ENOC-2008, Saint Petersburg, Russia, June, 30–July, 4 2008

ASYMPTOTIC MODEL OF PROPAGATION AND INTERACTION OF NONLINEAR LONGITUDINAL WAVES IN ELASTOPLASTIC SOLIDS *

Nikolay N. Myagkov
Institute of Applied Mechanics, Russian Academy of Sciences,
32a Lenin Avenue, GSP-1, 119991 Moscow, Russia
NN_Myagkov@mail.ru

Abstract

Both single- and multi-wave processes in the shock-loaded elastoplastic solids are modeled on the basis of the proposed approximate approach. We consider one- and two-dimensional problems on propagation of the longitudinal waves, arising at normal impact on the boundary of a half-space or plate, and problems on two-dimensional fracture of a plate of finite thickness, produced by a cylindrical impactor. The approximate approach under consideration allows one to simplify considerably the analysis of these and others similar problems taking into account an elastoplastic behavior of solid.

Keywords

Nonlinear waves; Shock-loaded solids; Elastoplastic solids.

1 Introduction

It is known that exact solutions for nonlinear mechanics problems of practical interest are rather rare. Either numerical or approximate methods or their combinations are used to find the solution. This also entirely concerns to the problem of wave propagation in solids. Among approximate methods the methods of asymptotic expansions in small parameters are basic [Bogolyubov and Mitropol'skij, 1974; Nayfeh, 1973]. Known approximate methods for analysis of nonlinear wave processes may be divided into two groups. The first group comprises the perturbation methods where the problem is reduced to determination of small corrections to some “unperturbed” solution given by known family of functions. In mechanics of solids these methods were used, for instance, in works [Wallace, 1980a,b]. The second group includes the methods of reduction of the original equations to some model nonlinear wave equations with respect to new variables. These methods presume the possibility to introduce one or several small parameters into the problem at that one parameter among them takes into account a small nonlinearity. They are used when small corrections to the unperturbed solution have a very small region of validity (e.g., often expansion becomes invalid at distances of wave length order). The multiple scales method is the most commonly used method of reduction.

In nonlinear wave theory the multiple scales method is used to reduce a complicated original set of equations to one nonlinear evolution equation (or system of simpler nonlinear equations) which solution gives a uniformly valid approximation to a solution of the original equations [Taniuti and Wei, 1968; Oikawa and Yajima, 1974; Leibovich and Seebass, 1974]. The method is based on the assumption that the parameters (density, velocity, pressure etc.) of the flow caused by nonlinearity and kinetic processes in a medium change slowly on the distances of wavelength order. This method of reduction was used in many fields of physics and mechanics (see, for instance, [Tatsumi and Tokunaga, 1974; Engelbrecht and Nigul, 1981; Zabolotskaya, 1987; Petviashvili and Pokhotelov, 1992]). The most known equations obtained in this way are Burgers and Kortweg-de Vries equations for one-dimensional waves and Khokhlov-Zabolotskaya-Kuznetsov and Kadomtsev-Petviashvili equations in case of quasi-plane waves. As applied to the problems of propagation of nonlinear waves of deformation in the elastic solids these equations were obtained, for instance, in [Engelbrecht and Nigul, 1981; Zabolotskaya, 1987]. For the problems of shock loading of solids with elastoplastic behavior such kind approximate equations were derived in our works [Myagkov, 1994; Myagkov 2003a].

An evolution analysis of the nonlinear waves generated by an explosive or impact loading of solids traditionally is of interest for a large number of

* Suppoted by ISTC grant 3412.

scientific and technical applications [Zel'dovich and Raizer, 1966; Orlenko, 2004]. This is connected with the fact that the study of mechanical and physical-chemical transformations under high-velocity loading is based on their influence on profile and evolution of the pulse of the shock loading in the sample under consideration. In spite of considerable progress in numerical modeling due to improvement of computers, the development and using of approximate equations describing dynamic processes in elastoplastic mediums seem to be useful because of their relative simplicity. It is worth noting that possibility for analytical investigation of approximate equations is considerably higher. Moreover qualitative conclusions inferred with the help of analytical estimates or solutions of the approximate equations may perform the experimental study more effective.

The traditional studies with the shock loading of solid are usually performed under conditions corresponding to the stress range from several gigapascals to tens of gigapascals. In these cases, the stress amplitude is small in comparison with the bulk modulus but exceeds considerably the elastic limit (e.g. for most metals); therefore the asymptotic method of multiple scales known in the general theory of nonlinear waves may be extended to these problems. The ratio of the stress amplitude to the bulk modulus determines a small parameter ε corresponding to the main term in asymptotic expansion of the solution. It is important to note that together with small parameters, which are usual ones for nonlinear acoustics, a small parameter $\nu = (C_l^2 - C_0^2)/2C_0^2$ is introduced (C_l is the phase velocity of longitudinal elastic wave and C_0 the bulk sound velocity). The parameter ν varies a little, like Poisson's ration, over the stress range under consideration. Introduction of this small parameter allows one to consider a stress deviator as a small value of second order relative to an acoustic approximation as well as nonlinearity and absorption. Over the stress range under consideration, for metals the dimensionless average stress amplitude $\varepsilon \sim 0.1$. For dimensionless components of stress deviator an estimation gives $|s'_{ij}| = \nu \cdot 3|s_{ij}|/(2G) \le \nu \cdot Y/G$, where Y is the yield stress and G is the shear modulus. For metals, we have $Y/G \sim 10^{-2} - 10^{-3}$; therefore, despite the fact that the introduced small parameter ν is rather "large" for metals ($\nu \le 0.3$), the dimensionless components of stress deviator will be small compared to the leading term of the expansion $\sim \varepsilon$.

In section 2 the approximate model, which was proposed to describe the wave processes in shock-loaded elastoplastic solids [Myagkov, 1994; Myagkov 2003a], is presented in more details. The possibilities of the analytical investigation based on our approximate approach have been shown in [Myagkov 2003b]. In sections 3 and 4 we consider one- and two-dimensional problems of pulse deformation and fracture of elastoplastic solids on basis of the equations proposed. Our purpose is to check the validity of these equations for the problems of this kind.

2 Approximate equations describing propagation and interaction of nonlinear longitudinal waves in elastoplastic solids

In [Myagkov, 1994; Myagkov 2003a] we proposed an asymptotic model of propagation and interaction of nonlinear longitudinal waves in elastoplastic solids. The approximate equations of the model have the form

$$\lambda_i \frac{\partial V_i}{\partial z} - \frac{1}{4}(\alpha+2)V_i \frac{\partial V_i}{\partial \xi_i} - 3\nu \frac{\partial \psi_i}{\partial \xi_i} - \frac{1}{2}\mu \frac{\partial^2 V_i}{\partial \xi_i^2} - \frac{1}{2}\varepsilon_\Delta \int^{\xi_i} \Delta_\perp V_i \, d\xi'_i = 0, \quad i = 1,2 \tag{1}$$

Here
$V_i = -\sigma'_{11} + \lambda_i u'_1$, $\lambda_{1,2} = \pm 1$; $\sigma'_{11} = \sigma_{11}/\rho_0 C_0^2$, $u'_1 = u_1/C_0$; $\xi_i = t' - \lambda_i^{-1} x'_1$ is the phase variable, $z = x'_1(1 + \mathrm{O}(\varepsilon))$ is the dimensionless Lagrange coordinate in the x_1 direction, $t' = t/t_0$, $x'_1 = x_1/C_0 t_0$, t_0 is the characteristic time, $\Delta_\perp = \partial^2/\partial r'^2 + \partial/r'\partial r'$ $(r' = \sqrt{x_2^2 + x_3^2}/r_0)$, r_0 is the characteristic transverse radius of the loading region, σ_{11} and u_1 are the stress and velocity in the x_1 direction, ρ_0 is the unperturbed density, ε, ε_Δ, ν, μ are small parameters: ε is determined as the ratio of the absolute value of the stress amplitude to the bulk modulus, ε_Δ characterizes the transverse divergence (wave diffraction) and is determined as $\varepsilon_\Delta = (C_0 t_0/r_0)^2$, the parameter ν has been determined above, μ characterizes the internal-friction viscosity and thermal conductivity, and the parameter α is determined from the equation of state.

Eqs. (1) are closed by the constitutive equation of the medium, which gives a functional relation between $\psi_i(z,\xi_i)$ and $V_i(z,\xi_i)$. This relation can be written as $\psi_i = \hat{R}[V_i]$, where $\hat{R}[V_i]$ is the result of the action of the nonlinear operator specified implicitly by the solution of the constitutive equation. To ensure uniform validity of Eq. (1) for $z \le \mathrm{O}[\min\{\varepsilon^{-1}, \mu^{-1}, \nu^{-1}, \varepsilon_\Delta^{-1}\}]$, it is sufficient to satisfy the conditions

$$\left|J_i^{(\varepsilon)}\right| = \mathrm{O}(\varepsilon) \quad \text{and} \quad \left|J_i^{(\nu)}\right| = \mathrm{O}(\varepsilon), \tag{2}$$

where $J_i^{(\varepsilon)} = \int_{-\infty}^{\xi_j} V_j \, d\xi_j$, $J_i^{(\nu)} = \int_{-\infty}^{\xi_j} \left(\frac{\partial \psi}{\partial \xi_i} - \frac{\partial \psi_i}{\partial \xi_i} \right) d\xi_j$,

$\psi = \hat{R}[V_1 + V_2]$, $\xi_j \in (-\infty; +\infty)$, $i, j = 1, 2, (i \ne j)$.

The first condition in (2) is satisfied by specifying appropriate boundary conditions. The form of the integrals $J_i^{(\nu)}$ implies that the second condition in (2) should hold, at least, for rapidly decaying solutions V_j as $|\xi_i| \to \infty$. It is easy to check that for an elastic medium this condition is fulfilled automatically. Generally, whether this condition is satisfied can be verified only for known functions V_j, i.e., when the problem is solved.

It is worth noting that the constitutive equation enters into system (1), which describes quasi-plane waves, in the same manner as into the system of equations for plane waves. To solve concrete problems, it is necessary to specify the form of the constitutive equation. For example, if dislocation concepts are used for this purpose, it is convenient to write the constitutive equation in the Malvern form [Hill, 1985; Orlenko, 2004], which can be written in our notation as

$$\frac{\partial \psi_i}{\partial \xi_i} = \frac{1}{3}\frac{\partial V_i}{\partial \xi_i} + \frac{4}{3}\frac{\partial \varepsilon^p[V_i, \psi_i]}{\partial \xi_i}, \ i = 1, 2, \tag{3}$$

where ε^p is the plastic shear strain. Based on the dislocation concepts, the plastic-strain rate of polycrystalline solids is usually determined by the Orowan relation [Gilman, 1968]. The dependence $\psi_i = \psi_i(V)$ closing Eqs. (1) can be specified using the constitutive relations of the theory of small elastoplastic strains [Hill, 1985]. If an elastoplastic-flow model of the Prandtl-Reuss type with the Mises yield criterion [Hill, 1985; Orlenko, 2004] is used, the constitutive equation can be written as

$$\frac{\partial \psi_i}{\partial \xi_i} = \frac{1}{3}\frac{\partial V_i}{\partial \xi_i} \quad \text{for} \quad |\psi_i| \le \frac{Y}{3G} \quad \text{and}$$

$$\psi_i = \frac{Y}{3G}\operatorname{sgn}(\psi_i) \quad \text{for} \quad |\psi_i| > \frac{Y}{3G} \tag{4}$$

During derivation of Eqs. (1), one can obtain "self-consistent" expressions that give the velocity-stress relations with accuracy to terms of the second order of smallness inclusively:

$$\begin{aligned} \sigma'_{11} &= -\lambda_i u'_1 - \frac{1}{4}(\alpha+2)u_1'^2 + \frac{1}{2}\mu\lambda_i \frac{\partial u'_1}{\partial \xi_i} - \frac{3}{2}\nu\psi, \\ \lambda_i u'_1 &= -\sigma'_{11} - \frac{1}{4}(\alpha+2)\sigma_{11}'^2 - \frac{1}{2}\mu\frac{\partial \sigma'_{11}}{\partial \xi_i} - \frac{3}{2}\nu\psi. \end{aligned} \tag{5}$$

Expressions (5) generalize the relations known in nonlinear acoustics [Rudenko and Soluyan, 1977] to the case of an inelastic medium.

To take into account the interaction between nonlinear waves, which refer to different characteristic directions, the phase variables, in contrast to (1), should be written in the form:

$$\xi_i = t' - \lambda_i^{-1}(x'_1 + \varepsilon\Phi_i(x'_1, t') + \nu\theta_i(x'_1, t')), \ i = 1, 2, \tag{6}$$

i.e., corrections of order ε and ν are introduced into the phase variables. Equations for the phase functions have the form

$$\varepsilon\frac{\partial \Phi_i}{\partial z} = -\frac{1}{4}(\alpha+2)V_j, \ i \ne j, \ i, j = 1,2 \tag{7}$$

$$\frac{\partial \theta_i}{\partial z} = -3\cdot\left(\frac{\partial\psi/\partial\xi_i}{\partial V/\partial \xi_i} - \frac{\partial\psi_i/\partial\xi_i}{\partial V_i/\partial\xi_i}\right)_z. \tag{8}$$

Here $V = V_1 + V_2$ and $\psi = \hat{R}[V]$. These equations take into account a change of a phase velocity caused by the square-law nonlinearity and the elastoplasticity. Physical meaning of the phase functions may be defined by means of the Lagrange phase velocity C_σ of fixed stress levels.

It is known [Fowles and Williams, 1970] that the plane waves in solid have two distinct phase velocity associated with the stress profile and the velocity profile. The former, C_σ, is defined as

$$C_\sigma / C_0 = (\partial z / \partial t')_\sigma = -(\partial\sigma'_{11}/\partial t')/(\partial\sigma'_{11}/\partial z),$$

the latter, C_u, is defined similarly. Moreover for given Lagrange particle one gets

$$\frac{C_u C_\sigma}{C_0^2} = -\frac{\rho^2}{\rho_0^2}\left(\frac{\partial\sigma'_{11}/\partial\xi_i}{\partial\rho'/\partial\xi_i}\right)_z \tag{9}$$

Using Eq. (5) taken at μ=0, we can obtain an estimation of C_u. Except for area at the point of profile inflection, where $(\partial\sigma'_{11}/\partial\xi_i)_z = o(\varepsilon)$, the estimated value is $C_u = C_\sigma(1 + \mathrm{O}(\varepsilon+\nu))$. Then from (7)-(9) as a result of transformations, taking into account that $\sigma'_{11} = \frac{1}{2}(V_i + V_j), i \ne j,$ we obtain

$$\frac{\partial(\varepsilon\Phi_i + \nu\theta_i)}{\partial z} = -\frac{\left(C_\sigma^2\right)_{|V_1+V_2} - \left(C_\sigma^2\right)_{|V_i}}{2C_0^2}, \tag{10}$$

The experiments [Zel'dovich and Raizer, 1966; Orlenko, 2004; Kanel, Razorenov and Fortov, 2004] with shock waves propagating through metals show that C_σ changes abruptly at the inflection point of stress profile, i.e., at transition from loading to unloading. Here relative change of C_σ in discontinuity is value of order $\mathrm{O}(\nu)$.

We note that the wave interaction is taken into account implicitly by nonuniform deformation of the phase variables in the solution that is constructed without regard to the interaction. To solve the problem of the interaction of two oppositely facing nonlinear waves one needs to get a solution of set of independent equations (1) and then calculate the phase functions (7), (8) using the values V_i obtained. Afterwards $V_i(\xi_i)$ is corrected by deformation of the variable ξ_i (6) to the initial form: $\xi_i \mapsto \xi_i + \lambda_i^{-1}(\varepsilon\Phi_i + \nu\theta_i)$.

It is necessary to note that the region of applicability of the equations (1), (6)-(8) is restricted. The solutions of these equations yield the first approximation, which is uniformly valid for $z \leq O\left[\min\left\{\varepsilon^{-1}, \mu^{-1}, \nu^{-1}, \varepsilon_\Delta^{-1}\right\}\right]$ only.

3 Propagation of single wave generated by an impulsive loading applied to a half-space boundary.

We consider propagation of single wave generated by a normal loading applied to boundary of an isotropic half-space. Suppose that the half-space is defined by $z \geq 0$. The homogeneous initial state of the half-space allows function V_2, corresponding to negative characteristic value $\lambda_2 = -1$, to be put to zero. In addition, obviously, $\Phi_i = \theta_i = 0$ $(i = 1, 2)$. It is easy to see that $V_2 = -\sigma_1' - u_1' = 0$ implies $V_1 = -2\sigma_1' + O(\varepsilon^2 + \varepsilon\mu + \varepsilon\nu)$.

3.1 Propagation of a plane shock wave produced by detonation of a layer of condensed explosive.

To describe the behavior of half-space, we employ the elastic-perfectly plastic model of solid, that is Eq. (4) with Y=const. The propagation of a plane shock wave in the half-space is described by the equation (1) with ε_Δ=0. The shock wave is produced by detonation of a layer of condensed explosive with parameters: density ρ_{BB}=1.6 g/cm^3, detonation velocity D =7.6 km/s. Thickness of the explosive layer is Δl=1.68 cm. The material of half-space is iron with parameters $\rho_0 = 7.85$ g/cm^3, $C_0 = 3.85$ km/sec, G = 81.4 GPa, Y = 0.93 GPa. The characteristic time is $t_0 = \Delta l / D = 2.21$ µs. Dependence of pressure on time at the contact boundary z'=0 we take from the analytical solution of a problem on reflection of the detonation wave from a deformable wall [Orlenko, 2004]. For the problem under consideration ν = 0.466 and $\varepsilon = P_K / \rho_0 C_0^2$ = 0.327 where P_K = 38 GPa is the pressure from solution of Riemann problem at z'=0.

The result of the numerical solution is shown in Fig. 1, where the wave profiles correspond to the instants of time 1 μs, 2 μs and 3 μs. Formation and evolution of an elastic precursor and elastic unloading are clearly visible in the figure. Thus, despite the relative simplicity of the approximate equations they allow one to consider the basic features of the shock wave propagation in elastoplastic material.

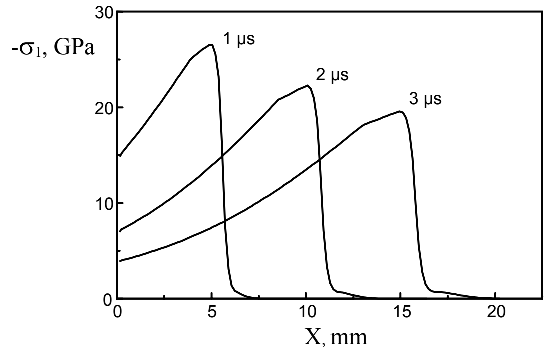

Fig.1. Evolution of the plane shock wave produced by detonation of a layer of condensed explosive.

3.2 Residual mass velocity in the central cross-section of a sample

Using (5), one can easily estimate residual mass velocity in the central cross-section of a sample after passage of a localized loading wave for which the normal stress tends to zero as $\xi_1 \to \pm\infty$. It occurs due to the hysteresis of the elastoplastic-deformation cycle and is given by $u_1' = -(3/2)\nu\psi_1(+\infty)$ where $\psi_1(+\infty) = -\psi_*$ (the velocity in (5) was taken for $\mu' = 0$ and $|\psi_*| = Y/(3G)$). The residual mass velocity was observed experimentally, for example, in [Kanel, Razorenov and Fortov, 2004]. If $(V_1)_{\max}/2 > 2\sigma_g / \rho_0 C_0^2$, where σ_g is the Hugoniot elastic limit, for the localized wave one gets from (10) for $\mu' \to 0$

$$I_1(z') = I_1(0) - \frac{3}{2}\nu\psi_* \cdot z, \qquad (11)$$

where $I_1(z') = \frac{1}{2}\int_{-\infty}^{+\infty} V_1 d\xi_1$ and $I_1(0)$ is the impulse transmitted during impact on boundary z = 0. In (11) it is taken into account that $\psi_1(-\infty) = 0$ and $\psi_1(+\infty) = -\psi_*$. The second term on the right hand side of Eq. (11), taken in absolute value, makes sense of dimensionless momentum stuck in the sample. From (11) one can see that a special feature of attenuation of the plastic shock wave is that it degenerates in a finite time (see, also, [Myagkov, 2003b]).

3.3 Propagation of a two-dimensional shock wave produced by detonation of a cylindrical tablet of explosive.

An axially symmetrical shock wave is produced by detonation of a cylindrical tablet of explosive located on a half-space. The shock-wave propagation in the half-space is described by the equation (1) for V_1 and the constitutive equation of the nonlinear viscoelasticity

$$\frac{\partial \psi_1}{\partial \xi_1} = \frac{1}{3}\frac{\partial V_1}{\partial \xi_1} - \frac{\psi_1}{\tau(V_1, \psi_1)} \tag{12}$$

Where $V_1 = -2\sigma_1'$, $\psi_1 = -(\sigma_1 - \sigma_2)/3G$, $\xi_1 = (t - z/C_0)/t_0$, $z \geq 0$, τ is a relaxation time for shear stresses. The parameters of the explosive (TNT) is as follows: density ρ_{BB}=1 g/cm^3, detonation velocity D =5.1 km/sec. A ratio of thickness of the tablet to its radius is equal to 0,4. The material of the half-space is the same as in section 3.1. Dependence $\tau(V_1, \psi_1)$ may be chosen in simple form as

$$\tau = \begin{cases} \tau_* /(|\psi_1| - \psi_*), & |\psi_1| > \psi_* \\ \infty, & |\psi_1| \leq \psi_* \end{cases}; \tag{13}$$

where $\tau_* = 10^{-4}$, $\psi_* (= Y/3G) = 3.8 \cdot 10^{-3}$. The initial and boundary conditions have the form

$$V_1(z = 0, \xi_1, r') = \varepsilon \upsilon(\xi_1) \mathrm{R}(r'), \tag{14}$$

$$V_1\big|_{\xi_1 \to -\infty} = \left(\frac{\partial V_1}{\partial \xi_1}\right)\bigg|_{\xi_1 \to +\infty} = 0,$$

$$V_1\big|_{r' \to \infty} = \left(\frac{\partial V_1}{\partial r'}\right)\bigg|_{r' = 0} = 0.$$

Here, we suppose that

$$\mathrm{R}(r') = \begin{cases} 1, \ r' \leq 0.75 \\ (1 - 4(r' - 0.75)^2)^2, 0.75 < r' < 1.25 \\ 0, \ r' \geq 1.25 \end{cases},$$

and $\varepsilon \upsilon(\xi_1)$ corresponds to solution of the problem on reflection of the plane detonation wave from a deformable wall and it is determined by expression (12). We also suppose that the detonation wave arrives at the contact boundary z'=0 at the moment $\xi_1 = t = 0$ and $\upsilon(0) = 1$. Radial scattering of the detonation products is not taken into account. The parameters ε = 0.145, ν = 0.31, $\varepsilon_\Delta = (C_0 t_0 / r_0)^2$ =0.16. The coefficient $\mu \leq 10^{-3}$ is chosen for reasons of stability of the numerical solution.

The result of the numerical solution is shown in Fig. 2, where the shock wave is depicted on the various distances (z = 0,5; 2; 4 and 6) from the half-space boundary. The influence of nonlinear viscoelasticity and two-dimensionality on attenuation of the shock wave is evident from figure. An elastic precursor before plastic shock front, elastic unloading behind wave crest and also a second elastic precursor propagating in negative phase are formed. For two-dimensional wave a generation of the powerful negative phase (region of the tensile stresses) caused by lateral divergence of the wave is indicated. One can see that the approximate equations describe correctly the stress-distribution evolution in both the elastic-flow and plastic-flow regions.

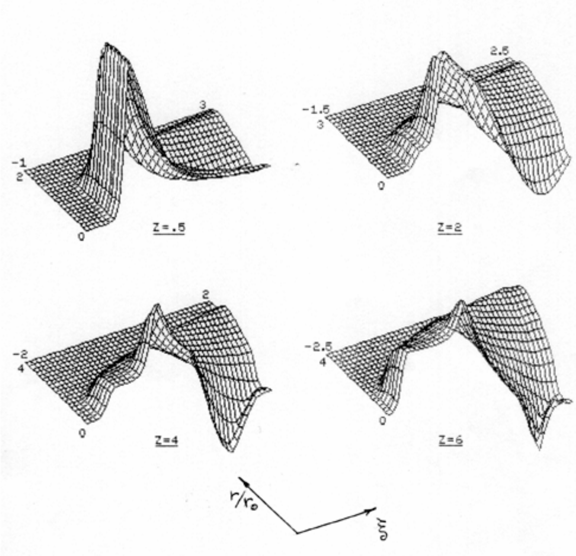

Fig. 2 . Evolution of two-dimensional shock wave produced by detonation of cylindrical tablet of explosive.

4 Multi-wave problems

In this section the equations (1), (6)-(8) are used for solving two problems. The first problem is one-dimensional problem of reflection of a shock wave from a free surface of plate. The second problem concerns two-dimensional damage of a plate of finite thickness, produced by a cylindrical impactor. It is known that the tensile stresses, arising from the interaction of waves inside the plate, can result in the damage and fracture of the plate.

4.1 Reflection of a shock wave from a free surface of plate.

When a shock wave arrives at a free surface of plate, its reflection occurs, i.e., the incident wave interacts with the reflected wave. This problem is of practical interest because the experimental measurements of a free surface velocity or a sample-window interface velocity are usually performed to determine the dynamic properties of metals.

Let us suppose that the plate is located in a layer 0 $\leq z \leq Z_0$. A rectangular (Π -shaped) wave is produced on the front surface (z = 0) of the plate due to the direct impact of a plate consisting of the same material. Let u_0 be an impact velocity, then the shock amplitude is $V_{1m} = u_0 / C_0$. The reflection of the incident wave V_1 occurs on the free surface z = Z_0, where the incident and reflected waves are connected by the boundary condition $(V_1 + V_2)\big|_{z=Z_0} = 0$. The free surface velocity is defined as $u_{fs}' = \frac{1}{2}(V_1 - V_2)\big|_{z=Z_0} = V_1(Z_0, \xi_1)$. The waves evolution in the plate is described by Eqs. (1) for i = 1, 2 (at $\varepsilon_\Delta = 0$) together with Eqs. (6)-(8). If the material of the plate is elastic-perfectly plastic, i.e. constitutive equation has the form (4) with Y = const, and $\mu = 0$ in (1), the solution taking into account interaction between incident and reflected waves may be obtained analytically [Myagkov, 1994]. The interaction has a qualitatively influence on the free-surface velocity history: the precursor becomes two-stepped, the elastic unloading is smoothed considerably and its shape changes. The two-stepped precursor in the profile of a free-surface velocity can be found, for example, in the experiment of Taylor and Rice [Morris, 1982].

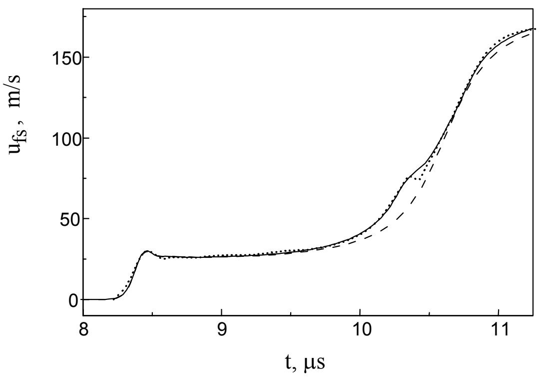

Fig 3. Velocity of the free surface versus time: the solid curve refers to the numerical solution taking into account the interaction, dashed curve refers to the numerical solution ignoring this interaction, and the dotted curve refers to the experimental data obtained by Taylor and Rice.

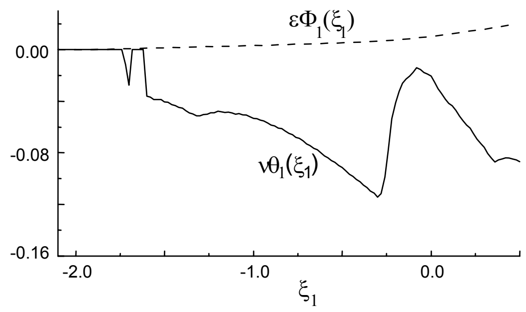

Fig. 4. The dimensionless phase functions which were obtained during the numerical simulation taking into account interaction between the incident and reflected waves (it is shown in Fig.3). Here $\xi_1 = t' - z$.

We model the experimental data obtained by Taylor and Rice [Morris, 1982, p. 29] using the dislocation model proposed by Gilman [Gilman, 1968]. Fig. 3 shows the velocity of the free surface u_{fs} versus time for an armco-iron plate 50.8 mm thick, which was loaded by the normal impact with a velocity of 170 m/sec (the impactor was made of the same material as the target). One can see that the numerical solution that takes into account the interaction between the incident and reflected waves agrees well with the experimental data. The dimensionless phase functions $\Phi_1(\xi_1)$ and $\theta_1(\xi_1)$, which are obtained during the numerical simulation, are shown in Fig. 4. It is worth to note the considerable difference between them. For the phase function $\theta_1(\xi_1)$ taking into account a change of a phase velocity caused by the elastoplasticity, the non-monotonic character of behavior is a peculiar property.

4.2 Simulation of a two-dimensional fracture of an extended plane plate produced by a cylindrical impactor.

4.2.1 Impact with a velocity of 185 m/sec

The problem of the normal-impact damage of a plane plate produced by a cylindrical impactor with a velocity of 185 m/sec is solved numerically. The waves evolution in the plate is described by Eqs. (1), (4) for i=1,2. The phase shifts caused by interaction of the waves are ignored. The material of the impactor and target is aluminum with parameters ρ_0=2.61 g/cm^3, C_0=5.3 km/sec, C_l=6.4 km/sec and the dynamic yield stress Y = 0.18 GPa. The impactor thickness is l=1.14 mm and target thickness is L=2.8$\cdot l$ or Z_0 =2.8. The radius of the impactor r_0=6l. Here ν=0.229, ε_Δ=0.028 and ε = 0.036 in Eq. (1). The characteristic time is $t_0 = l / C_0$ = 0.215 μs. Equations governing the evolution of the material damage are taken the same form as in [Seaman, Curran and Shockey, 1976]. The specific volume of microdefects (voids) is used as the damage measure. It is assumed that the material of the plate fails when the damage reaches a critical level. The critical level of the damage ω_{cr} is accepted to be equal to 0.12 in accordance with the data of Seaman, Curran and Shockey [1976].

The initial and boundary conditions have the form (14) where

$$\mathrm{R}(r') = \begin{cases} 1, r' \le 1{,}0 \\ 0, r' > 1{,}0 \end{cases},$$

$$\upsilon(\xi_1) = \begin{cases} 1, \xi_1 \in [0, 2 - \frac{1}{8}(\alpha+2)\frac{u_0}{C_0}] \\ 0, \xi_1 \notin [0, 2 - \frac{1}{8}(\alpha+2)\frac{u_0}{C_0}] \end{cases}, \tag{15}$$

and u_0 is the impact velocity.

Fig. 5 shows snapshots of right hand side of the plate cross-section for t = 3,92t_0 and 3,98t_0 (the plate is loaded from bottom at 0<r/r_0<1). Contour lines correspond to the specific volume of microdefects ω > 0.01. The central region enclosed by the lines corresponds to the failed material, i.e. $\omega = \omega_{cr}$. Fracture begins under the impactor (Fig. 5a) and then propagates towards the center forming a disc-shaped crack (Fig. 5b). In Fig. 5b, the dashed line shows the location of the spalling-fracture line of the one–

dimensional analogue of the problem. In this case according to performed calculations the minimum time of the spall surface formation is $3{,}98 t_0$. Peculiarities of the fracture connected with two-dimensionality of the process are qualitatively in an agreement with experiment (see, e.g., [Orlenko, 2004]).

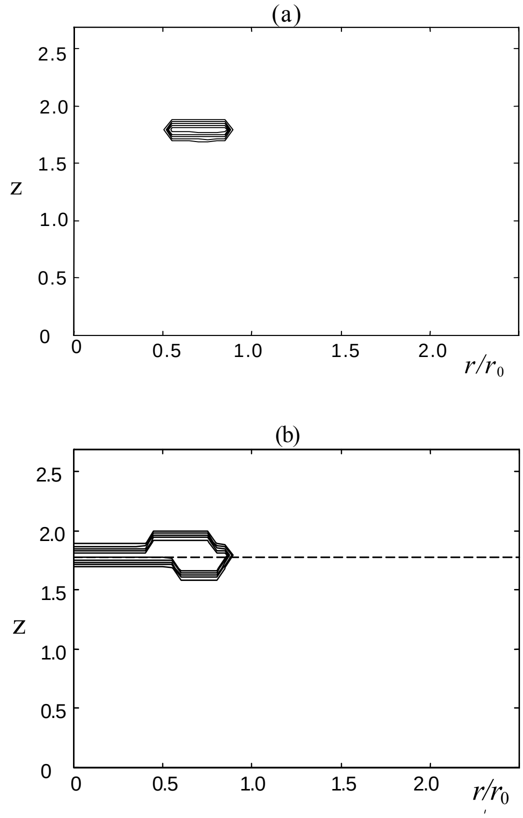

Fig. 5 a, b. Damage regions of the plate for $t = 3{,}92 t_0$ (a) and $3{,}98 t_0$ (b). The impact is performed along the lower surface $0<r/r_0<1$.

4.2.2 Impact with a velocity of 2 km/sec.

The problem of the normal-impact fracture of a plane plate produced by a cylindrical impactor with a velocity of u_0 = 2 km/sec is solved numerically. The radius r_0 of the impactor is equal to its thickness, and the impactor mass is equal to 9 g. The material of the impactor is steel and the material of the plate is aluminum. During the numerical solution the thickness Z_0 of the plate varied within the limits of $0< Z_0/r_0 <1$. The waves evolution is described by Eqs. (1), (4) for i =1,2. In the case under consideration we have $\varepsilon = 0.722$ and $\varepsilon_\Delta = 1.0$ in Eqs. (1).

In fact the fracture of the plate occurs during characteristic time which is equal to two passage times of a sound wave along the plate thickness. This simplifies considerably the problem of finding the numerical solution. The fracture criterion was chosen in the form of the modified impulse criterion [Morozov, Petrov and Utkin, 1990].

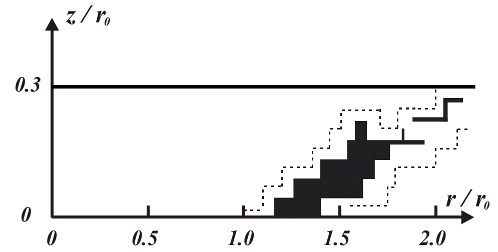

Fig. 6 Region of fractures of the plate at Z_0/r_0 = 0.3

Fig. 6 shows the calculation results for the two-dimensional fracture of plate at Z_0/r_0 = 0.3 in coordinates (r/r_0, z/r_0). Figure shows right-hand side of the plate cross-section (the plate is loaded from bottom at $0<r/r_0<1$, z = 0). The painted over area and continuous lines correspond to full fracture of the plate. Thus, two kinds of fragments are present in figure. This is a central plug and a ring zone of strongly failed material ("dust"). The calculation results allow one to determine the mass and the average starting velocity of these fragments. The plug will go ahead with higher velocity, the debris of the impactor and the dust will fly behind the plug. It is worth noting that the similar fracturing and fragmentation are observed in experiments [Finnegan, Schulz and Heimdahl, 1990; Vishnyakov, 1991].

Evidently the proposed approximate approach yields adequate results for sufficiently large velocity of impact up to 2 km/sec. It is unexpected enough because the parameter ε, which should be small in Eqs.(1), is value of order O(1) in this case.

5 Conclusion

The wave processes in shock-loaded elastoplastic solids was modeled on the basis of the approximate equations (1) proposed in [Myagkov, 1994; Myagkov 2003a]. The base assumptions at this approximate approach are as follows. The waves, in which the average stress amplitude divide by a bulk modulus is small (~ε) but sufficiently large to involve both elastic and plastic regions, are considered. The stress deviator is considered as the small value of next order relative to the average stress. The two-dimensionality is taken into account in so-called quasi-plane approximation, i.e. we suppose that the characteristic length of the wave is small in comparison with a linear dimension of the loading region.

The approximate equations (1) are obtained by making use of a modification of the method of multiple scales. Difference from standard procedure consists in the conditions (2) ensuring uniform validity of the equations (1). Alternative to the conditions (2) is representation of the phase variables in the form of (6) containing the corrections of order ε and ν. These corrections allow one to describe the interaction of two oppositely facing longitudinal waves due elastoplastic kinetics (8) and hydrodynamic nonlinearity (7). The physical meaning of the phase functions may be defined by means of Lagrangian phase velocity (see equation (10)).

We considered one- and two-dimensional problems of pulse deformation and fracture of elastoplastic solids on the basis of the approximate equations (1) and we showed a possibility to include different constitutive equations corresponding to different models of elastoplastic media: the elastoplastic model of the Prandtl-Reuss type, the dislocation model, and the model of the nonlinear viscoelasticity. Main purpose was to check the validity of these equations for the problems of this kind including both propagation and interaction waves in shock-loaded elastoplastic material.

References

Bogolyubov, N. N., Mitropol'skij, Yu. A. (1974). *Asimptoticheskie Metody v Teorii Nelinejnykh Kolebanij*. Nauka. Moscow (in Russian).

Engelbrecht, Y.K., Nigul U.K. (1981). *Non-linear Waves of Deformation.* Nauka, Moscow

Finnegan S.A., Schulz J.C., Heimdahl, O.E. (1990). Spatial fragment mass and velocity distributions for ordnance and ultra-ordnance speed impacts. *Int. J. Impact Engng.* **10**, 159-170

Fowles, R., Williams, R.F. (1970). Plane stress wave propagation in solids. *J. Appl. Phys*., **41**, pp. 360-363.

Gilman, J. J. (1968). Dislocation dynamics and the response of materials to impact. *Appl. Mech. Rev*., **21**, pp. 767-783.

Hill, R. (1985). *The Mathematical Theory of Plasticity*. Clarendon Press, Oxford.

Kanel, G.I., Razorenov, S.V., Fortov, V.E. (2004). *Shock-Wave Phenomena and the Properties of Condensed Matter*. Springer-Verlag, New York.

Leibovich, S., Seebass, A.R. (1974). Examples of dissipative and dispersive systems leading to the Burgers and the Korteweg-de Vriea equations. In *Nonlinear waves (ed. Leibovich, S. and Seebass, A.R.).* Cornel University Press, Ithaca.

Morozov N., Petrov Y., Utkin A. (1990). *Doklady Akademii nauk SSSR*, **313**, №2, pp. 276 (in Russian).

Morris, C. E. (ed) (1982). *Los Alamos Shock Wave Profile Data.* Univ. of California press, Berkeley.

Myagkov, N.N. (1994). Nonlinear waves in shock-loaded condensed matter. *J. Phys. D: Appl. Phys*., **27**, pp. 1678-1686;

Myagkov, N.N. (2003a). Asymptotic modeling of nonlinear wave processes in shock-loaded elastic-plastic solids. *J. Appl. Mech. Techn. Phys*., **44** (2), pp. 249-254.

Myagkov, N. N. (2003b). Propagation of quasi-acoustic pulses in an elastoplastic medium. *J. Appl. Mech. Techn. Phys*., **44** (1), pp. 122-128.

Nayfeh, A. H. (1973). *Perturbation Methods*. Wiley. New York.

Oikawa, M. Yajima, N. (1974). Generalization of the reductive Perturbation Method to multi-wave systems. *Suppl. Progr. Theor. Phys.*, **55**, pp. 36-51.

Orlenko, L.P. (ed) (2004). *Physics of Explosion, Vol. 2*. Fizmatlit, Moscow (in Russian).

Petviashvili, V. I., Pokhotelov, O. A. (1992). *Solitary Waves in Plasmas and in the Atmosphere.* Gordon and Breach, London.

Rudenko, O.V., Soluyan, S.I. (1977). *Theoretical Foundations of Nonlinear Acoustics*. Plenum, New York.

Seaman L., Curran D.R., Shockey, D. J. (1976). Computational models for ductle and brittle fracture. *Appl. Phys*., **47**, pp. 4814-4826.

Taniuti, T., Wei, C.C. (1968). Reductive perturbation method in nonlinear wave propagation. *J. Phys. Soc. Japan*, **24**, pp. 941-946.

Tatsumi. T., Tokunaga H.(1974). One dimensional shock turbulence in a compressible fluid. *J. Fluid Mech*., **65**, pp. 581-601.

Vishnyakov, A. N. (1991). Experimental ignition time determination for a particle formed during metal disintegration in oxygen. *Combustion, Explosion and Shock Waves*, **27**, 522-527.

Wallace D.C. (1980a). Irreversible thermodynamics of flow in solids. *Phys. Rev*., **B 22**, pp. 1477-1486.

Wallace D.C. (1980b). Flow process of weak shocks in solids. *Phys. Rev*., **B 22**, pp. 1487-1494.

Zabolotskaya, E.A. (1987). Nonlinear propagation of sound beam in solid. In *Proceedings of IUPAP-IUTAM symposium on nonlinear acoustics, part 1.* Novosibirsk, USSR, August 24-28. pp. 355-359.

Zel'dovich, Y.B., Raizer, Y.P. (1966). *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*. Academic, New York.