LMI Optimization of Sensors System for Elastic Vehicle Control Design Based on the Quadratic Performance Index

Sergey A. Brodsky, Alexander V. Nebylov, Alexander I. Panferov

International Institute for Advanced Aerospace Technologies
of State University of Aerospace Instrumentation,
67, Bolshaya Morskaya, 190000, Saint-Petersburg, RUSSIA
(Tel: +7(812)4947016; e-mail: iiaat@aanet.ru)

Abstract: The statement of problem of optimization of sensors system in the control system of the elastic aerospace vehicle under condition of the stochastic system control design on the base of quadratic performance index is considered. The desired control system is the linear-quadratic-Gaussian (LQG) regulator which consists from the steady-state Kalman estimator and the optimal state-feedback gain. The LQG regulator minimizes some quadratic cost function that trades off regulation performance and control effort. This regulator is dynamic and relies on noisy output measurements to generate the control. Magnitudes of time-average values of quadratic performance indexes and maximum feasible values of dispersion of estimation of state vector are considered as linear matrix-inequalities-restrictions in the sensor system optimization problem on the base of minimization of offered goal function related with the number, type and accuracy of sensors. The uniqueness of the solution and high performance of the suggested method are typical for the convex programming problems.

Keywords: measurement optimization, flight control design, flexible vehicle, linear matrix inequalities

1. INTRODUCTION

Presently in the statement of stochastic dynamic systems optimal control problem the number, structure and disposition of sensors as a rule are considered as the set-up parameters of the measurements model. The solution of the problem is parameters and structure of the state-vector observer and the regulator (the control law), satisfying to the defined criteria of transient process (the optimal trajectory of motion). It is supposed to expand the statement of the problem of the stochastic dynamic system control law synthesis, considering these specified parameters of measurements model as unknown variables.

The flexible aerospace vehicle control is connected with the estimation of motion parameters and structural deformations of the construction. Usually, the requirements for these parameters changing dynamics define statements of stabilization and tracking control problems and can be presented in the form of the set of criteria which minimization leads to the solution of the problem of control system synthesis. Independently of it, the problem of the state-space vector estimation with taking into account the aprioristic information about stochastic properties of external disturbances and errors of measurements is solved.

As a result the parameters of the measuring system, defined by the set of gauges and their arrangement, have an effect on accuracy of the control problem solution; in the extreme case the observability of some components of the state-space vector defines an opportunity of the control problem solution. In particular the position of sensors determines influence of elastic oscillations on the measured parameters of motion of the vehicle as a solid body. Parameters of this dependence are determined by values of the shapes of own elastic oscillations in the points of sensors locations. The effective analytical approach for the optimal choice of number of gauges, their type and positioning on the basis of linear matrix inequalities method is considered in this article.

2. PLANT MODEL

2.1. Accountable effects

State-space model of aeroelastic vehicle includes the dynamic equations of solid body motions, models of flexible relative displacements of construction, actuators dynamics from one side and from other side the cross relations defined by aerodynamic and trust forces and closed loop feedback control. Such effects as sloshing, stochastic models of non-stationary aerodynamic forces may be included also.

2.2. Elasticity equations

Discrete form of flexible forced oscillations in node displacements q at body axes frame is next

$$\Delta \mathbf{M}\ddot{\mathbf{q}} + \Delta \Xi \dot{\mathbf{q}} + \mathbf{q} = \Delta \mathbf{f}, \tag{1}$$

where $\mathbf{M}$ is diagonal mass matrix of lumped masses m_i, Δ is the inverse stiffness symmetrical matrix and Ξ is damping symmetrical matrix, $\mathbf{f}$ is vector of lumped loads in each node. Matrix Δ is calculated with tacking to account free boundaries and dynamic equilibrium conditions. It implies that the matrix Δ is singular, and pair of singular values corresponds to linear displacement and rotation of vehicle as solid body. In other words, the stiffness matrix ignores the part of distributed loads which do not cause the deformation. The solution of homogeneous ordinary differential equation

$$\Delta\mathbf{M}\ddot{\mathbf{q}}+\mathbf{q}=\mathbf{0}, \tag{2}$$

without damping $\Xi=0$, corresponds to free oscillations with natural frequencies ω_i, and mass normalized shapes $\varphi^{\langle j\rangle}=\{\Phi_{i,j}\}$, as columns of matrix Φ in the next eigenvalue problem for symmetrical matrix:

$$\mathbf{M}^{1/2}\Delta\mathbf{M}^{1/2}\left(\mathbf{M}^{1/2}\Phi\right)=\left(\mathbf{M}^{1/2}\Phi\right)\Lambda\ ,\quad \left(\mathbf{M}^{1/2}\Phi\right)'\left(\mathbf{M}^{1/2}\Phi\right)=\mathbf{E}\ , \tag{3}$$

$$\Omega=diag\left[\omega_i\right]=\Lambda^{-1/2}=diag\left[\lambda_i^{-1/2}\right], \tag{4}$$

The dimension of equation is defined by the number of node points. The displacements of forced flexible oscillations can be represented as linear combination of shapes of free oscillations. The components of vector ξ are known as modes of flexible oscillations (generalized coordinates).

$$\mathbf{q}=\Phi\xi\ . \tag{5}$$

Appropriate form of flexible forced oscillations in flexible modes is next

$$\Phi'\mathbf{M}\Phi\ddot{\xi}+\Phi'\Xi\Phi\dot{\xi}+\Omega^2\Phi'\mathbf{M}\Phi\xi=\Phi'\mathbf{f}\ . \tag{6}$$

The diagonal elements M_i of $\Phi'\mathbf{M}\Phi$ are known as general masses and the components of vector $\Phi'\mathbf{f}$ are known as general forces. In common case the matrix of general masses is diagonal, but for mass-normalized shapes (3) it is an identity matrix.

Theoretically the number of modes is equal to the number of local masses. Practically it is possible to decrease dimension of equation by eliminating the non-dominant harmonics. If one eliminates corresponding components of $\mathbf{q}$ and columns of matrix Φ, one obtains the reduced equation.

For analogy with pendulum equation the transformed damping matrix is approximately assumed as diagonal with elements equal to

$$\Phi'\Xi\Phi=diag(2\zeta_i M_i\omega_i)\ . \tag{7}$$

2.3. Aeroservoelasticity

Rigid body model of vehicle and model of elasticity are interconnected via distributed aerodynamic forces, which depend on parameters of body motion and local angle of attack at *i* nodes. For mathematical simplicity the so-called strip theory as a first approximation is used. In this theory it is assumed that the local force is proportional to the local angle of attack.

The lumped loads $\mathbf{f}$ include aerodynamic and thrust forces, applied to points of body in various directions. These forces depend from flexible displacements of construction and control law $\mathbf{u}$, which defines value and direction of thrust and positions of aerodynamic control surfaces. The total forces and moment are formed from lumped loads.

2.4. LTI model

State-space model of object with consideration of all factors may be created by applying linearization procedure to system of all nonlinear and time-varying equations about points of calculated base trajectory. Approximately one can separate the motion of object to translation and rotation and research motion in one plane. For simplicity let us investigate the longitudinal motion of vehicle in pitch channel and use beam flexibility model of bending oscillations. It is reasonable to use minimal realization of system where all uncontrollable or unobservable modes have been removed.

State-space model of aeroservoelastic object may be represented in the following matrix form:

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}+\mathbf{w}\ , \tag{8}$$

$$\mathbf{y}=\mathbf{C}\,\mathbf{x}+\mathbf{D}\,\mathbf{u}+\mathbf{v}\ , \tag{9}$$

where $\mathbf{x}=col(\mathbf{x}_s,\xi)$ is a state vector includes the solid body and actuators state parameters $\mathbf{x}_s$ and modes of oscillations ξ. The input of system contains deterministic control $\mathbf{u}$, process noise $\mathbf{w}$ and measurement noise $\mathbf{v}$. The output of system is measurement vector y.

The matrix $\mathbf{A}_{nxn}$ is called the dynamic coefficient matrix, and $\mathbf{B}_{nxm}$ is the input coupling matrix. The matrix $\mathbf{C}_{kxn}$ is the measurement sensitivity matrix, and $\mathbf{D}_{kxm}$ is the input-output coupling matrix.

3. MEASUREMENTS, ESTIMATION AND CONTROL

3.1. Shapes based model of measurements

The output of sensors, measuring linear or angle parameters of motion, includes matched parameters of flexible displacements. Influence of oscillations depends on the positions of sensors. It is necessary to perform the estimation of state space vector and design the control law considering this information. The optimization of measuring and control systems for flexible aerospace vehicles is not separated from estimator and regulator optimization.

The main feature of matrix $\mathbf{C}$ is that the rows of $\mathbf{C}$ contain information about sensors and their position. This is used to formalize and solve the problem of sensors choice and their accommodation.

For the pitching motion and two bending modes the state vector is given by $\mathbf{x}=col(\alpha,\theta,\dot{\theta},q_1,\dot{q}_1,q_2,\dot{q}_2)$. The corresponding matrix $\mathbf{C}=col\{\mathbf{C}_\theta,\mathbf{C}_\omega,\mathbf{C}_\alpha\}$ for sensors measuring angle, angular velocity and linear acceleration and for all available nodes *n* of elastic body, where it is possible to set the above types sensors, is given by

$$\begin{aligned}\mathbf{C}_\theta&=\left(\mathbf{0}\quad \varphi_{00}\quad \mathbf{0}\quad \partial\varphi^{\langle 1\rangle}/\partial x\quad \mathbf{0}\quad \partial\varphi^{\langle 2\rangle}/\partial x\quad \mathbf{0}\right),\\ \mathbf{C}_\omega&=\left(\mathbf{0}\quad \mathbf{0}\quad \varphi_{00}\quad \mathbf{0}\quad \partial\varphi^{\langle 1\rangle}/\partial x\quad \mathbf{0}\quad \partial\varphi^{\langle 2\rangle}/\partial x\right),\\ \mathbf{C}_a&=\left(\mathbf{0}\quad \mathbf{0}\quad \varphi_{01}\quad \mathbf{0}\quad \varphi^{\langle 1\rangle}\quad \mathbf{0}\quad \varphi^{\langle 2\rangle}\right),\end{aligned} \tag{10}$$

where measurement sensitivity matrix $\mathbf{C}_\theta$ corresponds to tangage, $\mathbf{C}_\omega$ corresponds to angular velocity , and $\mathbf{C}_a$ corresponds to acceleration. The vectors φ_{00} and φ_{01} are so-called solid body shapes for translation and rotation. The shapes of bending oscillations of homogenous beam $q(x,t)=\sum \phi^{\langle i \rangle}(x)\xi(t)$ and its derivatives $\partial\varphi^{\langle 1 \rangle}/\partial x$, $\partial\varphi^{\langle 2 \rangle}/\partial x$ corresponds to free bending oscillations.

The row-vectors of matrix **C** correspond to sensors positions. The elimination of rows of matrix **C** in equation (9) is adequate to the elimination of sensors. It is reasonable to complete matrix **C** in consideration of $\| \mathbf{c}(_i) - \mathbf{c}(_j) \|$, thus it is possible to change nodes partitions and exclude the possibility of ambiguity correspondence from rows of **C** to points of sensors location. The elimination of a priori not suitable points decreases the dimension of measurements optimization problem.

3.2. Kalman estimation and optimal LQ regulator

The Linear Quadratic Gaussian (LQG) control system contains optimal linear-quadratic regulator or tracking controller and stationary Kalman filter for estimation of state vector. Let us investigate LQG control system purposely to optimize measurements satisfying requirements for control and estimation. The system (8),(9) must be completely controllable and observable. Let us determine stochastic properties of unbiased white noises

$$E(\mathbf{w})=E(\mathbf{v})=\mathbf{0},\ E\left(\mathbf{ww}'\right)=\mathbf{Q}, E\left(\mathbf{vv}'\right)=\mathbf{R},\quad E\left(\mathbf{wv}'\right)=\mathbf{0}. \tag{11}$$

In this case, the optimal estimation of state-vector is

$$\dot{\mathbf{x}}_e=\mathbf{A}\mathbf{x}_e+\mathbf{Bu}+\mathbf{L}(\mathbf{y}-\mathbf{C}\mathbf{x}_e-\mathbf{Du}), \tag{12}$$

$$\mathbf{L}=\mathbf{SC}'\mathbf{R}^{-1}, \tag{13}$$

where **S**.>0 is a solution of associated with estimator Riccati equation

$$\mathbf{AS}+\mathbf{SA}'-\mathbf{SC}'\mathbf{R}^{-1}\mathbf{CS}+\mathbf{Q}=\mathbf{0} \tag{14}$$

The control is implemented using observer state variables

$$\mathbf{u}=-\mathbf{F}\mathbf{x}_e,\ \mathbf{F}=\mathbf{R}_\mathbf{r}^{-1}\mathbf{B}'\mathbf{P}, \tag{15}$$

where P.>0 is a solution of associated with regulator Riccati equation

$$\mathbf{A}'\mathbf{P}+\mathbf{PA}-\mathbf{PBR}_\mathbf{r}^{-1}\mathbf{B}'\mathbf{P}+\mathbf{Q}_\mathbf{r}=\mathbf{0}, \tag{16}$$

for the following quadratic performance index with weighting matrixes $\mathbf{Q}_\mathbf{r}.\geq 0, \mathbf{R}_\mathbf{r}.>0$, which condense requirements for dynamic properties of closed-loop system

$$J=E\int_0^\infty\left(\mathbf{x}'\mathbf{Q}_\mathbf{r}\mathbf{x}+\mathbf{u}'\mathbf{R}_\mathbf{r}\mathbf{u}\right)dt\ . \tag{17}$$

The time-averaged value of quadratic performance index is equal to

$$\lim_{\substack{t_0\to-\infty\\ t_1\to\infty}}\frac{1}{t_1-t_0}E\left\{\int_{t_0}^{t_1}\left(\mathbf{x}'\mathbf{Q}_\mathbf{r}\mathbf{x}+\mathbf{u}'\mathbf{R}_\mathbf{r}\mathbf{u}\right)dt\right\}= \tag{18}$$

$$\mathrm{tr}\left(\mathbf{PLRL}'+\mathbf{SQ}_\mathbf{r}\right)=\mathrm{tr}\left(\mathbf{PQ}+\mathbf{SF}'\mathbf{R}_\mathbf{r}\mathbf{F}\right),$$

$$\overline{\sigma}=\mathrm{tr}\left(\mathbf{PQ}+\mathbf{SF}'\mathbf{R}_\mathbf{r}\mathbf{F}\right). \tag{19}$$

The value of $\overline{\sigma}$ linearly depends from covariance matrix of state error estimation **S**, which in one's turn is linked with covariance matrix **R**, defining error dispersion of measurements. Let us assume that the measurement noises are not correlated variables and therefore the matrix **V** is diagonal matrix with $v_{i,j}$ elements corresponding to dispersions of noises of sensors in *i* nodes. The diagonal elements of inverse matrix $\mathbf{V}^{-1}$ equaling zero can be interpreted as absence of sensors in the corresponding node.

Let us impose a responsibility for dynamical properties of closed-loop system with state-feedback law (15) to choice weight matrix **Q**r and **R**r and fix this by setting minimal value of time-averaged quadratic performance index $\overline{\sigma}$. The matrix of state error estimation **S** defines the accuracy of estimation.

4. MATHEMATICAL PROGRAMMING PROBLEM FORMULATION

4.1. Restrictions

The main question in the measurement optimization is where, which and how many sensors one should use to provide a necessary accuracy of state estimation and to realize the desired control system. Let us formulate the main requirements as

$$\overline{\sigma}(\mathbf{S})\leq\sigma^*, \tag{20}$$

$$\mathbf{S}(\mathbf{R})\ .\leq\mathbf{S}^*\ . \tag{21}$$

The last inequality for solution of (21) defines that the difference is not a positive-definite matrix.

The restrictions may be not so stringent if the accuracy is declared only for some of components of state vector or their linear combination $\mathbf{v}'\mathbf{x}_e$

$$\mathbf{v}'\,\mathbf{S}(\mathbf{R})\,\mathbf{v}\leq d^*\ . \tag{22}$$

Fulfillment of these inequalities for various weight matrixes of a functional (19) one shall use as restrictions. Performance of these restrictions by some composition of sensors provides permissible nonoptimal solution **R**o.

4.2. Goal function

Let us examine equation (14). All information about sensors condensed in diagonal matrix $\mathbf{R}^{-1}$. Let us define $\mathrm{x}_i=\mathrm{R}_{i,i}^{-1}$. The goal function for **x** can be written as

$$f(\mathbf{x})=\rho'\mathbf{x}\,, \tag{23}$$

where ρ is weight vector. The physical meaning of this measurements cost function minimization can be explained by the following features:

$x_i \geq 0$ it is condition for dispersions,

$x_i = 0$ there are no sensor in *i* node,

$x_i = x_k + x_m$ there are two sensors in *i* node.

The last equality assumes that the signals from two sensors were processed as least squares solution $\mathbf{x_e}$ in the presence of known covariance diagonal matrix $\mathbf{R} = E\{\mathbf{ww}'\} = diag([x_k^{-1} x_m^{-1}])$ and $\mathbf{C} = [1 \;\; 1]$. The descriptions of least squares solution is as following

$$\mathbf{y} = \mathbf{Cx} + \mathbf{w},$$
$$\mathbf{x_e} = \left(\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}'\mathbf{R}^{-1}\mathbf{y}, \tag{24}$$
$$E\{(\mathbf{x}-\mathbf{x_e})(\mathbf{x}-\mathbf{x_e})'\} = \left(\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\right)^{-1}.$$

The coefficients of weight vector ρ are specified under the assumption about priority of applied sensors (cost of the sensor, its weight, reliability, etc.) and setting points which can differ by variance of noise of measurements.

4.3. Linear programming problem

Let matrix $\mathbf{S}$ satisfies restrictions (20) and (21), then the equation (14) defines the restriction for $\mathbf{x}$

$$\mathbf{SC}'diag(\mathbf{x})\mathbf{CS} = \mathbf{AS} + \mathbf{SA}' + \mathbf{Q}. \tag{25}$$

With taking into account the requirement to minimize goal function (23) $\rho'\mathbf{x} \to \min$ the problem can be represented in the following form

$$\min_{\mathbf{x}}\left\{\rho'\mathbf{x} \middle/ \begin{array}{l}\mathbf{x} \geq 0,\\ \mathbf{SC}'diag(\mathbf{x})\mathbf{CS} = \mathbf{AS} + \mathbf{SA}' + \mathbf{Q}\end{array}\right\} \tag{26}$$

This is complete setting of linear programming problem. Reduction of the equations to a canonical form justifies an optimum amount of sensors, and the outcome of a solution determines a locations and parameters of sensors. In other words *the number of active restrictions defines a number of nonzero components of vector **x**, that equal to number of sensors*.

The solving problem with equalities-restrictions may exclude minimal solution for goal function, which increase precision of estimation. The following problem statement with extended vector of controlled variables $col(\mathbf{x}, \tilde{\mathbf{x}})$ formally compensates accuracy advantage

$$\min_{\mathbf{x},\tilde{\mathbf{x}}}\left\{\rho'\mathbf{x} \middle/ \begin{array}{l}\mathbf{x} \geq 0, \tilde{\mathbf{x}} \geq 0,\\ \mathbf{SC}'diag(\mathbf{x}-\tilde{\mathbf{x}})\mathbf{CS} = \mathbf{AS} + \mathbf{SA}' + \mathbf{Q}\end{array}\right\} \tag{27}$$

5. LMI APPROACH

5.1. Properties of Linear matrix inequalities

In view of the available limitations determined by the physical sense, and also in view of available limitations on minimum feasible dispersions of errors of sensors, the problem can be formalized in the form of a system of linear matrix inequalities (LMI) in which the ration "more" between matrixes is considered in the sense of positive definiteness of their difference.

A linear matrix inequality (LMI) has the form

$$\mathbf{F}(\mathbf{x}) \stackrel{\Delta}{=} \mathbf{F}_o + \sum_{i=1}^{m} x_i \mathbf{F}_i > 0 \tag{28}$$

where $x \in \mathbf{R}^m$ is the variable and the symmetric matrices $\mathbf{F}_i = \mathbf{F}_i' \in \mathbf{R}^{nxn}$ is a basis for symmetric *n*x*n* matrices (*m*= *n*(*n*+1)/2). The inequality symbol in (28) means that F(x) is positive- definite, i.e

$$\mathbf{u}'\mathbf{F}(\mathbf{x})\mathbf{u} > 0 \text{ for all nonzero } \forall \mathbf{u} \neq \mathbf{0} \in \mathbf{R}^n. \tag{29}$$

Multiple LMIs $\{\mathbf{F}^{(1)}(\mathbf{x}) > 0,...,\mathbf{F}^{(n)}(\mathbf{x}) > 0\}$ can be expressed as the single LMI $\{diag(\mathbf{F}^{(1)}(\mathbf{x}),....\mathbf{F}^{(n)}(\mathbf{x})) > 0$

Nonlinear and convex inequalities can be converted to LMI form using *Schur complements*. The basic idea is as follows:

$$\text{the LMI} \quad \begin{bmatrix}\mathbf{Q}(\mathbf{x}) & \mathbf{S}(\mathbf{x})\\ \mathbf{S}'(\mathbf{x}) & \mathbf{R}(\mathbf{x})\end{bmatrix} > 0, \tag{30}$$

where

$$\mathbf{Q}(\mathbf{x}) = \mathbf{Q}(\mathbf{x})', \mathbf{R}(\mathbf{x}) = \mathbf{R}(\mathbf{x})' \tag{31}$$

is equivalent to

$$\mathbf{R}(\mathbf{x}) > 0, \mathbf{Q}(\mathbf{x}) - \mathbf{S}(\mathbf{x})\mathbf{R}(\mathbf{x})^{-1}\mathbf{S}(\mathbf{x})' > 0 \tag{32}$$

In other words, the set of nonlinear inequalities (32) can be represented as the LMI (30).

Let's consider the quadratic matrix inequality

$$\mathbf{A}'\mathbf{P} + \mathbf{PA} + \mathbf{PBR}^{-1}\mathbf{B}'\mathbf{P} + \mathbf{Q} < \mathbf{0}, \tag{33}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{Q} = \mathbf{Q'}$, $\mathbf{R} = \mathbf{R'} > 0$ are given matrices of appropriate sizes, and $\mathbf{P} = \mathbf{P'}$ is the variable. Note that this is a quadratic matrix inequality in the variable $\mathbf{P}$. It can be expressed as the linear matrix inequality

$$\begin{bmatrix}-\mathbf{A}'\mathbf{P} - \mathbf{PA} - \mathbf{Q} & \mathbf{PB}\\ \mathbf{B}'\mathbf{P} & \mathbf{R}\end{bmatrix} > \mathbf{0} \tag{34}$$

This representation also clearly shows that the quadratic matrix inequality (33) is convex in $\mathbf{P}$.

Use of these properties allows considering a solution of the problem (27) in view as a linear programming problem with restrictions (20) and (21), which can be represented as a set of linear inequalities of the infinite order with taking to account property (29).

5.2. LMI form of restriction for least-squares state estimation in presence of known covariance

First of all, let's consider, how linear matrix inequalities for restrictions on dispersions of the errors of static least-squares estimation $\mathbf{x_e}$ for separate components of a state-space vector in presence of known covariance, given by matrix $\mathbf{D_o}$ are formed

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{w},$$
$$\mathbf{x_e} = \left(\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}'\mathbf{R}^{-1}\mathbf{y}, \tag{35}$$
$$\mathbf{D} = E\{(\mathbf{x}-\mathbf{x_e})(\mathbf{x}-\mathbf{x_e})'\} = \left(\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\right)^{-1}.$$

Except for restrictions on dispersions the positive definiteness of the matrixes entering into the equations determined by their physical sense is considered.

$$\begin{cases} \mathbf{C}'\mathbf{R}^{-1}\mathbf{C} > \mathbf{D}^{-1} \\ \mathbf{D} > 0 \\ \mathbf{x}^{(i)\prime}\mathbf{D}\,\mathbf{x}^{(i)} < \mathbf{x}^{(i)\prime}\mathbf{D}_O\,\mathbf{x}^{(i)},\ i = 1...k \end{cases} \tag{36}$$

The multiple matrix inequalities can be presented as

$$\begin{bmatrix} \mathbf{M_{01}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M_{02}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M_k} \end{bmatrix} > 0, \tag{37}$$

$$\mathbf{M_{01}} = \mathbf{R}^{-1},$$
$$\mathbf{M_{02}} = \begin{bmatrix} \mathbf{C}'\mathbf{R}^{-1}\mathbf{C} & \mathbf{I} \\ \mathbf{I} & \mathbf{D} \end{bmatrix}, \tag{38}$$
$$\mathbf{M_i} = \left[\mathbf{x}^{(i)\prime}\mathbf{D_o}\,\mathbf{x}^{(i)} - \mathbf{x}^{(i)\prime}\mathbf{D}\,\mathbf{x}^{(i)}\right].$$

The set of problem is minimization of linear goal function $\mathbf{h} = \arg\min(\rho'\mathbf{h})$ for linear matrix inequalities (37) and (38) when the matrix variables $\mathbf{R}^{-1}$ и $\mathbf{D}$ has next fixed structure

$$\mathbf{D} = \begin{bmatrix} d_1 & d_2 & \cdots & d_{n(n+1)/2-n+1} \\ * & d_3 & \cdots & \vdots \\ * & * & \ddots & d_{n(n+1)/2-1} \\ * & * & * & d_{n(n+1)/2} \end{bmatrix} \tag{39}$$

$$\mathbf{R}^{-1} = \begin{bmatrix} h_1 & & & \mathbf{O} \\ & h_2 & & \\ & & \ddots & \\ \mathbf{O} & & & h_m \end{bmatrix} \tag{40}$$

This result directly follows from (30), (31) and (32).

5.3. LMI form of restriction for Kalman estimation

The more complex linear matrix inequalities satisfy to restrictions for estimation of state-space vector by using steady state Kalman filter

$$\dot{\mathbf{x}}_e = \mathbf{A}\mathbf{x}_e + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\mathbf{x}_e - \mathbf{D}\mathbf{u}), \tag{41}$$
$$\mathbf{L} = \mathbf{S}\mathbf{C}'\mathbf{R}^{-1}, \tag{42}$$
$$\mathbf{A}\mathbf{S} + \mathbf{S}\mathbf{A}' - \mathbf{S}\mathbf{C}'\mathbf{R}^{-1}\mathbf{C}\mathbf{S} + \mathbf{Q} = \mathbf{0} \tag{43}$$
$$\mathbf{D} = E\{(\mathbf{x}-\mathbf{x_e})(\mathbf{x}-\mathbf{x_e})'\} = \mathbf{S}. \tag{44}$$

The set of restrictions is given by

$$\begin{cases} -\mathbf{S}^{-1}\mathbf{A} - \mathbf{A}'\mathbf{S}^{-1} + \mathbf{C}'\mathbf{R}^{-1}\mathbf{C} - \mathbf{S}^{-1}\mathbf{Q}\mathbf{S}^{-1} > 0 \\ \mathbf{R}^{-1} > 0, \mathbf{Q} > 0, \mathbf{S} > 0, \mathbf{S}^{-1} - \mathbf{D}^{-1} > 0 \\ \mathbf{x}^{(i)\prime}\mathbf{D}\,\mathbf{x}^{(i)} < \mathbf{x}^{(i)\prime}\mathbf{D}_O\,\mathbf{x}^{(i)},\ i = 1...k \end{cases} . \tag{45}$$

The multiple matrix inequalities can be presented as

$$\begin{bmatrix} \mathbf{M_{01}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M_{02}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M_{03}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M_k} \end{bmatrix} > 0, \tag{46}$$

$$\mathbf{M_{01}} = \mathbf{R}^{-1}$$
$$\mathbf{M_{02}} = \begin{bmatrix} \mathbf{S}^{-1} & \mathbf{I} \\ \mathbf{I} & \mathbf{D} \end{bmatrix}$$
$$\mathbf{M_{03}} = \begin{bmatrix} -\mathbf{S}^{-1}\mathbf{A} - \mathbf{A}'\mathbf{S}^{-1} + \mathbf{C}'\mathbf{R}^{-1}\mathbf{C} & \mathbf{S}^{-1} \\ \mathbf{S}^{-1} & \mathbf{Q}^{-1} \end{bmatrix}. \tag{47}$$
$$\mathbf{M_i} = \mathbf{x}^{(i)\prime}\mathbf{D}_O\,\mathbf{x}^{(i)} - \mathbf{x}^{(i)\prime}\mathbf{D}\,\mathbf{x}^{(i)}$$

The matrix variables are the matrixes $\mathbf{R}^{-1}$ and $\mathbf{D}$ and matrix variable $\mathbf{S}^{-1}$ with the same fixed structure

$$\mathbf{S}^{-1} = \begin{bmatrix} p_1 & p_2 & \cdots & p_{n(n+1)/2-n+1} \\ * & p_3 & \cdots & \vdots \\ * & * & \ddots & p_{n(n+1)/2-1} \\ * & * & * & p_{n(n+1)/2} \end{bmatrix} \tag{48}$$

5.4. The expansion of LMI for the more general statement of the sensor system optimization problem

In most cases for the problem of optimization of the sensors system it is enough to consider the examined restrictions on dispersions of estimations and exactitude of control. Installation of additional restrictions raises the order of system of inequalities and reduces efficiency of calculations.

The system of linear matrix inequalities can be expanded in view of following additional conditions of optimization of system of sensors:

- the joint optimization for sensors of various types with necessity to solve the more general Riccati equation;
- the additional restrictions on a minimum errors of sensors;
- the common optimization for all stages of motion, in view of a modification of parameters of model of plant and measurements and a modification of parameters of quadratic performance index;
- the optimization within the bounds of robust control design;
- the interpretation of an error of identification of forms as multiplicative error of measurements;
- the interpretation of not identified shapes of high-frequency oscillations as the distributed noise

The increase of number of restrictions-inequalities in most cases does not lead to substantial growth of number of active restrictions. The elimination of redundant inequalities and reduction of a problem to canonical form still allows determines the optimum amount of sensors equal to number of active limitations, and the outcome of minimization of goal function determines localization and parameters of sensors. It is proved, that in this statement the problem concerns to convex programming that ensures high effectiveness of a method. The using of standard programs of LMI Toolbox of package Matlab simplifies application and research of the suggested approach to measures optimization for various methods of control systems design for flexible aerospace vehicles.

6. CONCLUSIONS

Optimization of measuring system for an elastic construction demands the full information about mathematical model of motion of plant and elastic oscillations and cannot be solved separately from optimization of control.

The suggested approach based on minimization of chosen criterion, connected with the number, type and location of sensors and determining the expenditures on measurements, allows solving the given problem of optimization of sensors system. The execution of the set of restrictions corresponds to the fulfillment of requirements for the feasible accuracy of the estimation of the separate components of the state-space vector and the accuracy of control, which can be determined by the set of minimum values of the time-average values of criteria of quality, for the various stages of motion.

The modification of parameters of plant and measurements, and also modification of requirements to accuracy of the estimation and control can be included in consideration.

The additionally developed methods and algorithms for solving this problem, such as choice of controllable variables and elimination of surplus inequalities, guarantee the convex programming conditions for goal function and restrictions. This implies uniqueness of solution and good performance and convergence.

The offered approach is used in the specialized program for modeling and research of a broad class of elastic plants, developed in IIAAT SUAI.

REFERENCES

Besekersky, V.A., and A.V. Nebylov (1983). *Robust Systems of Automatic Control,* Nauka, Moscow (in Russian).

Boyd, S., L. El Ghaoui, E. Feron, and V. Balakrishnan , (1994). *Linear Matrix Inequalities in Systems and Control Theory*, SIAM books, Philadelphia.

Brodsky, S.A., A.V. Nebylov and A.I Panferov (2004). Mathematical models and software for flexible vehicles simulation. *Proceedings of 16th IFAC Symposium on Automatic Control in Aerospace*, Oxford , vol. 2, 416 – 421.

Brodsky, S.A., A.V. Nebylov and A.I Panferov (2008). Measuring Optimization in Optimal Control of Flexible Aerospace Vehicles. *Proceedings of 17th IFAC World Congress*, Seoul, 13305 – 13309.

Caldwell, B.D., R.W. Pratt, R. Taylor and R.D. Felton (2000). Aeroservoelasticity. In: *Flight control systems* (Roder W. Pratt (Ed)).

Dewey, H. Hodges, G. Alvin Pierce (2002). *Introduction to structural dynamics and aeroelasticity.* Cambridge.

Kuzovkov, N.T. (1976). *Control Systems of Flying Vehicles,* (in Russian).. Machinostroenie, Moscow.

Mishin, V.P (1990). *Rocket Dynamics,* (in Russian). Mashinostroenie, Moscow

Nebylov, A.V. (2004). *Ensuring control accuracy*. Springer-Verlag, Heidelberg.

Nebylov, A.V., A.I. Panferov and S.A. Brodsky (2005a). Flexible aerospace vehicles simulation and nonlinear control synthesis. *Proceedings of 16th IFAC World Congress*, Prague.

Nebylov, A.V., S.A. Brodsky and A.I. Panferov (2005b). Peculiarities of Stabilization and Guidance Systems for Flexible Vehicles and Software Package for Such Systems Simulation and Design. *Proceedings of II European conference for aerospace sciences*, TsAGI-ONERA, Moscow.

Nebylov, A.V., S.A. Brodsky and A.I. Panferov (2005c). Mathematical Models, Designing, Analysis and Synthesis of Control Systems of Complex Flexible Objects. *Proceedings of International Conference "Physics and Control"*, Saint Petersburg.

Rauw, M (1998). FDC 1.2 – *Simulink Toolbox for Flight Dynamics and Control Analysis*.

Tomita, N., Nebylov A.V., et al (1999). Performance and Technological Feasibility of Rocket Powered HTHL-SSTO with Take-off Assist. *Astronautica*, vol. 45, No.10, 629-637.