaranteed. Dead-locks (permanent absence of conditions that would allow the system to change state) are a common and undesired situation and at the opposite end is the Zeno phenomenon - an infinite number of discrete transitions occurring in finite time. There is an identical phenomenon called chattering, which differs by not converging.

The case of non-uniqueness of solution, although allows uncertainty modeling, "requires additional care when designing controllers for such systems, or when developing arguments about their performance", [Lygeros, 2004], since the arguments have to be true for all solutions.

Some concepts already presented, or presented hereafter, are not exclusive of hybrid systems. For example, the existence and uniqueness of solution is also a concern in continuous dynamical systems but it is known that if f is **Lipschitz continuous** then $\dot{x} = f(x)$, $x(0) = x_0$ has a solution and it is unique.

Fundamental to finding the conditions of existence and uniqueness of solution is the concept of Reachability. A state is reachable if it is possible to get to in finite time from the current state. All the states reachable from a particular state form the reach sets, which are very important in control because they indicate the motion capability of the dynamical system. There is also the notion of backward reachability, which defines the set of all states the system can be in at a time t ¡ t 0 that allow the system to get at t = t 0 to a state belonging to a target set. The concept of attainability is slightly different because it does not include all the states visited in the past. Other two important properties are safety and liveness: while safety describes the ability to maintain in the set a solution that already belongs to it, liveness refers to the aptitude to bring into the set a solution that did not belong to it. Stability is naturally a safety property.

2.2 Limit cycle

Focusing now on the periodic motion that characterizes legged locomotion, limit cycle and related concepts will be clarified.

A limit cycle is an asymptotically stable or unstable periodic orbit with no other periodic orbits nearby.

An orbit x is considered of period T if $\exists\ T > 0 :$ $x(t+T) = x(t)\ \forall\ t$ and $x(t+n) \neq x(t)$ for any $n \neq kT$, k being a positive integer.

Poincaré-Bendixson Theorem, claims that if a closed region of phase space which does not contain any fixed points can be defined, then it must contain a closed-orbit" and since gradient potential fields - a category in which Lyapunov functions are included - can not have a closed-orbit, it is impossible to use unmodified Lyapunov analysis to examine limit cycle stability.

A way to study limit cycle stability is using **Poincaré map** (or return map). This method transforms the study of continuous-time stability of a limit cycle into the study of discrete-time fixed-point stability by defining a surface of section S of dimension n-1. This section can not be defined parallel to the trajectories. To obtain the discrete-time system, the continuous-time dynamics is sampled whenever S is crossed with the instant of the n^{th} crossing being $t_c[n]$, so $\dot{x} = f(x)$ becomes $x_p[n] = x(t_c[n])$. The Poincaré map is a mapping from S to itself defined by $x_p[n+1] = P(x_p[n])$.

P is hard to obtain analytically ([Mark W Spong and Vidyasagar, 2006]), but can be analyzed numerically. Once the map is obtained, eigenvalues can be inspected to infer stability. A limit cycle will always have an eigenvalue of magnitude 1, corresponding to the direction of perturbation which allows the system to flow along the orbit and all the others should have a magnitude smaller than the unity.

3 Proposed Approach

In spite of the limited scope of the analysis of the required motion performance presented here, which was based in key literature references as [Bell, 1998][et al., 1998b][Levangie and Norkin, 2005], it is clear that the following requirements have to be considered:

1 Articulation of long time (say, as defined by the scope of the available a priori or sensed data, which might well be infinite) and short time horizon 'optimal' control strategies in spite of the, possibly conflicting, goals to be considered in the different time horizons.

2 System's integrity. The control system should be designed in such a way that all the constraints to be satisfied by the various subsystems are satisfied. These constraints arise from the external environment due to a priori known features but also due to perturbations and to the, usually unexpected, associated variability. This may lead to control references that need to be adjusted "on-the-fly".

3 Scalability in time and space. Scalability is required not only to deal with complexity and the heterogeneity of subsystems with very diverse process dynamics to be considered, and multiple goals to be targeted and performance criteria to be optimized, but also with the fact that these might be relevant over different time scales. Modularity is an important feature enabling this requirement to be fulfilled. As it has been recognized in the some of the surveyed literature, for example [et al., 2009], a multi-stage structure is required to coordinate the various modules in order to ensure local strategies contributing to common goals.

4 Coordinated decentralization of the decision and control system. This requirement emerges from the need to take into account local specific issues to be addressed by exploiting local degrees of freedom, at a given time scale with shared constraints that arise from the other subsystems and the environment. It should enable the organization of the system in a discrete set of 'independent' nodes, each one acting with partial information but also coordinating indicators to enable automatic adaptation of control references.

5 Adaptivity to take into account trends associated with environment changes, such as ground morphology and physical properties, and weather conditions. By incorporating the most update perception provided by the user or the overall system sensors, the optimization underlying the control synthesis will yield results better adjusted to the user expectations.

6 Robustness of the solution with respect to modeling uncertainties and perturbations. Data gathering, sensing and computational limitations as well as human factors entail the omnipresence of modeling uncertainties and perturbations. This requirement is fulfilled by appropriate feedback control systems designed at subsystem level as well as appropriate choices of targets and performance criteria.

Focussing on the characterization and synthesis of control strategies, we single out:

- Maximum Principle of Pontryagin, [L. Pontryagin., 1962; Vinter, 2000; et al., 1998a; Arutyunov, 2000; A. Arutyunov, 2011], which yields an open loop control strategy by maximizing the so-called Pontryagin function. This involves, an adjoint function which have the useful interpretation of propagating back in time the gradient of the cost functional at the optimum, and can be regarded as the gradient of the Value Function (the optimum cost to go) along the optimal trajectory almost everywhere with respect to time.
- Dynamic programming, [Vinter, 2000], which provides both a technique for verification of optimality, as well as, a means for the synthesis of the optimal control strategy in a state feedback form.

Any one of these classes of optimal control results can be used to provide the control synthesis required in blocks of the above described structure.

Optimal control provides a powerful framework for formulating control problems. This led to the selection of a Model Predictive Control (MPC) scheme to the, whose optimal control foundations are outlined next.

An MPC requires the specification of two slider horizons - one input (or control) horizon and one output (or prediction) horizon - and tries to predict the future evolution of the system (over the output horizon) to optimize the control signal. This means it solves an optimal problem for N future iterations at time t and repeats the optimization at time t + 1 (the next iteration) based on the new sensors' measurements.

The control horizon N_u is typically 10 times smaller than the output horizon N,[Bemporad, 2009], which, despite causing loss of performance, decreases computation time and allows the feasibility to be kept.

The high level MPC will generate a number of indicators that will translate into targets or constraints to be satisfied or approximated by the control problems of the Local Subsystems at the low level for a given finite time horizon. Remark that the specification of changes in models, functionals and targets - for example the desired long term equilibrium for the high level - can be incorporated in optimization processes at both levels of the structure as a result of the evolution of knowledge and of the effectiveness of the deployed control strategies. These changes in the formulation of the optimization problems can be either event-driven in the case of disruptive developments, or the result of a periodic review, being the rate at which these changes take place such that the overall stability of the scheme is maintained, and promote the adaptivity requirement of the overall system. On the other hand, the feedback nature of the MPC scheme will endow the overall system with robustness to perturbations.

Essentially, the very basic MPC scheme consists in a recursive procedure in which, once sampled the state of the system at the initial time, say $t = t_i$ and optimal control problem (P_T) is solved in a given optimization horizon $[t_i, T]$, and then applied during a time subinterval $[t_i, t_i + \bar{T}]$ with $\bar{T} \ll T$. At this point, the state of the system is sampled, the whole optimization horizon slides of $\bar{T}$ time units, and the whole process is repeated.

Remark, that even if the solution to the optimal control problem is open loop, the periodic sampling of the state variable together with the computation of the associated solution to (P_T), ensures the closure of the control loop to the required extent. If sampling at step (4) reveals no significant deviations of the sampled state $\bar{x}$ from the expected optimal value $x^*(t_0 + \Delta)$, then step (2) can be skipped. Other variations of the scheme may include the possibility of using the sampled data to upgrade the estimate of the model dynamics, changing time horizons as a function of the scope of the data provided by the system sensors. this information can be used to change "on the fly" constraint functionals and sets, and performance functionals. All these elements might be required to specify the optimal control problem (P_T), whose simplest formulation can be stated as follows:

$$
\begin{aligned}
(P_T)\ \text{Minimize}\ \ & g(x(T)) + \int_{t_0}^{t_0+T} l(s, x(s), u(s))ds \\
\text{subject to}\ \ & \dot{x} = f(t, x, u),\ \text{a.e.} \\
& x(t) \in X_t, \\
& u \in \mathcal{U} \text{ and } x(t_0) \text{ is given},
\end{aligned}
$$

where $f : [t_0, t_0+T]\times R^n\times R^m \to R^n$ defines the controlled dynamics of the system, $g : R^n\times R^n \to R$ is the endpoint cost functional, $l : [t_0, t_0+T] \times R^n \times R^m \to R$ is the running cost function, and $X_t \subset R^n$, and $U_t \subset R^m$ are, respectively, the pointwise state and control constraints. Optimality conditions are currently available for this problem under substantially weak assumptions on its data.

For further details on the MPC scheme, we point out [et al., 2000; Fontes, 2001; F. Fontes, 2007; F. Fontes, 2012] and the references therein.

Since one key objective of the proposed resources optimization framework is to reconcile long term goals with short term goals, the MPC scheme proposed to the high level of the control structure should generate strategies that asymptotically approximate the solution to an "T_∞-horizon" optimal control problem that drives the system to the desirable equilibrium, that is, that solves a problem of the type

$$
\begin{aligned}
\text{Minimize}\ \ & g_\infty(\xi) + \int_{t_i}^{T_\infty} l(t, x(t), u(t))dt \\
\text{subject to}\ \ & \dot{x} = f(t, x, u)\ \ \text{a.e.} \\
& \xi \in C_\infty,\ \lim_{t\to T_\infty} x(t) = \xi \\
& u \in \mathcal{U}.
\end{aligned}
$$

The function $g_\infty(\cdot)$ is the term in the performance functional that forces the system to be driven to the desired long term equilibrium. In order for the MPC scheme to yield solutions approximating the ones of the infinite horizon optimal control problem, the associated optimization problem (P_T) is defined as follows:

$$
\begin{aligned}
\text{Minimize}\ \ & V(t_0 + T, x(t_0 + T))+ \\
& \int_{t_0}^{t_0+T} l(s, x(s), u(s))ds \qquad (3) \\
\text{subject to}\ \ & \dot{x} = f(t, x, u),\ u \in \mathcal{U},\ x(t_0)
\end{aligned}
$$

where the function $V(\cdot,\cdot)$ is a value function defined by

$$
V(\tau, z){:=}\min_{u\in\mathcal{U},\xi\in C_\infty} \Big\{ g(\xi) + \int_\tau^{T_\infty} l(t, x(t), u(t))dt{:}\ \dot{x}{=}f(t, x, u),\ x(\tau){=}z,\ x(t){\to}\xi \Big\} \qquad (4)
$$

Under appropriate assumptions, the value function can be obtained by solving an Hamilton-Jacobi partial differential equation

$$
\begin{cases}
\dfrac{\partial}{\partial t}V(t, x) + \min\limits_{u\in\Omega}\langle \dfrac{\partial}{\partial x}V(t, x), f(t, x, u)\rangle = 0 \\
V(T, x(T)) = g(x(T)).
\end{cases}
$$

For a good reference see [Vinter, 2000]. In general, a solution to this partial differential equations in the conventional sense fails to exist, and the type of solution and the notion of derivative that have to be considered may depend on the ingredients of the problem. Moreover, the huge difficulties arising in the computational tractability in solving this equation are well known (for computational approaches and tools, see [J. Silva, 2011] and references therein). This constitutes a huge challenge in the current state-of-the-art in Optimal Control Theory.

In order to investigate an alternative approach to this problem, necessary conditions of optimality for a class of infinite horizon optimal control problems appears to be particularly well suited for the applications considered here, [F. Pereira, 2011]. Consider the problem

$$
\begin{aligned}
\text{Minimize}\ \ & h(x(0), \xi) \\
\text{such that}\ \ & \dot{x}(t) = f(t, x(t), u(t)) \quad \mathcal{L} - a.e.\ [0, \infty) \\
& x(0) \in C_0,\ x(t) \to \xi \in C_\infty \text{ as } t \to \infty \\
& u(t) \in \Omega \subset R^m,\ \forall t \in [0, \infty),
\end{aligned}
$$

where C_0 and C_∞ are compact sets and the remaining ingredients are as above. In spite of the significant body of literature on this class of problems (see [Caputo, 2005; F. Pereira, 2011] and references therein), the degenerative effect of very long time horizons still constitutes a huge challenge. The goal consists in deriving a maximum principle exhibiting boundary conditions at the final endpoint with maximal information. This should enable the appropriate propagation of a suitable Value Function from the final time to the current time. For this purpose, we consider $\xi \in R^n$ to be an equilibrium point as $t \to \infty$, i.e., there exists a feasible control process $(x(\cdot), u(\cdot))$ such that $\lim_{t\to\infty} x(t) = \xi$, and $0 \in \lim_{t\to\infty} intf(t, x(t), \Omega(t))$, and, introduced the notion of directional inclusion at infinity.

Let $y(t) \in R^n$, $y(t) \neq 0$ a.e. in $[0, \infty)$ and $K \subset R^n$ be a pointed cone.We say that $y \in K$ directionally at infinity, i.e., $y \in^d_\infty K$, if $\hat{Y} \subset K_1$ where $K_1 =$ conehull$(K) \cap B_1(0)$ and

$$\hat{Y} = \left\{ \hat{y} \in R^n : \exists t_i \to \infty, \lim_{i \to \infty} \frac{y(t_i)}{\|y(t_i)\|} = \hat{y} \right\}.$$

Below, we will denote by $y \in_\infty K$ either $y \in K$ or $y \in^d_\infty K$.

Then, the necessary conditions of optimality in the form of a maximum principle derived in [F. Pereira, 2011] can be stated as follows:

Let the control process (x^*, u^*) be a solution to (P_c). Then, there exists a multiplier $(\lambda, p) \in [0, \infty) \times AC([0, \infty), R^n)$ satisfying, $\lambda + \|p(\cdot)\| > 0$ (non-triviality)

$$\begin{aligned} -\dot{p}^T(t) &\in p^T(t)\partial_x f(t, x^*(t), u^*(t)), \quad \mathcal{L} - a.e. \\ p(0) &\in \lambda \partial_1 g(x^*(0), \xi^*) + N_{C_0}(x^*(0)) \\ -p &\in_\infty \lambda \partial_2 g(x^*(0), \xi^*) + N_{C_\infty}(\xi^*). \\ u^*(t) & \text{ maximizes } v \to p^T(t) f(t, x^*(t), v)\ \mathcal{L} - a.e. \text{ in } \Omega \end{aligned}$$

4 Use-Cases and Experimental Results

To evaluate the controller behavior, two systems, both non-linear, were used

- a continuous-time system of dimension 2
- a hybrid system of dimension 4

This allowed us to evaluate scalability and to verify the consequences of the hybrid character.

Figure 1 shows the response to a square wave, while figure 2 shows the corresponding control effort.

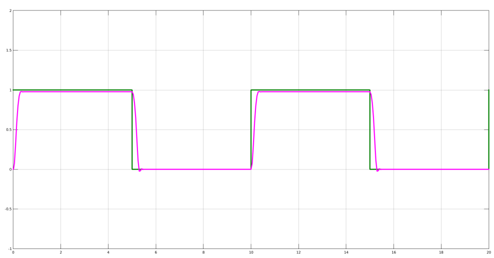

Figure 1. MPC on continuous-time system: small amplitude square wave reference

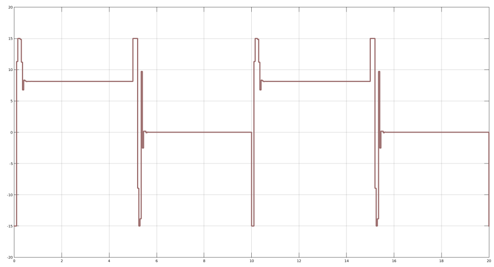

Figure 2. MPC on continuous-time system: large amplitude square wave reference, perturbation and constraints

As it can be seen, the system is able to keep close track of the reference signal without response overshoot.

However, such reference signal does not fully show the usefulness of the MPC, once the system is kept on the neighborhood of its equilibrium point, unperturbed and unconstrained. In such conditions any controller suited for linear systems would behave well. In a scenario were where this conditions don't apply, the MPC can still have good performance. Figure 3 shows the system being forced to move far away from its equilibrium point (x=0) and cumulatively facing a perturbation, which occurs at time $t = 5s$, and while subjected to a constraint. As it can be seen the system is still able to closely follow the reference.

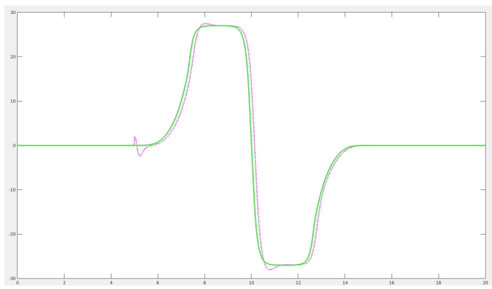

Figure 3. MPC on continuous-time system: large amplitude square wave reference, perturbation and constraints

Moving now to a more complex use-case. The discontinuities impose a challenge to the controller, similar to significant perturbation and issues like sampling rate/instant become very important. Also the system dimension plays a role: the more variables the controller as to deal with, the harder it is to tune, also due to coupling. If on top of that a perturbation is introduced, the system may not be able to follow all of references. This is what figure 4 and figure 5 display: on top are the reference signals and on the bottom is the output. You can see that the state variables in blue and magenta are closely tracked but the others, present bigger error, specially on the hybrid domain transition moments and at instant $t = 1.85s$ where a perturbation occurs.

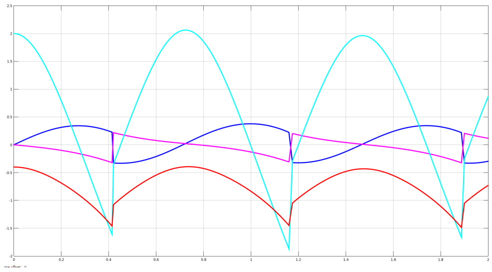

Figure 4. MPC on hybrid system : reference signals

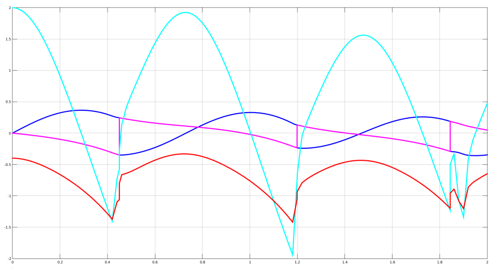

Figure 5. MPC on hybrid system: output signals with disturbances

5 Conclusion

This paper reports the study of a Model Predictive Controller has a strategy to control Nonlinear and Hybrid systems. Designing a controller for target systems is challenging, specially for systems of higher dimension and where discrete events, such as ground impacts, which are intrinsic to legged locomotion and whose dynamics must also be taken into account, are frequent.

The results show that optimal control techniques are very powerful, but also nontrivial and sensitive to practical issues such as choices regarding tunning and computational time and power requirements, particularly in case of higher system complexity and if discrete transitions occur.

As future work, it will be explored the combination of different techniques that can enhance the controller's performance.

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