STATE FEEDBACK CONTROL OF CHAOTIC SYSTEMS AND ELECTRONIC IMPLEMENTATIONS

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Abstract
This work designs a state feedback chaos control approach in which it is possible to use only one state variable with a control parameter for controlling chaos. The method utilizes the controllability condition and Routh's stability criterion to find stable and periodic solutions for a chaotic system. Numerical and an electronic circuit implementation on a Rössler-like system is provided to show effectiveness, validity and feasibility of the chaos control approach.

Key words
Chaos, chaos control, electronic circuits, feedback control, Rössler system.

1 Introduction

Controlling chaos involves perturbing a chaotic system in order to stabilize a given unstable periodic orbit embedded in a chaotic attractor with a chosen small perturbation (Fradkov and Evans, 2005; Kapitaniak, 1996; Sanjuan and Grebogi, 2010). Such a process allows utilizing a chaotic dynamical system for the production of a large number of different periodic behaviors, with flexibility in different yields.

The first control method known as OGY method was proposed by Ott et al. (Ott, Grebogi and Yorke, 1990). In the OGY method, a perturbation is applied when the system state is close to the fixed point. The time lapse for a natural passage of the flow within the fixed neighborhood and thus for switching on the control process can be very large. The control application time can be minimized with a technique of targeting (Shinbrot et al., 1993). Another approach is a one-dimensional version of the OGY method, known as occasional proportional control (OPF) (Galias et al., 1996; Hunt, 1991). In the implementation of the OPF control, one either uses the peaks of one of the system variables or takes derivative of the input signal and generates a pulse when it passes through zero in order to provide the one-dimensional map. An interesting method is the delayed feedback control which forces the dynamical behavior of the chaotic system toward the desired periodic dynamics whenever the system becomes close to such a periodic behavior (Pyragas, 2006, 1992). The delayed feedback control can be considered as a high-pass filter. A similar method designed in frequency domain called washout filter is based on the insertion of a selective filter within a feedback loop (Tesi et al., 1996; Zhou, Lin and Li, 2011). The other chaos control methods include periodic perturbations (Braiman and Goldhirsch, 1991; Lei et al., 2004; Li, Xu and Li, 2006; Lima and Pettini, 1990), stochastic perturbations (Fahy and Hamann, 1992), adaptive control (Boccaletti and Arecchi, 1996; Hua and Guan, 2004), minimum entropy control (Sadeghpour, Salarieh and Alasty, 2013; Salarieh and Alasty, 2008) and impulsive control (Li, Chen and Aihara, 2008; Sun, 2004). In addition, many control methods, such as variable structure control (Yu, 1997), linear feedback control (Liao and Yu, 2006), nonlinear feedback control (Ren and Liu, 2006) and sliding mode control (Ablay, 2009), are applied to chaos for obtaining asymptotic stability or tracking a reference signal. These types of control approaches are aiming to remove chaotic structure completely rather than the stabilization of unstable periodic orbits.

There are also many application and implementations of control of chaos in various scientific fields, which shows the possibility of controlling chaos for useful goals in practice. The first experimental chaos control application of OGY was the stabilization of a chaotic gravitationally buckled, amorphous magnetoelastic ribbon (Ditto, Rauseo and Spano, 1990). Then, the OPF based experimental technique was demonstrated on a chaotic diode oscillator (Hunt, 1991). Many other applications and experimental systems have followed these prototypes and provided successful examples of chaos control in electrical and electronic systems, communication systems, information systems, physics (e.g. lasers, chaos in plasma), mechanical systems (e.g. control of vibroformers, vessels and beams), chemical and processing industries (e.g. stirring of fluid flows), medicine, biology, ecology, economics, and spacecraft (Andrievskii and Fradkov, 2004; Arecchi et al., 1998; Aslanov and Yudintsev, 2012;

Behnia et al., 2013; Chen, 1999; Ferreira, de Paula and Savi, 2011; Ablay, 2015; Kapitaniak, 1996).

In this study, a state feedback control approach is designed for chaos control and an electronic circuit implementation of the method is given on a Rössler-like system. The method needs design of a single control parameter which is obtained systematically by the use of controllability condition and Routh's stability criterion. A practical implementation of the method is illustrated on a Rössler-like system whose nonlinear term is described to be the diode's equation. It is shown that unstable periodic orbits of the chaotic systems can be stabilized up to many periods in an efficient way with the given approach.

The following sections cover the state feedback based chaos control approach (Section 2), an application of the method on a Rössler-like system (Section 3), electronic circuit implementations and numerical simulations (Section 4), and finally, a conclusion to the paper (Section 5).

2 State Feedback for Controlling Chaos

Controlling chaos means application of a small perturbation to a chaotic system to achieve a desirable behavior (i.e., periodic, stationary or chaotic). Consider an *n*th-order chaotic system defined by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \ , \quad \mathbf{x} \in \Re^n , \ \mathbf{f} : \Re^n \to \Re^n \qquad (1)$$

Let x_e be a fixed point of (1). It is aimed to stabilize an unstable periodic orbit embedded in a chaotic attractor and occurring around the fixed point x_e. The controlled system can be given as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{k}(\mathbf{x}) \qquad (2)$$

where $\mathbf{k}(\mathbf{x})$ is an n-dimensional control vector described by $\mathbf{k}(\mathbf{x}) = \begin{bmatrix} k_1 & \cdots & k_n \end{bmatrix}^T$ with feedback gains $k_1, \cdots, k_n$. Here, it is aimed to use only a single non-zero feedback control element of $\mathbf{k}(\mathbf{x})$ (e.g., $k_1 \neq 0$ and $k_2, \cdots, k_n = 0$) for controlling chaos. To determine the non-zero feedback element (or elements if necessary), controllability condition of the linearized chaotic system can be utilized. Linearization of the system (1) at the fixed point x_e results in

$$\dot{\mathbf{x}} = (\mathbf{A} + \mathbf{K})\mathbf{x} \qquad (3)$$

where $\mathbf{A} = \partial \mathbf{f} / \partial \mathbf{x}\big|_{x_e}$ is the linearized system matrix, and $\mathbf{K} = \partial \mathbf{k} / \partial \mathbf{x}\big|_{x_e}$ is the linearized feedback control matrix (for simplicity only one of the diagonal elements of $\mathbf{K} = \mathrm{diag}(k_1, \cdots, k_n)$ can be selected as non-zero). To determine suitable non-zero feedback element (or elements), the controllability matrix of (3) can be used

$$\mathbf{C} = \begin{bmatrix} \mathbf{K} & \mathbf{AK} & \cdots & \mathbf{A}^{n-1}\mathbf{K} \end{bmatrix} \qquad (4)$$

For controllability equation (4) must have full rank, $\mathrm{rank}(\mathbf{C}) = n$, with the selected non-zero feedback element. After selection of the feedback element k_j, from the Routh's stability criterion, a suitable value for this feedback gain can easily be obtained. Firstly, the characteristic equation of (3) is written as

$$\begin{aligned} P(s) &= \det(s\mathbf{I}_n - (\mathbf{A} + \mathbf{K})) \\ &= a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n \end{aligned} \qquad (5)$$

If the controllability matrix of the system (3) is of full rank, then the characteristic polynomial (5) can always be made Hurwitz stable, i.e. negative real valued roots for (5), with an appropriate feedback element k_j. Then, to determine the value of the feedback element, the Routh's table from (5) is written as in the form of

$$\begin{array}{c|ccc} s^n & a_0 & a_2 & \cdots \\ s^{n-1} & a_1 & a_3 & \cdots \\ s^{n-2} & a_2 - a_0 a_3 / a_1 & a_4 - a_0 a_5 / a_1 & \cdots \\ \vdots & \vdots & \vdots & \\ s^0 & \alpha & & \end{array} \qquad (6)$$

Finally, from Routh's stability criterion which states that all values of the first column of the Routh's table (6) must have the same sign for stability (Ogata, 2009), the value of the single control parameter k_j can be determined. If one of the elements of the first column of (6) is equal to zero, there exists a periodic solution. Therefore, the approximate value of k_j for periodic solutions can also found from the Routh's stability criterion by using the boundary value of k_j.

The state feedback based chaos control method is illustrated in Figure 1. The method can also be implemented as model independent with some trial and error in the parameter adjustments.

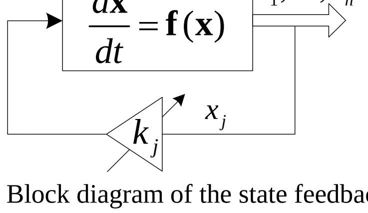

Figure 1. Block diagram of the state feedback based chaos control method.

3 A Rössler-like Chaotic system and its Control

3.1 System description and analysis

Rössler systems (Rössler, 1979) were introduced in 1970s as prototype equations with minimum ingredients for chaos. These chaotic systems contain six terms with a quadratic nonlinearity. There also exist Rössler-like systems for various applications and structures (Chen et al., 2006; Larptwee and San-Um, 2013; Pisarchik et al., 2012; Sprott, 2000). Due to non-dissipative feature of the Rössler systems, their robust electronic implementations are difficult. To deal with these problems, a new Rössler-like system is described by

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= -z + b(e^{cx} - 1) \end{aligned} \tag{7}$$

where nominal values of the system parameters are selected as $a = 0.35$, $b = 0.3$ and $c = 0.03$. The last term in (7) is the equation of a diode, which means that a diode can directly be used in electronic circuit realizations. The system (7) has a fixed point $x_e = (0,0,0)$ and linearization at the fixed point results in the following characteristic equation:

$$P(s) = s^3 + (1-a)s^2 + (bc + 1 - a)s + (1 - abc) \tag{8}$$

When the nominal system parameters are inserted in (8), the following eigenvalues are found

$$\gamma = -0.99, \quad \sigma \pm j\omega = 0.14 \pm 0.99i \tag{9}$$

which means that the fixed point x_e is a saddle focus and satisfies the following inequality

$$|\gamma| > |\sigma| > 0 \tag{10}$$

Therefore, according to Shil'nikov theorem (Turaev and Shilnikov, 1998; Shilnikov, 1965), the chaotic system produces a Shil'nikov chaos. Furthermore, the existence of the chaotic attractor can be given by the divergence of flows as

$$\frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} = a - 1 \tag{11}$$

For $0 < a < 1$, the Rössler-like system is dissipative, i.e. all system orbits will be confined to a specific limit set of zero volume and asymptotic motion will converge to an attractor (Hunt, 2004).

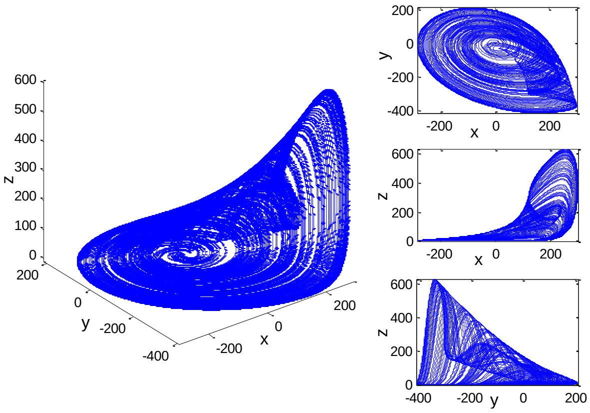

Figure 2. Chaotic attractor of the Rössler-like system from all perspectives for a=0.4, b=0.3 and c=0.03.

A bifurcation diagram exhibiting a period-doubling route to chaos for the Rössler-like system is shown in Fig. 3 for the parameter b in the range of $0.01 \le b \le 3.5$. There is a rich interleaving of chaos and order for $b \le 2.4$. While it is not shown in the bifurcation diagram, there are also periodic and stable solution regions when b is greater than 3. Figure 4 illustrates the rich nonlinear phenomenon and gradual forming of the compound structures.

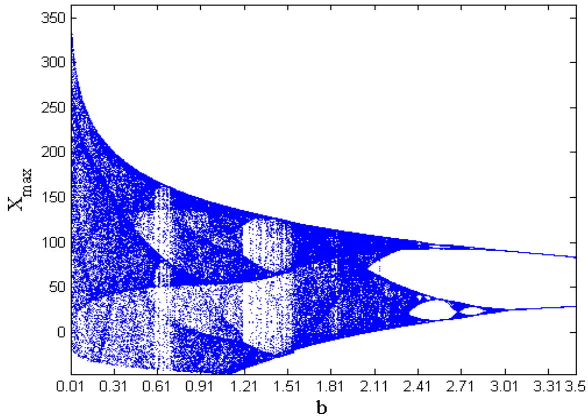

Figure 3. Bifurcation diagram displaying a periodic route to chaos of the peak of x versus parameter b.

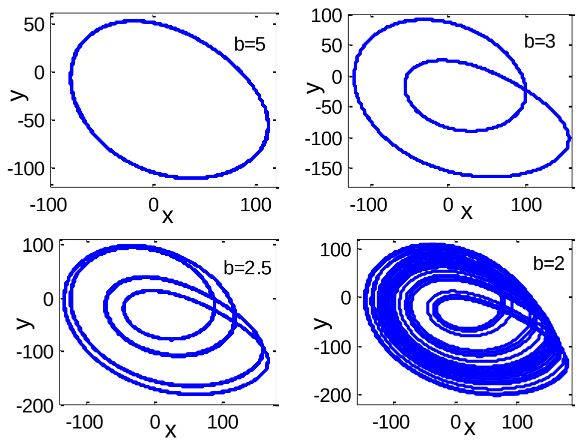

Figure 4. Phase portraits: (a) limit cycle for b=5, (b) period-2 for b=3, (c) period-4 for b=2.5 and (d) chaos for b=2.

3.2 Feedback control design

By considering the general control equation (2) and the Rössler-like system (7), a control matrix can be designed to be $\mathbf{K} = \mathrm{diag}(0, k_2, 0)$, i.e. only $k_2 \neq 0$. Since the system has a fixed point $x_e = (0,0,0)$, the linearized system with feedback control can be written as

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay + k_2 y \\ \dot{z} &= bcx - z \end{aligned} \qquad (12)$$

It is clear that the controllability matrix given in (4) of the system (12) has full rank under any of non-zero diagonal feedback gains, but in here k_2 is selected to illustrate the proposed chaos control method. The main advantage of the use of a diagonal feedback element is due to simple design and implementations (see Section 4). For system parameters $a = 0.35$, $b = 0.3$ and $c = 0.03$, the characteristic equation of the controlled system (12) is obtained as

$$\begin{aligned} P(s) = s^3 + (0.65 - k_2)s^2 + (0.66 - k_2)s \\ + (0.997 - 9\mathrm{x}10^{-3} k_2) \end{aligned} \qquad (13)$$

Creating Routh's table for (13) yields

$$\begin{array}{c|cc} s^3 & 1 & 0.65 - k_2 \\ s^2 & 0.66 - k_2 & 0.997 - 9\mathrm{x}10^{-3} k_2 \\ s^1 & \alpha & 0 \\ s^0 & 0.997 - 9\mathrm{x}10^{-3} k_2 & 0 \end{array} \qquad (14)$$

where
$\alpha = (0.65 - k_2) - (0.997 - 9\mathrm{x}10^{-3} k_2)/(0.66 - k_2)$
which gives the stability condition by $k_2 < -0.35$. Then, periodic solutions can be obtained if $k_2 \approx -0.35$ because there will be a root on the imaginary axis. Due to the effects of the nonlinear term of the chaotic system, the value of k_2 should be selected slightly smaller for stability of fixed points and slightly larger for periodic solutions.

4 Electronic Circuit Implementations

Analog electronics is one of the most useful engineering tools available for design and analysis of linear and nonlinear systems. It is significant to understand that analog electronics solution is simply a voltage wave form whose time dependency is the same as that of the desired variable. It is a common practice that the magnitudes of reference voltages on an analog device are normalized to avoid saturation. In general, op-amp outputs are confined to ±10 volts, so the magnitude of the system is systematically scaled. For experimental realizations, dissipative systems are preferable for robust electrical circuits (Sprott, 2000).

An analog electronics realization of the Rössler-like system is given in Figure 5. Electronic elements of the circuit include quad TL08x JFET-input op-amps and 1N4001 diode as the nonlinear element of the circuit. The controller is a potentiometer connected in parallel to the capacitor of an integrator and is an example of the simplest implementation of the chaos control.

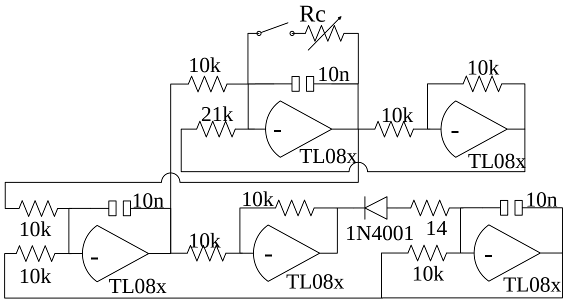

Figure 5. An analog electronic realization of the new Rössler-like chaotic system and an application of the state feedback control.

Figure 6 illustrates uncontrolled behavior of the Rössler-like system. Numerical and Pspice circuit realization results show the same characteristics while their initial conditions are different. In numerical simulations, the nominal values of the Rössler-like system is used, i.e. $a = 0.35$, $b = 0.3$ and $c = 0.03$. The electronic circuit elements and their values are shown in Fig. 5.

Figure 7 displays the stabilization of the fixed point of the Rössler-like system with the proposed chaos control method. As developed in Section 3, the stability condition is obtained by $k_2 < -0.35$ and thus, the feedback control gain is selected as $k_2 = -0.5$ for numerical simulations. In electronic circuit implementation, the value of the potentiometer can be taken as $R_c = 20\,k\Omega$ or less for fixed point stabilization since feedback gain k_2 is proportional to $1/R_c$. It is seen from figure that the stabilization of the fixed point is achieved for both numerical and circuit realizations efficiently.

In Figure 8, a period-1 unstable periodic orbit is of concern. As calculated in Section 3, stabilization for periodic solutions can be obtained for $k_2 = -0.35$ and greater control values. Stabilization for a period-1 behavior is illustrated in Figure 9 for $k_2 = -0.2$ and $R_c = 40\,k\Omega$ for numerical and electronic circuit realizations, respectively.

Similarly, higher order unstable periodic orbits are stabilized with proposed control method. Figure 9 shows stabilization of a period-2 behavior with $k_2 = -0.13$ for numerical simulations and $R_c = 80\,k\Omega$ for circuit realizations. By decreasing the control parameter k_2 (or increasing potentiometer value R_c), stabilization of a period-4 and a period-8 unstable periodic orbit are accomplished as seen in Figures 10 and 11. It is also possible to stabilize more complex unstable periodic orbits in a similar manner.

When the control parameter is selected much smaller than the periodic solution's condition $k_2 = -0.35$, various controlled chaotic behaviors are obtained. Similarly, by selecting the potentiometer value R_c very large, different controlled chaotic motions are observed in circuit implementations.

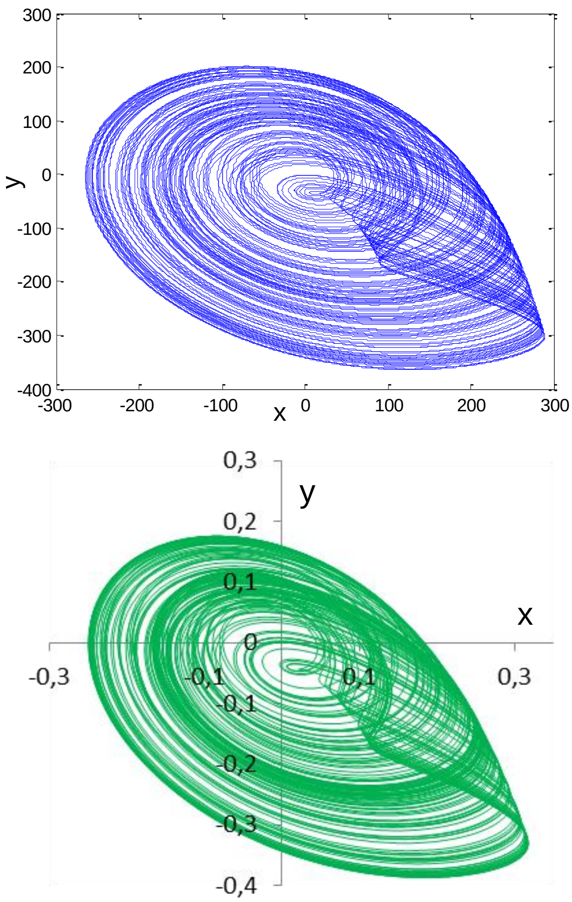

Figure 6. Chaotic system response without any control action: (a) numerical, (b) circuit realizations.

5 Conclusion

A state feedback control method is designed for controlling chaos with an implementation on a Rössler-like system. The study set out to develop an efficient feedback chaos control approach for stabilization of unstable periodic orbits embedded in chaotic attractors. The feedback controller is designed with an adjustable resistor to electronic realization circuit of Rössler-like chaotic system whose nonlinear term is defined and designed with diode. It is shown that the linear chaos control approach is able to stabilize unstable periodic orbits with only one control parameter and without needing any waiting time. Model-independent implementation of the controller is also possible, which can make practical realizations reasonably easy and cheap.

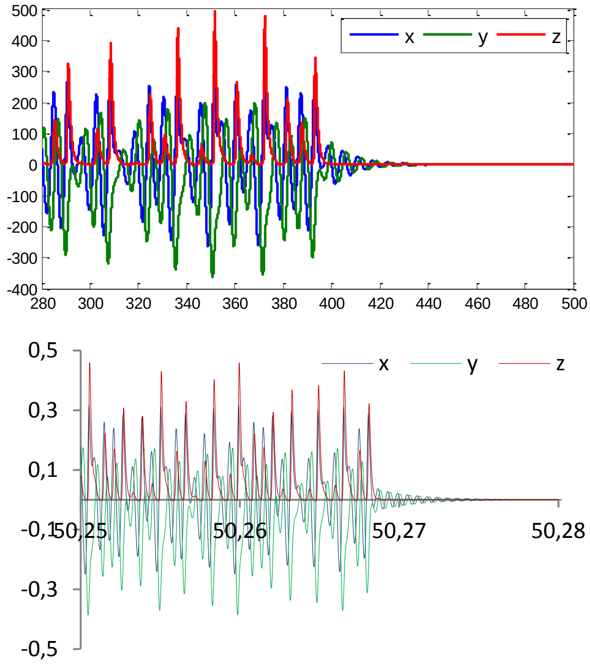

Figure 7. Fixed point stabilization, (a) numerical result for k=-0.5, (b) circuit realization for Rc=20kΩ.

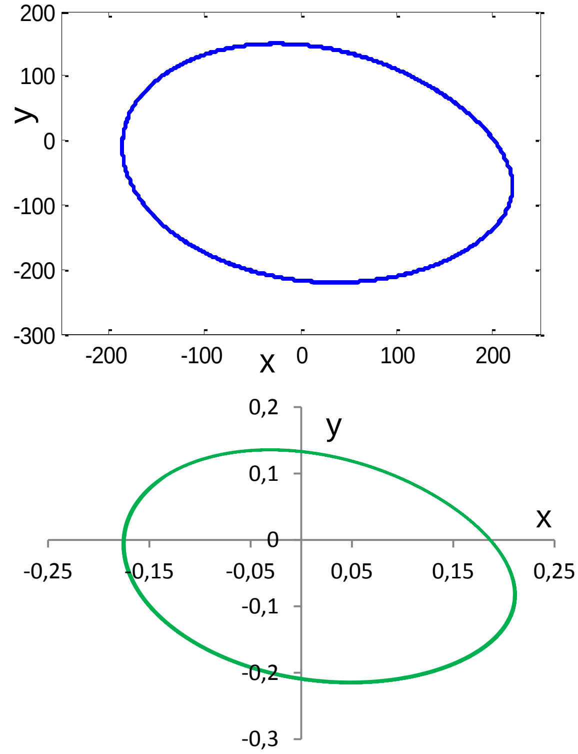

Figure 8. Controlled limit cycle response: (a) numerical result for k=-0.2, (b) circuit realization for Rc=40kΩ.

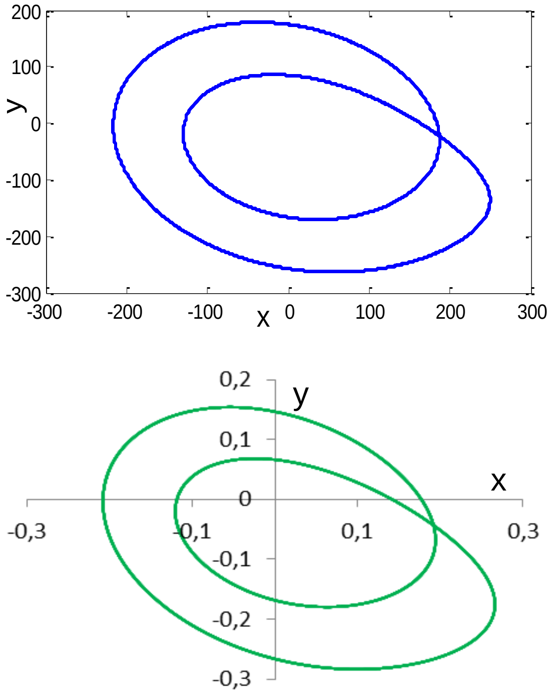

Figure 9. Controlled period-2 response, (a) numerical result for k=-0.13, (b) circuit realization for Rc=80kΩ.

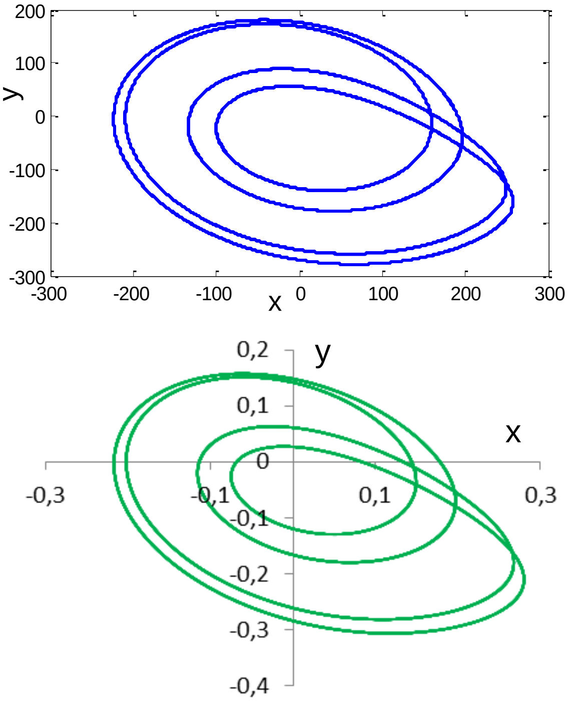

Figure 10. Controlled period-4 response, (a) numerical result for k=-0.108, (b) circuit realization for Rc=110kΩ.

Acknowledgements

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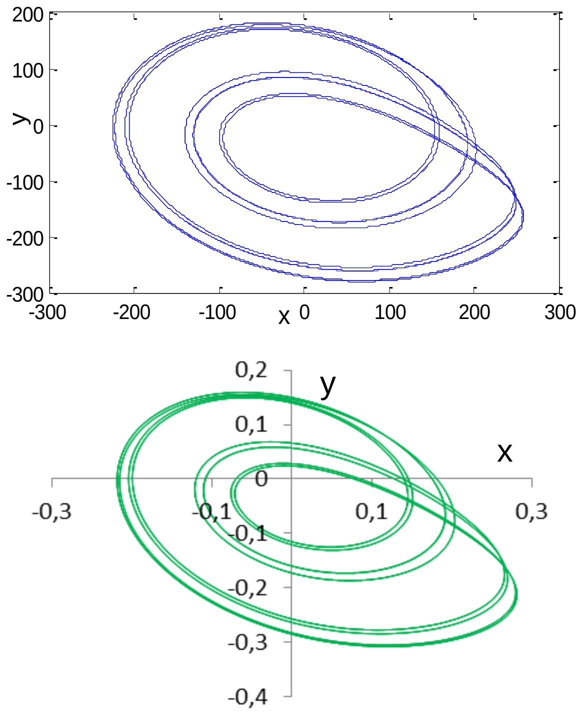

Figure 11. Controlled period-8 response, (a) numerical result for k=-0.105, (b) circuit realization for Rc=112kΩ.

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