BURSTING DYNAMICS IN A MIXED POPULATION OF ACTIVE AND EXCITABLE RESISTANCE-CAPACITATIVE SHUNTED JOSEPHSON JUNCTIONS

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Abstract

We report the bursting in a globally coupled network of mixed population of Active and inactive Josephson junctions. We find the parameter space the parameter regime of the junction where its dynamics is governed by the saddle-node on invariant circle (SNIC) bifurcation. We check the parameter regime where the dynamics of the junction governed by the saddle-node on invariant circle (SNIC) bifurcation. In this SNIC regime, the bursting appears in a broad parameters space of the ensemble of mixed junctions.

Key words

Nonlinear Dynamics, Bifurcation Theory, Synchronization, Comeplex Network,.

1 Introduction

It is usually modeled as a resistive-capacitive-shunted junction (RCSJ) which has its mechanical analog in a damped pendulum with a constant torque. A RCL-shunted junction (resistive-capacitive-inductively-shunted junction) model ([Lobb 1998], [Strogatz2 1998], [Dana 2001], [Kurt 2009], [Pikovsky 2013]) was also used to include an inductive loading effect in an array of junctions where more complex dynamics including chaos was seen. Interestingly, the superconducting device shows some typical spiking and bursting behaviors ([Dana 2006],[Lynch 2012]) most commonly seen in a Type I excitability neuron [Izikevich 2000]. The bursting dynamics was also found prominent ([Dana 2001], [Dana 2006], [Crotty 2010]) in a periodically forced junction. This is due to the intrinsic SNIC characteristic of the junction in a selected parameter space ([Strogatz1 1998], [Dana 2006], [Mackay 2013]), which typically governs a class of bursting dynamics in Type I excitability neurons.

Spiking is a repetitive firing state and bursting is a state of recurrent switching between a firing state or oscillatory state and a resting state. The minimal condition for bursting in a system necessitates the presence of an intrinsic slow-fast dynamics ([Izikevich 2000],[Hindmarsh 1989], [Rinzel 1986], [Ermentrout 1986]). As example, in biological neurons, the simplest ionic processes involved in spiking are due to the flow of Na^+ and K^+ ions across the cell membrane, while the bursting may be observed when the fast spiking (FS) is controlled by a slow process like Ca^{++}-gated K^+ ion movement across the membrane. The slow dynamics controls the firing or start of the oscillation and intermittently stops it when the trajectory of the dynamics moves slowly towards a steady state. Alternatively, an excitable system when coupled to an oscillatory system, was found [Chakraborty 2010] to induce a slow dynamics and thereby originates a type of chaotic bursting.

On a different context, a mixed population of globally coupled inactive or excitable and active or oscillatory units was investigated earlier ([Daido 2006],[Pazo 2006],[Sinha 2012], [Luke 2013]) in search of synchrony and global oscillation. Such a global oscillation is practically important, particularly, in the context of a desired synchrony of the pacemaker cells [Kalsbeek 2007]. It is also important to know, in the event of a growing cell death, how robust are the pacemaker cells in the heart or the suprachiasmatic cells in the brain to sustain a globally synchronized oscillation? In the dynamical sense, a death of a cell is considered as a passive or an excitable state. In the situation of a progressive cell death, in other words, increasing number of passive oscillators, a population of globally coupled oscillators showed a type of aging transition [Daido 2006]. Such aging transition or death state is not the focus of this current study. We emphasize rather on the synchronized state (1:1 or higher phase-locking) of global oscillation of the mixed population as shown earlier ([Daido 2006], [Pazo 2006]) where the type of oscillatory dynamics was not given appropriate attention.

In this backdrop, we consider the superconducting RCSJ model to construct a globally coupled network of mixed population of excitable and oscillatory junctions and, particularly, focus on its collective coherent dynamics. Each individual junction is governed by the SNIC bifurcation to limit cycle oscillation. We distinguish the RCSJ units as excitable when they are in a stable steady state for a selected constant current bias less than a critical value and oscillatory when biased by a higher constant current to cross the SNIC bifurcation point. As a result, we find that the presence of a fraction of excitable units generates bursting in the whole network although the uncoupled oscillatory junctions never show bursting dynamics. For a coupling above a threshold, the whole network starts synchronous firing with single spiking, and for further increase of coupling, periodic bursting appears with increasing number of spikes in a single burst and finally which clearly emerges as a parabolic bursting. During the spiking and bursting above a coupling threshold, the whole network splits into two synchronous clusters, one forming a synchronization manifold of the excitable units and another of the oscillatory units, however, they are phase-locked. We reduce the network model using the two synchronization manifolds of the oscillatory and excitable units and explain the bursting mechanism and furthermore, numerically verify the bursting dynamics of the whole network.

2 Single junction model

A single RCSJ model is described by,

$$\ddot{\theta} + \alpha\dot{\theta} + sin\theta = I. \tag{1}$$

where θ is phase difference of the junction, $\dot{\theta}$=v is the voltage across the junction, α=$[h/2\pi eIR^2C]^{1/2}$ is the damping parameter, h is the Planck's constant, e is the electronic charge and I is a constant bias current. It has an equilibrium solution of sin θ=I_0 in a cylindrical space. The stability of the equilibrium is obtained from the $f'(\theta^*) = cos\theta^* = (1-I_0^2)^{1/2}$ where f'=$df/d\theta$ at equilibrium $\theta = \theta^*$. For $I_0 < 1.0$, the model has clearly two equilibrium points, a node for $f'(\theta^*) < 0$ and a saddle for $f'(\theta^*) > 0$. They coalesce at $I_0 = 1.0$ via SNIC bifurcation ([Levi 1978], [Strogatz1 1998]) for a choice of $\alpha > 1.19$. For $\alpha < 1.19$, a fold bifurcation is recorded at $I_0 = 1.0$. In addition there is a bistable region for $I_0 < 1.0$ and $\alpha < 1.19$. We focus here on the SNIC regime for $I_0 > 1.0$ and $\alpha > 1.19$, where the stable equilibrium is separated from the oscillatory regime by a bifurcation line ($I_0 = 1.0$).

3 Network of junctions

We consider a population of N globally coupled RCSJ units in which p number of oscillators are in excitable mode ($I_e < 1.0$), in general, and $(N-p)$ units are self-oscillatory ($I_s > 1.0$). The network consists of two subpopulations and its dynamics is described by

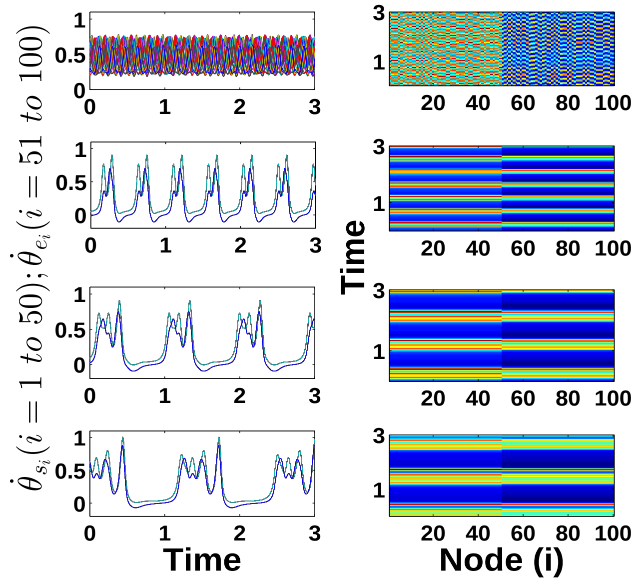

Figure 1. (Color online) Bursting dynamics in a network of Josephson junctions. Temporal dynamics shown in the left panels and spatio-temporal dynamics in the right panels for N=100. Fraction of excitable units in the network, $p/N = 0.5$. $I_e = 0.5, I_s = 1.5, p = 0.5, \alpha = 1.5$. Asynchronous network for $\epsilon = 3.7$ (panels in the uppermost row), two synchronous clusters with bursting in the second row ($\epsilon = 5.0$), third row ($\epsilon = 8.0$), bottom row ($\epsilon = 9.7$).

two sets of equations,

$$\ddot{\theta}_e + \alpha_e\dot{\theta}_e + sin\theta_e = I_e + \frac{\epsilon}{N}\sum_{j=1}^{N}(\dot{\theta}_j - \dot{\theta}_e). \tag{2}$$

$$\ddot{\theta}_s + \alpha_s\dot{\theta}_s + sin\theta_s = I_s + \frac{\epsilon}{N}\sum_{j=1}^{N}(\dot{\theta}_j - \dot{\theta}_s). \tag{3}$$

where $e = 1, 2, ..., p$ and $s = p+1, p+2, ..., N$ denote the excitable and self-oscillatory units respectively. The $\alpha = 1.5$ is chosen identical for all the oscillators to restrict our current study in the SNIC regime [Levi 1978; **?**]. The bias currents to the excitable and oscillatory units are assumed as, $I_e = 0.5$ and $I_s = 1.25$ respectively.

For numerical simulations, we first consider a network of size N=100 with equal number of oscillatory and excitable units. . Initial conditions are generated using random numbers between 0.2 and 0.3. Figure 1 reveals a sequence of bursting oscillation in the whole network for increasing coupling strength in the upper to the lower panels except the uppermost panels. The panels in the uppermost row show no phase-locking for coupling strength $\epsilon = 3.7$. For $\epsilon > 3.7$ in rest of the panels, the whole population forms two clusters as seen from the time series plot of all the oscil-

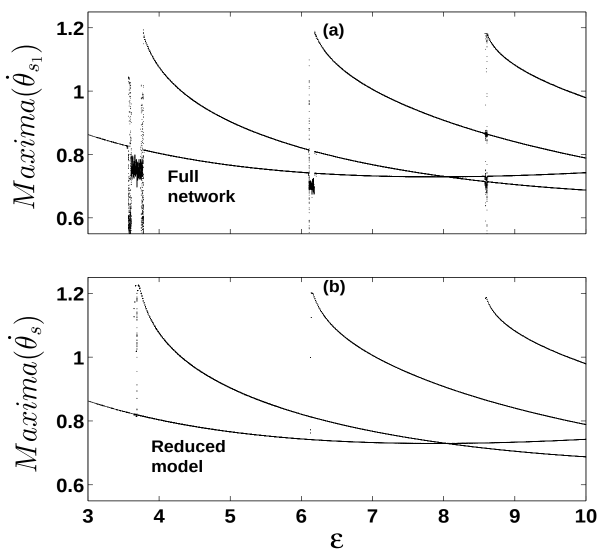

Figure 2. Bifurcation diagram of a Josephson junction unit in the network. One oscillatory unit randomly chosen from the whole population and shown its bifurcation in the upper panel (a), and dynamics of the reduced model at lower panel (b). $I_e = 0.5, I_s = 1.5, \alpha = 1.5$.

lators ($\dot{\theta}_e$, $e = 1, ..., 50$ and $\dot{\theta}_s$, $s = 51, .., 100$) in each panel. The excitable and the oscillatory units form two separate clusters above a threshold coupling (panels in lower three rows), and the two subgroups are also seen phase locked. In fact, the first phase-locked firing in the whole network starts with single spiking dynamics (not shown here) above a coupling threshold and then appears the bursting for larger coupling strength and adds on one after another spike in each burst (left panels in lower three rows). The number of spikes could be even larger for further increase of coupling strength as shown later, in the text, when we are able to recognize the parabolic nature of the bursting. Each of the right panels describes a temporal pattern of all the oscillator nodes ($e = 1, ..., 50, s = 51, ..., 100$); lower three panels clearly show formation of two clusters. These are in perfect match with the nature of the time series at their immediate left panels. This allows a reductionism approach [[Daido 2006], [Nadan 2014]] to the large network dynamics and restrict them into two synchronization manifolds, $\theta_1 = \theta_2 = ... = \theta_p$ representing the original excitable units and $\theta_{p+1} = \theta_{p+2} = ... = \theta_N$ representing the original oscillatory units when we represent the network by two oscillators,

$$\ddot{\Theta}_e + \alpha_e \dot{\Theta}_e + sin\Theta_e = I_e + \epsilon(1-p)(\dot{\Theta}_s - \dot{\Theta}_e) \quad (4)$$
$$\ddot{\Theta}_s + \alpha_s \dot{\Theta}_s + sin\Theta_s = I_s + \epsilon p(\dot{\Theta}_e - \dot{\Theta}_s). \quad (5)$$

where p/N denotes the fraction of excitable junctions in the whole population.

Figure 2 presents the bifurcation diagram of the dynamics of a single oscillatory unit arbitrarily chosen from the whole network and its reduced model (4)-(5) as well. Maxima of $\dot{\theta}_s$ of the junction node (say, $s = 1$) is plotted with coupling strength (ϵ) in the upper panel which represents the original oscillatory units (s). It shows periodic bursting with the number of spikes increasing in a burst one after another with coupling strength. Each period-adding regime is intercepted by a complex bursting window. The maxima of $\dot{\Theta}_s$ of the reduced model is shown in the lower panel and its bifurcation is in agreement with the upper panel. The windows of complex dynamics are also found matching, which also shows complex bursting pattern but here we do not focus on this feature . The reduced model thereby perfectly represents the dynamics of the whole network. The excitable units (e) also show similar bifurcation diagram (not shown here) and match with the reduced model of the excitable units as expected since they are all phase-locked with the oscillatory units (s).

4 Conclusion

In summary, we investigated a mixed population of oscillatory and excitable Josephson junctions under all-to-all global coupling when we observed bursting in a broad parameter range of the junction and the coupling strength. We produced numerical evidence of the phenomenon using a network of N=100 oscillators and taking two equal populations of oscillatory and excitbale junctions. The whole network splits into two clusters for our chosen range of coupling strength that helps reduce the system into a two-oscillator model. Results of the reduced model were found perfectly matching with the numerical results of the whole network. We found that the number of spikes increases with coupling strength which we supported with a bifurcation diagram of the whole network and its reduced model. The bursting dynamics had been a dominant feature of the mixed population such that it existed for different percentage of excitable units although we have only detailed the case of fifty-fifty populations of oscillatory and excitable units.

References

Levi, M., Hoppensteadt, F., Miranker W.L, (1978) *Q. Appl. Math.* **36**, pp. 167

Salam, F.M.,, Sastry, S., (1968) *IEEE Trans. Cir. Systs.* **32**, pp. 784.

Strogatz S.H., (1998) *Nonlinear dynamics and Chaos*, (Westview Press, USA).

Cawthorne, A., Whan, C.B., Lobb, C.J., (1998) *J.Appl.Phys.* **84** (2), pp. 1126.

Wissenfeld, K., Colet, P., Strogatz, S., (1998) *Phys. Rev. E* **57**, pp. 1563 ; *ibid*, (1996) *Phys. Rev. Lett.* **76**, pp. 406.

Dana, S.K., Sengupta, D.C., Edoh K., (2001) *IEEE Trans. Cir. Systs .-I* **48** (8), pp. 950.

Kurt, E., Canturk, M., (2009) *Physica D* **238**, pp. 2229.

Vlasov, V., Pikovsky, A., (2013) *Phys.Rev. E* **88**, pp. 022908.

Dana, S.K., Sengupta, D.C., Hu, C.K., (2006) *IEEE Trans. Cir. Systs. -II* **53** (10), pp. 1031; Dana , S.K., Roy P.K., Sengupta, D.C, Sethia, G.,and Sen, A., *IEE Proc. Cir. Systs.and Devices* (2006) **153** (5), pp. 453.

Borresen J., Lynch S., (2012) *PLoS ONE* **7(11)**, pp. e48498.

Izhikevich E.M., (2000) *Int. J. Bifur. Chaos* **10**, pp. 1171; Sherman, A., and Rinzel J., (1992) *Proc. Natl. Acad. Sci. USA* **89**, pp. 2471.

Crotty, P., Schult, D., Segall K., (2010)*Phys. Rev. E* **82**,pp. 011914.

Baesens, C., MacKay, R.S., (2013)*Nonlinearity* **26**, pp. 3043.

Ermentrout G.B., Kopell N., (1986) *SIAM J. Appl. Math.* **46** pp. 233; Kopell N., Ermentrout G.B., (1986) *Math. Biosci.* **78**, pp. 265.

Rinzel J., (1986) *Proc. Int. Congress of Mathematicians, Berkely, California, USA*, pp. 1578.

Rose, R.M., Hindmarsh, J., (1989) *Proc. R. Soc. Lond. Ser. B Biol.Sci.* **237**, pp. 267.

Chakraborty, S., Dana, S.K., (2010) *Chaos* **20**, pp. 023107; Dana, S. and Roy, P.K., (2007) *Int. J. Bifur. Chaos* **17**,pp. 3437.

Daido, H., Nakanishi, K., (2006) *Phys.Rev.Letts.* **96**, pp. 054101.

Pazo, D. and Montbrio, E., (2006) *Phys. Rev. E* **73**, pp. 055202(R).

Singh, R., Xu, J., Garnier, N., Pumir,A., and Sinha S., (2012) *Phys. Rev. Letts.* **108**,pp. 068102.

Luke, T., Barreto E., So, P., (2013)*Neur. Comput.* **25**, pp. 3207.

Kalsbeek A., Kreier, F., Fliers, E., Sauerwein, H., Romijn J., and Buijs R.M, (2007) *Endocronology* **148**, pp. 5635.

Nandan, M., Hens, C., Pal, P., Dana, S.K., (2014) *Chaos* **24**, pp. 043103.