

# NONLINEAR ROBUST ADAPTIVE EKF FOR IDENTIFICATION OF EMAs PARAMETERS IN THE PRESENCE OF SENSOR FAULTS

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**Abstract.** An Extended Kalman filter (EKF) is designed to estimate the parameters of the electro-mechanical actuator (EMA). If the measurements are not reliable because of any kind of malfunction in the estimation system, the filter gives inaccurate results and diverges by time. For the presence of measurement faults, a Nonlinear Robust Adaptive EKF with the filter gain correction based on the evaluation of the posterior probability of the normal operation of system, given for current measurement is proposed. This probability is proposed to calculate via the posterior probability density of the normalized innovation sequence at the current estimation step. In the proposed filtration algorithm, the filter gain is corrected by multiplying with the mentioned posterior probability, which plays the role of the weight coefficients to the innovation vector. As a result, faults in the estimation system are corrected by the system, without affecting the good estimation behavior. The developed Nonlinear Robust Adaptive EKF is applied for the parameter identification process of an EMA. The performance of the proposed filter is tested for the different types of measurement faults; instantaneous abnormal measurements, continuous bias at measurements, measurement noise increment and fault of zero output.

**Keywords:** Electro-Mechanical actuator, Extended Kalman filter, Robust Kalman Filtering, Measurement Faults, Parameter Identification.

## 1. Introduction

Electro-mechanical actuators (EMAs) are widely utilized in marine and aerospace applications. Actuators are safety-critical components of motion control systems and an actuator failure can lead to serious consequences. However, EMAs still lack of the knowledge for the other actuator types, particularly with regard to fault detection and fault tolerant control.

For EMAs monitoring and control, parameter estimation based approach can be used [1,2]. When there are sufficient measurements, necessary EMA parameters can be estimated via a Kalman Filter. That is a desired

procedure since it is important to know the parameters of actuator precisely. When these parameters of the actuator are obtained without any problem, actuator control can be achieved successfully. On the other hand, that is a dependent process to the accuracy of the measurements. If the measurements are not reliable because of any kind of malfunction in the estimation system, the filter gives inaccurate results and diverges by time. Since achieving fault tolerance in the design of a control system is important, filter should be built robustly in order to overrun such problems.

The Kalman filter approach to the state estimation is quite sensitive to any measurement malfunctions (abnormal measurements, sudden shifts in the measurement channel, and other difficulties such as decrease of instrument accuracy, an increase of background noise, etc.). If the condition of the operation of the measurement system does not correspond to the models, used in the synthesis of the filter, then these changes resulting from some possible failures at the measurement channels significantly decrease the effectiveness of the estimation systems. When dealing with the measurement faults in previous estimation steps, rather than the current one, Adaptive Kalman Filters can be used so as to recover the possible malfunctions.

KF can be made adaptive and hence insensitive to the priori measurements or system uncertainties by using various different techniques. The basic approaches to the adaptive Kalman filtering problem are Multiple-model-based adaptive estimation (MMAE) [3,4], Innovation-based adaptive estimation (IAE) [5,6], and Residual-based adaptive estimation (RAE) [7]. While in the first approach bank of Kalman filters run in parallel under different models for the filter's statistical information, in the rest the adaptation is done directly to the covariance matrices of the measurement and/or system noises based on the changes in the innovation or residual sequences.

In methods described in [3,4], the faults are assumed to be known, and the Kalman filters are designed for the known sensor/actuator faults. As the MMAE approach requires several parallel Kalman filters, and the faults should be known, it can be used in limited applications. Estimation of the covariance matrices by IAE and RAE requires the usage of the innovation vectors or residual

vectors of  $m$  epoch. This increases the storage burden and presents the determination of the width of the moving window  $m$  as another problem. Furthermore, IAE and RAE estimators require that the number, type and distribution of the measurements for all epochs within a window should be consistent. If they do not, the covariance matrices of the measurement noises cannot be estimated based on the innovation or the residual vectors.

Another concept is to scale noise covariance matrix by multiplying it with a time dependent variable. One of the methods for constructing such algorithm is to use an adaptive factor as a multiplier to the process or measurement noise covariance matrices [8,9]. This algorithm, which may be named as Adaptive Fading Kalman Filter (AFKF), can be both used when the information about the dynamic process or the priori measurements is absent. However, when the point at issue is the current measurements, another technique to scale measurement noise covariance matrix and make filter robust (insensitive to measurement faults) should be proposed. Therefore, if there is a malfunction in the measurement system, Robust Kalman Filter (RKF) algorithm can be utilized and by the use of a measurement noise scale factor (MNSF) as a multiplier on the measurement noise covariance matrix insensitiveness of the filter to the current measurement faults can be satisfied. As a consequence, via a correction applied to the filter gain, good estimation behaviour of the filter will be secured without being affected from faulty measurements [10].

A new Nonlinear Robust Adaptive EKF with the filter gain correction at the each estimation step based on the evaluation of the posterior probability of the normal operation of system, given for current measurement is presented in this study. This approach does not require a priori statistical characteristics of the faults and knowledge of historical information. Furthermore the computational burden is not heavy. The EMAs parameter identification based on the developed Nonlinear Robust Adaptive EKF is performed.

## 2. EKF for EMAs Parameter Estimation

Based on the measurements of the input and output coordinates of the electro-mechanical actuator, estimation of the actuator states and parameters will be performed. The electro-mechanical actuator is stationary and defined by the differential equation below [11]:

$$T y'(t) + y(t) = Kx(t) \quad (1)$$

Eq.(1) is transformed to discrete form to calculate the predicted values of the  $y$  coordinate,  $T$  and  $K$  parameters:

$$y(k) = \left(1 - \frac{\Delta t}{T(k-1)}\right) y(k-1) + \frac{K(k-1)\Delta t}{T(k-1)} x(k-1) \quad (2)$$

The following condition is valid since the electro-mechanical actuator is stationary.

$$K(k) = K(k-1), T(k) = T(k-1) \quad (3)$$

When the measurement equation is concerned, it can be expressed as:

$$z(k) = y(k) + \delta(k), \quad \delta(k) \sim N(0, \sigma^2) \quad (4)$$

where  $\delta(k)$  is the normal Gaussian noise with zero average and variance  $\sigma^2$ . The predicted values of  $x(k), y(k), K(k), T(k)$  are calculated according to the a priori data in  $(k-1)^{\text{th}}$  step.

The extended vector  $U^T(k) = [y(k) \quad K(k) \quad T(k)]$  is formed and by using stationary conditions, expanding Eq.(2) into Taylor series we can write:

$$U(k) = F_U(k-1)U(k-1) + F_G(k-1)x(k-1) \quad (5)$$

where

$$F_U(k-1) = \begin{bmatrix} 1 - \frac{\Delta t}{\hat{T}(k-1)} & \frac{\Delta t \hat{x}(k-1)}{\hat{T}(k-1)} & \Delta t \hat{y}(k-1) - K(k-1)\Delta t \hat{x}(k-1) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_G^T(k-1) = \begin{bmatrix} \frac{\hat{K}(k-1)\Delta t}{\hat{T}(k-1)} & 0 & 0 \end{bmatrix}$$

The estimate vector  $U$  can be evaluated by means of the Kalman filter as:

Estimation equation:

$$\hat{U}(k) = f[\hat{U}(k-1)] + K(k)\Delta(k) \quad (6)$$

Innovation:

$$\Delta(k) = z(k) - H(k)f[\hat{U}(k-1)] \quad (7)$$

Gain matrix of the filter:

$$K(k) = M(k)H^T(k) \left[ D_y(k) + H(k)M(k)H^T(k) \right]^{-1} \quad (8)$$

Covariance matrix of the estimation error:

$$P(k) = [I - K(k)H(k)]M(k) \quad (9)$$

Covariance matrix of the extrapolation error:

$$M(k) = F_U(k)P(k-1)F_U^T(k) + F_G(k)D_x(k)F_G^T(k) \quad (10)$$

where,  $H = [1 \quad 0 \quad 0]$  is the measurement vector,  $D_y(k) = \sigma^2$  is the variance of the measurement error of the actuator output coordinate,  $D_x(k)$  is the variance of the input signal,  $I$  is the identity matrix.

### 3. Robust Adaptive EKF for Parameter Identification of EMA

In normal operation conditions, where any kind of measurement malfunction is not observed, EKF (6)-(10) gives sufficiently good estimation results. However, when the measurements are faulty because of malfunctions in the estimation system such as abnormal measurements, sudden shifts or step-like changes in the measurement channel etc. filter estimation outputs become inaccurate. Therefore, a robust adaptive EKF algorithm, which brings the fault tolerance to the filter and secures accurate estimation results in case of faulty measurements without affecting the remaining good estimation characteristic, should be introduced.

In case of the abrupt faults in the estimation system such as computer malfunctions, abnormal measurements, step-like changes in the measurement channel, measurement noise increment, fault of zero output etc., the following suboptimal filter algorithm is proposed:

$$\hat{U}(k) = f[\hat{U}(k-1)] + p(1/k)K(k)\Delta(k) \quad (11)$$

Here,  $p(1/k)$ , is the posterior probability of the normal operation of the estimation system, given for measurement result at the  $k^{\text{th}}$  time step. The other filter parameters are the same as the expressions (6)-(10). When  $p(1/k)=1$ , this filter will be exactly same with the EKF, but when  $p(1/k)=0$  it disregards the new measurements, acting as an extrapolator.

During normal operation of the measurement channel,  $\tilde{\Delta}(k)$  normalized innovation sequence of EKF

$$\tilde{\Delta}(k) = [H(k)M(k)H^T + D_y(k)]^{-1/2} \Delta(k) \quad (12)$$

will satisfy the normal distribution  $N(0,1)$  [11].

Two hypotheses are assumed:

$$\begin{aligned} H_0 : \tilde{\Delta}(k) \in \Omega, \text{ fault free} \\ H_1 : \tilde{\Delta}(k) \notin \Omega, \text{ with fault} \end{aligned} \quad (13)$$

where  $\Omega$  is the allowable domain (confidence interval in the one dimensional case) for the normalized innovation sequence  $\tilde{\Delta}(k)$ .

The a prior probability of the normal operation of system is calculated by the formula

$$p_0 = \int_{\Omega} f(\tilde{\Delta})d\tilde{\Delta}, \quad (14)$$

where  $f(\tilde{\Delta})$  is the probability density of the parameter  $\tilde{\Delta}$ . In the investigated case the prior probability density of the normalized innovation sequence  $\tilde{\Delta}$  can be written as

$$f(\tilde{\Delta}) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{\tilde{\Delta}^2}{2}\right\}. \quad (15)$$

Than the prior probability of normal operation of system is calculated by the formula

$$p_0 = \int_{-\tilde{\Delta}_{th}}^{\tilde{\Delta}_{th}} f(\tilde{\Delta})d\tilde{\Delta}, \quad (16)$$

where  $\pm\tilde{\Delta}_{th}$  are the threshold values (confidence interval). It is clear that the inequality

$$p(-\tilde{\Delta}_{th} < \tilde{\Delta} \leq \tilde{\Delta}_{th}) = p_0$$

is true. The threshold value  $\tilde{\Delta}_{th}$  for given  $p_0$  is determined from the table of probability density of standardized normal distribution.

The posterior probability density of the parameter  $\tilde{\Delta}(k)$  at the estimation step  $k$  is

$$f[\tilde{\Delta}(k)/z(k)] = \frac{1}{\sqrt{2\pi}\sigma_{\tilde{\Delta}}(k)} \exp\left\{-\frac{[\tilde{\Delta}(k) - \hat{\tilde{\Delta}}(k)]^2}{2\sigma_{\tilde{\Delta}}^2(k)}\right\} \quad (17)$$

It is proposed to calculate the posterior probability of the normal operation of system  $p(1/k)$ , given for  $z(k)$  measurement via the posterior probability density of the normalized innovation sequence  $\tilde{\Delta}$  by the following formula

$$p(1/k) = \int_{-\tilde{\Delta}_{th}}^{\tilde{\Delta}_{th}} f[\tilde{\Delta}(k)/z(k)]d\tilde{\Delta} \quad (18)$$

The calculation of probabilities  $p_0$  and  $p(1/k)$  is explained in the Figure 1.

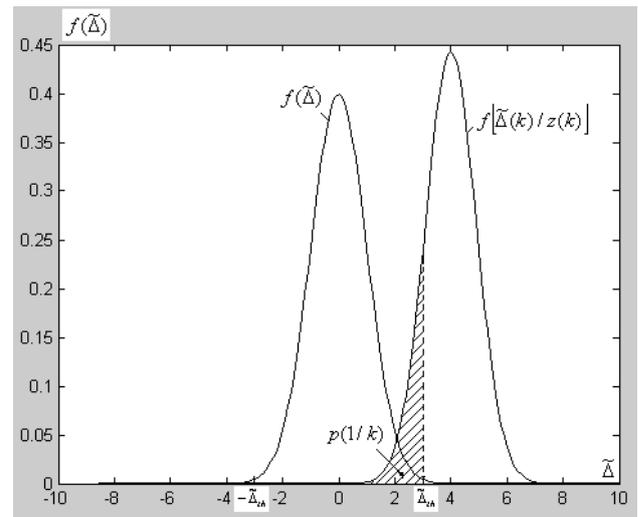


Figure 1. Graphs for calculation of the posterior probability  $p(1/k)$

Consequently the robust adaptive filtration algorithm with the filter gain matrix correction can be presented in the form below

$$\hat{U}(k) = f[\hat{U}(k-1)] + p(1/k)K(k)\Delta(k)$$

$$p(1/k) = \frac{1}{\sqrt{2\pi}\sigma_{\tilde{\Delta}}(k)} \int_{-\tilde{\Delta}_n}^{\tilde{\Delta}_n} \exp\left\{-\frac{[\tilde{\Delta}(k) - \hat{\Delta}(k)]^2}{2\sigma_{\tilde{\Delta}}^2(k)}\right\} d\tilde{\Delta}$$

$$\Delta(k) = z(k) - H(k)f[\hat{U}(k-1)] \quad (19)$$

$$\tilde{\Delta}(k) = [H(k)M(k)H^T + D_y(k)]^{-1/2} \Delta(k)$$

$$K(k) = M(k)H^T(k)[D_y(k) + H(k)M(k)H^T(k)]^{-1}$$

$$P(k) = [I - p(1/k)K(k)H(k)]M(k)$$

$$M(k) = F_v(k)P(k-1)F_v^T(k) + F_g(k)D_x(k)F_g^T(k)$$

where  $p(1/k)$  is the posterior probability of the normal operation of system, given for measurement result at the estimation step  $k$ . The rest parameters in the above filter correspond to the same meaning as in EKF (6)-(10).

If the fault probability changes, then the gain coefficient of the filter is automatically changed. Using (18) in Kalman filter (19), gives the filter ability to adapt to change in operating conditions. If there is a fault in the measurement channel, the value  $p(1/k)$  decreases, consequently, the gain coefficient of filter decreases too. As a result, the correction effect of innovation sequence decreases. Differing from EKF, the current measurements have a considerable weight in the proposed algorithm, since the elements of gain matrix  $K(k)$  are corrected on each measurement result.

The correction value  $p(1/k)$  depends on the innovation value  $\Delta(k) = z(k) - H(k)f[\hat{U}(k-1)]$ ; when there is no divergence in the innovation process (the system operates normally), then  $z(k) \approx H(k)f[\hat{U}(k-1)]$ ,  $p(1/k) \rightarrow 1$  and its effect is insignificant. In this case the filter operates as the EKF. If there is a divergence in the innovation (the system operates faulty), then the correction value  $p(1/k) \rightarrow 0$  and decreases the correction of prediction considerably. However, in all cases it becomes nonlinear filter, because of the dependence of  $p(1/k)$  and consequently

$$K_A(k) = p(1/k)K(k) \quad (20)$$

to the current measurement  $z(k)$ , where  $K_A(k)$  is the gain matrix of the Nonlinear Robust Adaptive EKF (19).

Let us remark that if the gain matrix of the proposed Kalman filter changes according to (20), the accuracy of estimation value in the case of normal operation of the system in comparison to the theoretical accuracy decreases a little, because of regarded  $p(1/k)$ .

Therefore the proposed Nonlinear Robust Adaptive EKF in this study is not optimal.

Therefore, robust adaptive algorithm is operated only when the measurements are faulty and in all other cases procedure is run optimally with regular EKF (6)-(10). Process is controlled by the use of a kind of statistical information. To detect failures a statistical function may be defined as,

$$\beta(k) = \Delta^T(k)[D_y(k) + H(k)M(k)H^T(k)]^{-1} \Delta(k). \quad (21)$$

This statistical function has  $\chi^2$  distribution with  $n$  degree of freedom where  $n$  is the dimension of the state vector. If the level of significance,  $\alpha$ , is selected as,

$$P\{\chi^2 > \chi_{\alpha,n}^2\} = \alpha; \quad 0 < \alpha < 1, \quad (22)$$

the threshold value,  $\chi_{\alpha,n}^2$  can be found. Hence, when the hypothesis  $H_1$  is true, the statistical value of  $\beta(k)$  will be greater than the threshold value  $\chi_{\alpha,n}^2$ , i.e.:

$$H_0: \beta(k) \leq \chi_{\alpha,n}^2, \forall k$$

$$H_1: \beta(k) > \chi_{\alpha,n}^2, \exists k \quad (23)$$

#### 4. Simulations

During simulations, for testing proposed Nonlinear Robust Adaptive EKF algorithm, different kinds of measurement malfunction scenarios are taken into consideration; instantaneous abnormal measurements, continuous bias, measurement noise increment, fault of zero output. Besides, in case of measurement faults, the simulations are also achieved for conventional EKF so as to compare results with RAEKF algorithms and understand efficiency of the robust EKF in a better way.

The initial values and data below are used in the simulation:  $T(0) = 2$ ,  $K(0) = 1$ ,  $y(0) = 0.001$ , initial values of EKF:  $\hat{T}(0)=3$ ,  $\hat{K}(0)=1.5$ ,  $\hat{y}(0)=0.01$ , the initial value of the covariance matrix of the estimation error: the variance of input signal, and the variance of measurement error are assumed to be  $D_x = 0.0001$  and  $D_y = 0.001$  respectively. The input signal is  $x = 0.5\sin(t)$ . The actual values of  $T$ ,  $K$ , and  $y$  that form the mathematical model of the EMA are calculated for 400 steps with iteration time 4 s. The simulation of the measurements is carried out for 400 steps by adding noise of variance 0.001 to the output of the actuator  $y$ . The actuator parameters  $y(k), K(k), T(k)$  are estimated using the conventional EKF and the proposed RAEKF.

##### i) Continuous Bias at Measurements

Continuous bias term is formed by adding a constant term to the measurement in between 2 and 2.5 seconds. As Figures 2-3 shows, in this case Nonlinear RAEKF gives sufficiently good estimation outputs.

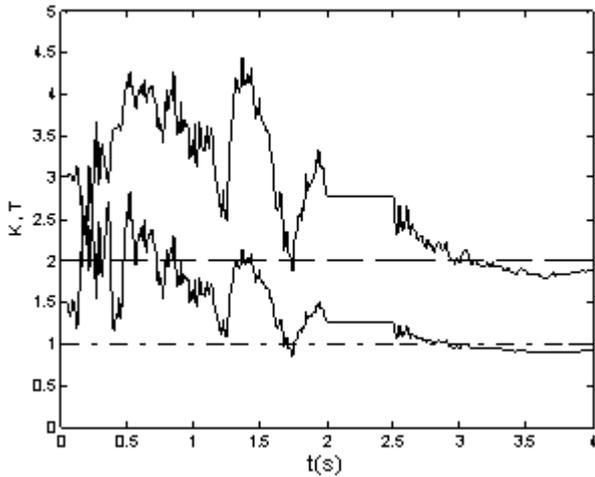


Fig.2. Actual (dashed line) and estimated (solid line) values of parameter T; Actual (dash and dotted line) and estimated (solid line) values of parameter K in case of continuous bias at measurements (RAEKF is used)

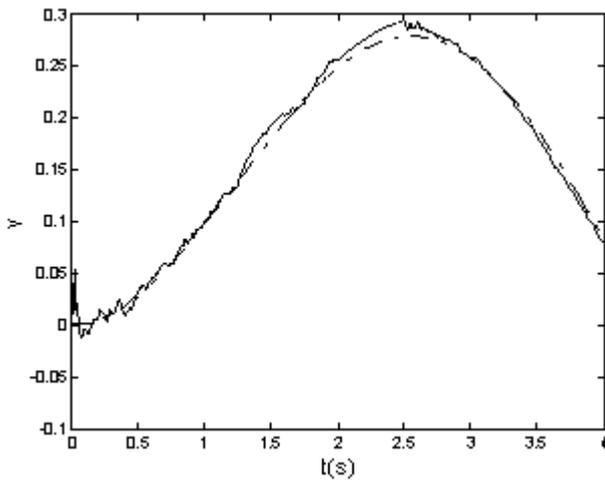


Fig. 3. Actual (dashed line) and estimated (solid line) values of parameter y in case of continuous bias at measurements (RAEKF is used)

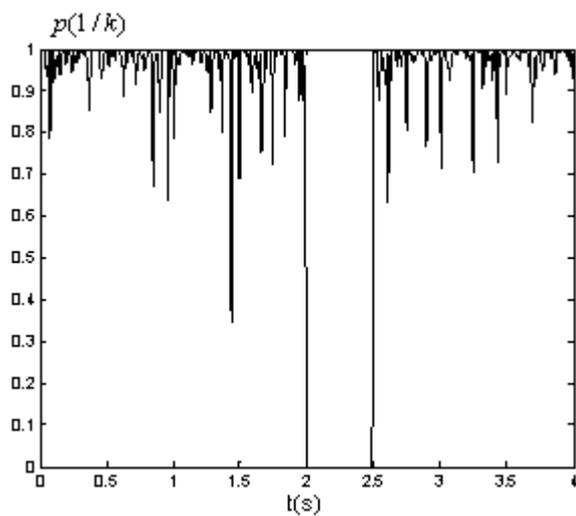


Fig.4. Behavior of the posterior probability values  $p(1/k)$  in case of continuous bias at measurements

The posterior probability values of normal operation of the estimation system  $p(1/k)$ , given for the measurement result at the  $k^{th}$  time step are presented in Fig. 4. As it is seen from the graphs, in the failure condition case (between 2 and 2.5 seconds) probability  $p(1/k)$  close to zero; as a result, the filter become insensitive to the introduced continuous bias. Part of the EKF simulation results in case of continuous bias at the measurements is shown in Appendix A (Figs. A.1, A.2). The presented figures show that, in the presence of continuous bias at the measurements, the estimation results of conventional EKF are not reliable.

#### ii) Measurement Noise Increment

In that third measurement malfunction scenario, measurement fault is characterized by multiplying the variance of the measurement noise with a constant term in between 2 and 2.5 seconds. Figures 5-6 show that Nonlinear Robust Adaptive EKF algorithm achieves estimation of the states accurately.

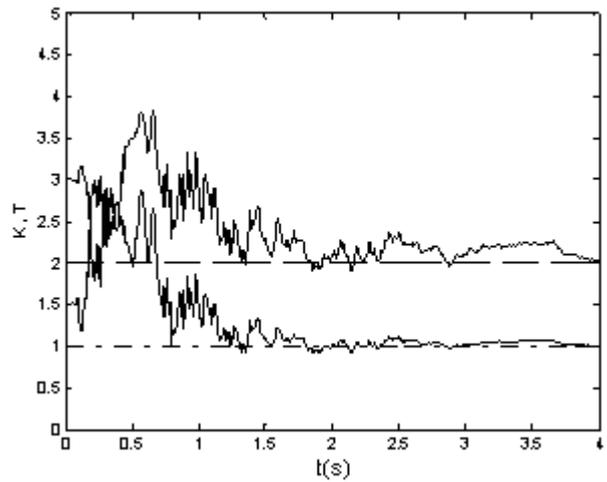


Fig.5. Actual (dashed line) and estimated (solid line) values of parameter T; Actual (dash and dotted line) and estimated (solid line) values of parameter K in case of measurement noise increment (RAEKF is used)

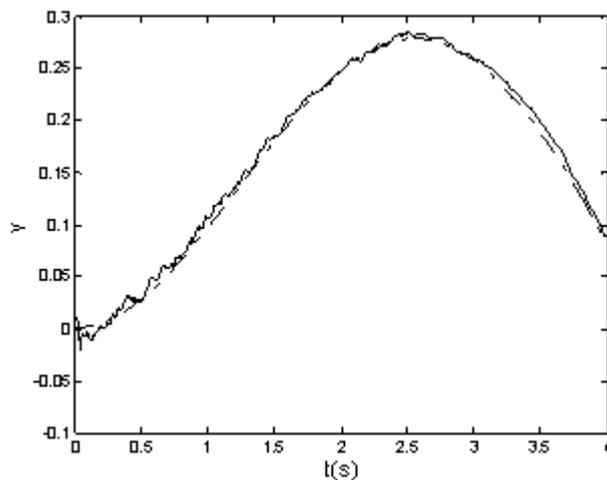


Fig. 6. Actual (dashed line) and estimated (solid line) values of parameter y in case of measurement noise increment (RAEKF is used)

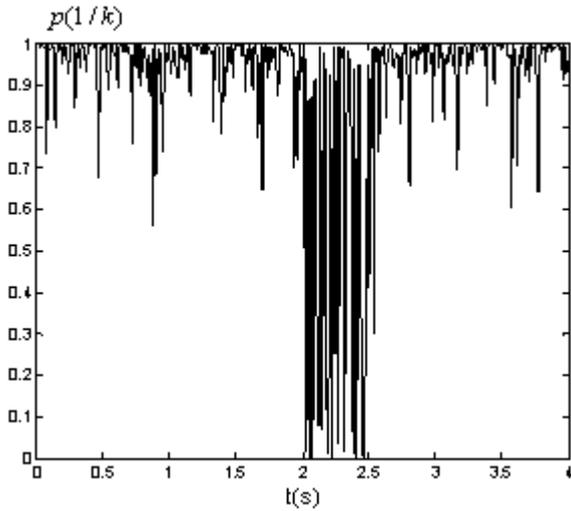


Fig.7. Behavior of the posterior probability values  $p(1/k)$  in case of measurement noise increment

The posterior probability values  $p(1/k)$ , corresponding to this case are presented in Fig. 7. As it is seen from graphs presented in Fig.7, in the measurement noise increment case (between 2 and 2.5 seconds) probability  $p(1/k)$  close to zero; as a result, the filter become insensitive to the introduced failure. The EKF simulation results which correspond to this malfunction scenario are considerably bad as it is seen from graphs presented in the Figs. A.3 and A.4 (Appendix A).

### iii) Instantaneous Abnormal Measurements

The whole algorithm is exactly the same with the one for the conventional EKF conditions, but errors are implemented into 100<sup>th</sup>, 200<sup>th</sup>, 300<sup>th</sup>, and 400<sup>th</sup> iterations. These abrupt errors, which represent the failure condition at the measurement channel, are formed by multiplying the variance of the measurement noise with a constant term. In the simulation of the EMA, system equipped with the EKF and Nonlinear Robust Adaptive EKF. The resulting data are given in Figures 8-9.

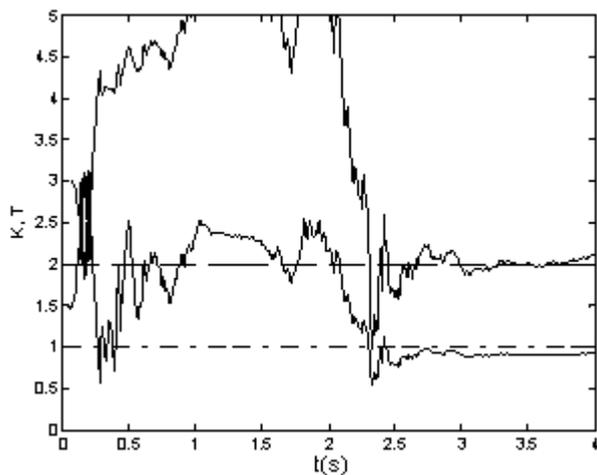


Fig.8. Actual (dashed line) and estimated (solid line) values of parameter T; Actual (dash and dotted line) and estimated (solid line) values of parameter K in case of instantaneous abnormal measurements (RAEKF is used)

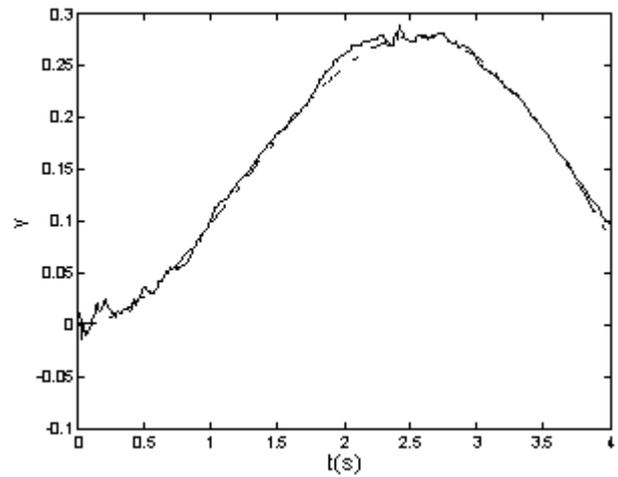


Fig. 9. Actual (dashed line) and estimated (solid line) values of parameter  $y$  in case of instantaneous abnormal measurements (RAEKF is used)

Figures 8 and 9 show that Nonlinear Robust Adaptive EKF algorithm achieves estimation of the states accurately. The EKF simulation results which correspond to this malfunction scenario are considerably bad as it is seen from graphs presented in the Fig. A.5 and A.6 (Appendix A).

### iv) Fault of zero output

In that fourth measurement malfunction scenario, it is assumed the EMA related sensor gives "0" as the output. Fault is simply simulated by taking output measurement as "0" for the filter algorithm in between 2 and 2.5 seconds. The simulation results for this case are given in Figures 10 and 11.

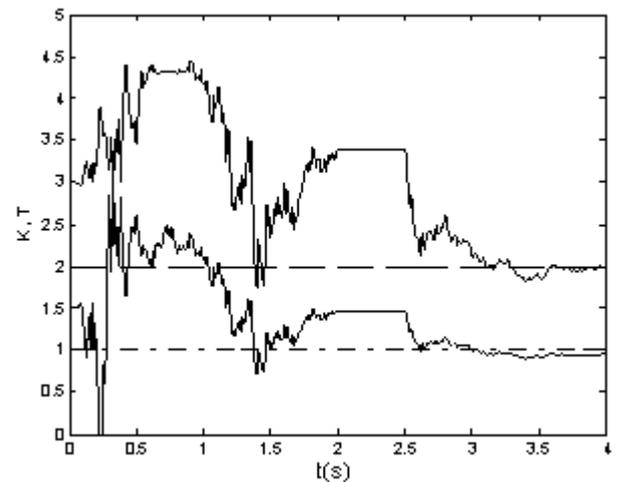


Fig.10. Actual (dashed line) and estimated (solid line) values of parameter T; Actual (dash and dotted line) and estimated (solid line) values of parameter K in case of fault of zero output (RAEKF is used)

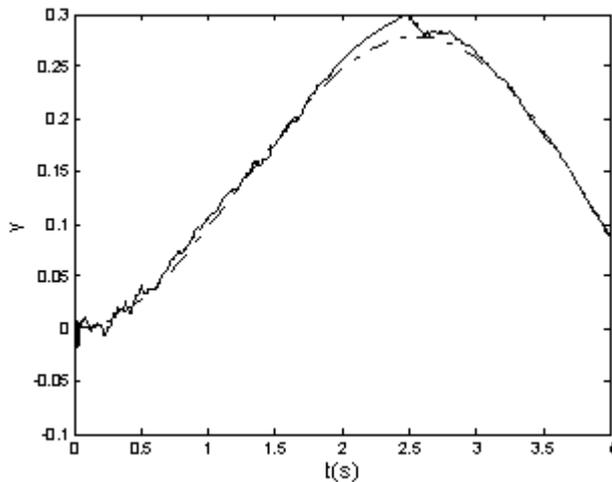


Fig. 11. Actual (dashed line) and estimated (solid line) values of parameter  $y$  in case of fault of zero output (RAEKF is used)

As it is seen, the results obtained by Nonlinear Robust Adaptive EKF are sufficiently good in case of fault of zero output of EMA. The simulation results show that, in case of zero output of EMA, the EKF estimation results, which are similar to the results in Fig. A.1-A.6, are not reliable.

## 5. Conclusion And Discussion

A new Nonlinear Robust Adaptive EKF algorithm with the filter gain correction at the each estimation step based on the evaluation of the posterior probability of the normal operation of the system, given for the current measurement is proposed. In the proposed filtration algorithm, this probability plays the role of the weight coefficients to the innovation vector. The faulty measurements are taken into consideration with small weights and the proposed EKF compensates the faulty results by decreasing the gain coefficients of the filter. As a result, measurement faults in the estimation system are corrected by the system, without affecting the good estimation behaviour. This approach does not require a priori statistical characteristics of the faults and knowledge of historical information. Furthermore the computational burden is not heavy.

Proposed RAEKF algorithm is tested in the case of EMA sensor faults, and results are compared with the outputs of conventional EKF for the same case.

During simulations, for testing the Nonlinear Robust Adaptive KF algorithm, four kinds of measurement malfunction scenario are taken into consideration; instantaneous abnormal measurements, continuous bias, measurement noise increment and fault of zero output. In the simulations two filtration algorithms are performed and are comprised: conventional EKF and Nonlinear Robust Adaptive EKF. In the case of healthy operation of the system the accuracy of EKF is slightly better than Nonlinear Robust Adaptive EKF, but the simulation results show that, the EKF becomes faulty while the introduced RAEKF algorithm stands robust to the measurement faults. The proposed approach does not

require a priori statistical characteristics of the faults and can be used for both linear and nonlinear systems. Furthermore the presented RAEKF algorithm is simple for practical implementation.

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**APPENDIX A: Conventional EKF Estimation Results**

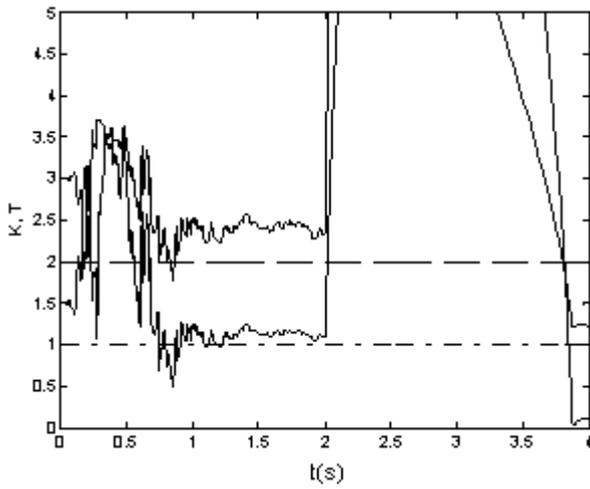


Fig.A.1. Actual (dashed line ) and estimated (solid line) values of parameter T; Actual (dash and dotted line) and estimated (solid line) values of parameter K in case of continuous bias at measurements (EKF is used)

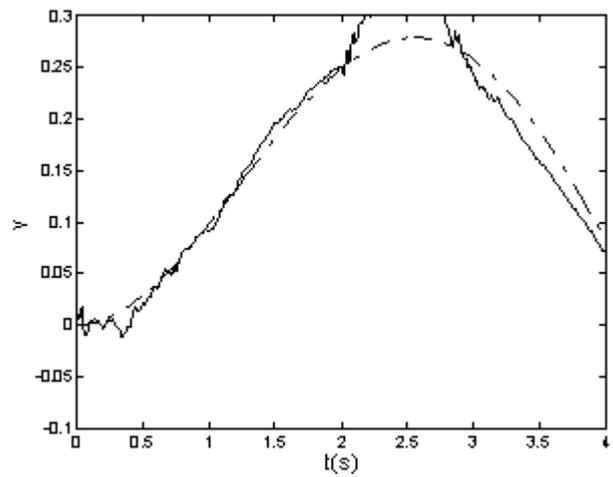


Fig. A.4. Actual and estimated values of parameter y in case of measurement noise increment (EKF is used)

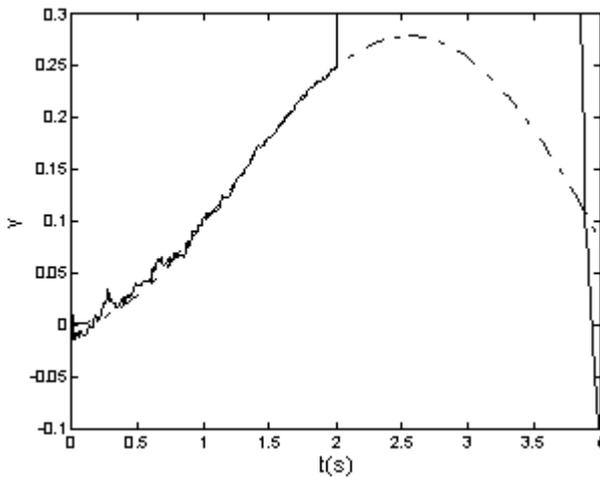


Fig. A.2. Actual (dashed line ) and estimated (solid line) values of parameter y in case of continuous bias at measurements (EKF is used)

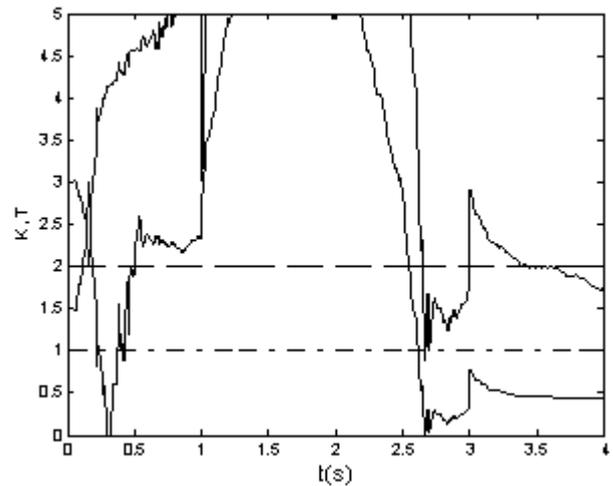


Fig.A.5. Actual and estimated values of parameters T and K in case of instantaneous abnormal measurements (EKF is used)

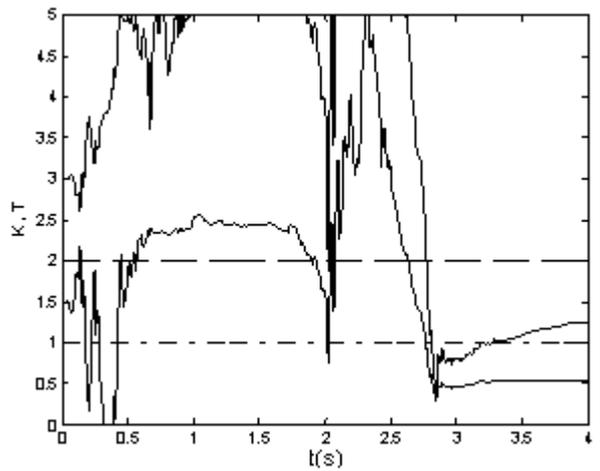


Fig.A.3. Actual and estimated values of parameters T and K in case of measurement noise increment (EKF is used)

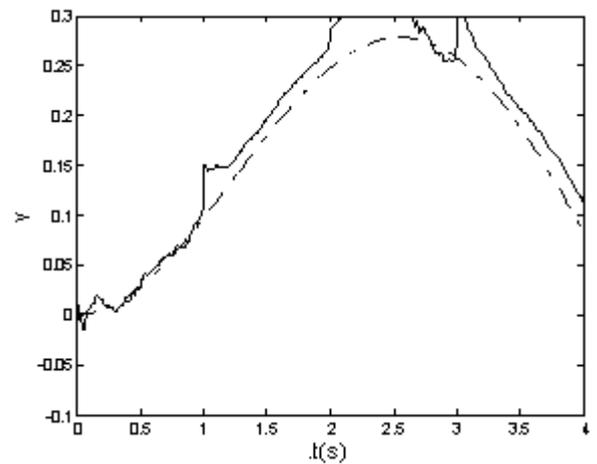


Fig. A.6. Actual and estimated values of parameter y in case of instantaneous abnormal measurements (EKF is used)