

# PRECISE CONTROL OF COMPLEX LAGRANGIAN SYSTEMS

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(Extended abstract)

**Abstract:** This paper deals with the problem of decomposition and precise control by complex objects. Decomposition is based on concepts of an object technical controllability and model reference adaptive control. Precise accuracy is attained with the help of adaptive and optimal control. A space robotic module is considered as the example of a complex object. Computer simulation demonstrates good results.

**Key words:** Complex Object, Lagrangian System, Technical Controllability, Decomposition, Adaptive Control, Lyapunov Function, Simulation

## 1. INTRODUCTION

As a complex we assume an object with some interconnected subsystems [1, 2]. In the wake of [3], by the Lagrangian systems are meant those where the controlled plant obeys a mathematical model (MM) of the form of Lagrange equations. A MM of such an object is usually multiconnected nonlinear and nonstationary one. Synthesis of control algorithms for such an object is not a simple problem. All the more it is difficult for precise control.

Qualitatively under precise control we mean the situation when the motion of any subsystem and the system in the whole are coincided with prescribed motions with prescribed accuracies.

Usual method for such an object control is decomposition [1, 2]. In this paper we assume that an object in the whole could be represented as a set of interconnected subsystems. For every subsystem a component of interconnections is selected and compensated on the base of adaptive or optimal control [4-6]. For this goal we use two approaches to the decomposition. The first one is based on the decomposition of an object MM. In this case the object has to satisfy to the property of technical controllability [7]. The second approach is based on the decomposition

of a system control MM. In this case special adaptive control algorithms are derived [8].

In this paper we assume that different subsystems could have actuators of two different nature: dc motors [9] or jet engines [10].

## 2. PROBLEM STATEMENT

We consider controlled plants with the MM in the form of Lagrange equation

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = Q, \quad (1)$$

where  $q = (q_i)$  and  $Q = (Q_i)$  ( $i = \overline{1, n}$ ) are vectors of generalized coordinates and forces;  $T = \frac{1}{2} \dot{q}^T A(q) \dot{q}$  is the kinetic energy;  $A(q) = (a_{ij}(q))$  ( $i, j = \overline{1, n}$ ) is a symmetrical positive definite matrix ( $T$  denotes transposition).

By performing differentiation in (1), we proceed to the equation [7]

$$A(q)\ddot{q} + \sum_{s=1}^n [\dot{q}^T D_s(q)\dot{q}] e_s = S(q)M,$$

where  $M = (M_i)$  ( $i = \overline{1, n}$ ) is a vector of control actions to the plant from a controller.

We assume that during the object operating:

- matrices  $A(q)$ ,  $D_s(q)$ ,  $S(q)$  ( $s = \overline{1, n}$ ) are known;
- vectors  $q = q(t)$ ,  $\dot{q} = \dot{q}(t)$  are measurable.

As actuators we consider dc motors [9] and jet engines [10].

For every  $q_i$  ( $i = \overline{1, n}$ ) there exists a function  $q_i^0(t)$  and an equation

$$\ddot{q}_i + d_i \dot{q}_i + k_i q_i = k_i q_i^0(t), \quad (2)$$

where the function  $q_i^0(t)$  and the numbers  $k_i > 0$ ,  $d_i > 0$  are prescribed in advance.

### The problem:

It is necessary to discover control algorithms

$$M = M(t, q, \dot{q})$$

that guarantees the motion (2).

Qualitatively the main idea of this paper is clear. Concrete material of the paper will include the next sections:

3. DECOMPOSITION OF AN OBJECT MATHEMATICAL MODEL
4. MATHEMATICAL MODEL OF A SUBSYSTEM WITH DC MOTOR ACTUATORS
5. MATHEMATICAL MODEL OF A SUBSYSTEM WITH JET ENGINE ACTUATORS
6. ADAPTIVE PROGRAMMED CONTROL OF AN OBJECT SUBSYSTEM WITH DC MOTOR ACTUATORS
7. MODEL REFERENCE ADAPTIVE CONTROL FOR SUBSYSTEM WITH DC MOTOR ACTUATORS
8. DECOMPOSITION OF THE OBJECT SUBSYSTEM WITH JET ENGINE ACTUATORS
9. PRECISE CONTROL OF AN OBJECT SUBSYSTEM WITH JET ENGINE ACTUATORS
10. SIMULATION RESULTS

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