CONTROL SYSTEM WITH ADDITIVE ADJUSTMENT ON BASIS OF VELOCITY VECTOR METHOD

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Abstract

This paper considers the use of a controller with additive adjustment to achieve system stability when time-varying uncertainty influence. Time-varying parameters are piece-wise perturbations, which change at arbitrary and unknown times. An adaptive controller and adaptor are designed by the reference equation and the velocity vector methods. The system has state variable derivative feedback that leads to two-time scale motions. The stability problem is studied with the help of the common Lyapunov function and singular perturbation method. A numerical example is given.

Key Words

adaptive control, time-varying perturbations, stability, common Lyapunov function

1. Introduction

In this paper we consider a MIMO plant as a system with parameter perturbations, which change at arbitrary and unknown times. Typical examples of these plants include intelligent vehicle highway systems and robotics, especially in the supervision and coordination of multiple mobile robots [Davrazos and Koussoulas, 2001], transmission systems, air conditioning systems. An adaptive controller is synthesized by the reference equation method, and we get an adaptive algorithm with the help of the velocity vector method [Vostrikov and Shpilevaya, 2004]. It is shown a new approach to synthesis reduces order of the direct adaptive control system. This approach is based on some properties of parameter perturbations. A full order system has a controller with parameters adjustment. If we use the proposed approach, the system will have a controller with an additive adjustment. The system has the feedback on the state variables derivatives, which leads to two-time scale motions. The proposed closed-loop system is nonlinear, and there are some subsystems with motions having different velocities. Therefore, the study of process convergence is not a simple problem. We study the system stability by the common Lyapunov function [Narendra and Balakrishnan, 1994] and the singular perturbation method. A numerical example to indicate the effectiveness of the suggested design approach is presented.

2. Problem statement

The plant model used in this paper is

$$\dot{x}(t) = A_l(t)x(t) + Bu(t), \quad y(t) = Cx(t), \tag{1}$$

where x, u and y are vectors of state, input and output variables accordingly, $x, u, y \in R^m$; $\det CB \neq 0$; the system matrix $A_l(t)$ describes parameter perturbations on a time interval $t_l < t < t_{l+1}$, t_l is a moment, in which the perturbations sharply change; l is a index, $l = \overline{1, L}$; $A_l(t) = \{a_{lij}(t)\}$ is a matrix with piecewise constituents, and $a_{lij}(t)$ is a smooth function for $t_l < t < t_{l+1}$, $t_{l+1} = t_l + \tau_l$, $t_0 \le t_l < t_f$, $(t_f - t_0) > \tau_l > t_n$. Here τ_l, t_n are active time of lth matrix and the transition time conformably; t_0, t_f are the initial and the final work time moments. We suppose that the smooth function $a_{lij}(t)$ can be defined as

$$a_{lij}(t) = a_{ij}^0 + \tilde{a}_{lij}(t), \tag{2}$$

where

$$\left|\tilde{a}_{lij}(t)\right| < \varepsilon_{1lij}, \left|\dot{\tilde{a}}_{lij}(t)\right| < \varepsilon_{2lij}, \; \varepsilon_{slij} = const < \infty \tag{3}$$

for $s = 1, 2$; $i, j = \overline{1, m}$; $t_l < t < t_{l+1}$; $l = \overline{1, L}$. We suppose that a_{ij}^0, ε_{sij} for $i, j = \overline{1, m}$ are known or can be estimated. According to (2) we have

$$\dot{x}(t) = \left(A^0 + \tilde{A}_l(t)\right)x(t) + Bu(t), y(t) = Cx(t), \tag{4}$$

where $\tilde{A}_l(t) = \left\{\tilde{a}_{lij}(t)\right\}$; $A^0 = \left\{a_{ij}^0\right\}$ is Hurwitz's matrix, which can characterize the rated duty or $A^0 = A_l(t_0)$. The control purpose is

$$\lim_{t \to \infty} \left| r - y \right| = 0, \tag{5}$$

where r is a constant reference signal. It is necessary to synthesize an adaptive controller to ensure (5).

3. Design of control system with additive adjustment

In this section we describe an approach to designing of the direct adaptive control system having minimal order. We write (4) in the following way

$$\dot{y}_l(t) = C\left(\left(A^0 + \tilde{A}_l(t)\right)x(t) + Bu(t)\right). \quad (6)$$

According to the perturbation properties (2) we use (6) in the next kind

$$\dot{y}_l(t) = C\left(A^0 x(t) + Bu(t)\right) + M_l(x,t), \quad (7)$$

where $M_l(x,t) = C\tilde{A}_l(t)x(t)$ is a new perturbation which has the velocity depending on velocities of $\tilde{A}_l(t)$ and $x(t)$ on interval $t_l \le t \le t_{l+1}$, $\dim M_l = m \times 1$. We have replaced $A_l \in \Im_{m\times m}$ on $M_l \in \Re_{m\times 1}$, and have reduced number of the perturbations. It is necessary to have fast adjustment algorithm to suppress influence of these new perturbations. But at the beginning of design we get a control law by using reference equation method. Let a reference model be described by the equation

$$\dot{y}_m(t) = C^*\left(A^* x_m(t) + B^* r\right), \quad (8)$$

where $A^* = \{a_{ij}^*\}$, $B^* = \{b_{ij}^*\}$ and $C^* = \{c_{ij}^*\}$ are received according to the given quality performance of the transient. Let $x_m = x$, equating right parts (7) and (8) we have the control law

$$u = (CB)^{-1}\left(F - CA^0 x - M_l\right).$$

After replacing M_l on K_M we get

$$u = (CB)^{-1}\left(F - CA^0 x - K_M\right), \quad (9)$$

where $\det CB \ne 0$, $F(x(t), r) = C^*\left(A^* x(t) + B^* r\right)$ and $\dim K_M = m \times 1$.

An adaptive algorithm is synthesized by the velocity vector method [Vostrikov and Shpilevaya, 2004]. For modified control plant (7) and the controller (9) an adaptive algorithm is

$$\dot{K}_M = -\Gamma \operatorname{sgn}\left(F - C\dot{x}\right). \quad (10)$$

Here $\Gamma = diag\{\gamma_i\} > 0$ and $\operatorname{sgn}(a) = \begin{cases} 1, & a > 0, \\ -1, & a < 0. \end{cases}$

We can see that the controller (9) has an additive adjustment (10). In the system (7), (9) and (10) we can estimate $\dot{x}$ with the help of the linear low-inertial filter [Vostrikov and Shpilevaya, 2005].

4. Stability of System without Linear Low-Inertial Filter

We consider convergence conditions for system (7), (9) and (10). Substituting the control law (9) into (7), we have

$$\dot{y}_l = C^* A^* x + C^* B^* r + M_l - K_M,$$

where

$$\dot{K}_M = -\Gamma \operatorname{sgn} \varepsilon_l, \; \varepsilon_l = F - C\dot{x}_l = K_M - M_l.$$

Let $\Gamma = \gamma I$, where I is the unity matrix. We assume that the intervals between consecutive times l and $l+1$ are large enough. Let $\tau > t_n$, $\tau_l > \tau$ for $l = \overline{1, L}$. Study system stability on the interval $t_l \le t \le t_{l+1}$ for all l with the help of the common Lyapunov function:

$$V = 0.5\varepsilon_l^T H \varepsilon_l, \; H = H^T > 0.$$

The researched function derivative is equal to

$$\dot{V} = \varepsilon_l^T H \dot{\varepsilon}_l = \varepsilon_l^T H\left(-\dot{M}_l - \gamma \operatorname{sgn} \varepsilon_l\right)$$

or $\dot{V} = -\varepsilon_l^T H \dot{M}_l - \gamma \varepsilon_l^T H \operatorname{sgn} \varepsilon_l$.

A condition of negative definiteness of the function $(\dot{V})$ is carried out if we choose adaptor gain as

$$\gamma > \delta_l, \; \delta_l = \max_t \left\|\dot{M}_l\right\|, t_l \le t \le t_{l+1}. \quad (11)$$

Using the conditions (3) we can determine γ.

Proposition 1: The system (7), (9) and (10) with $M_l(x,t) = C\tilde{A}_l(t)x(t)$ is asymptotically stable for $l = \overline{1, L}$ on the interval τ_l, $\left(t_f - t_0\right) > \tau_l > t_n$, if the conditions (3) and (11) are satisfied.

Using **Proposition 1** and the theorems given in [Mancilla-Aguilar, 2000], we can formulate the second proposition for $t_0 \le t \le t_f$.

Proposition 2: The asymptotically stable system (7), (9) and (10) on τ_l, $\left(t_f - t_0\right) > \tau_l > t_n$ is locally asymptotically stable on $t \in \left[t_0, t_f\right]$, if $\left(t_f - t_0\right) > \tau_l > t_n$, $\gamma > \delta_{\max}$, where $\delta_{\max} = \max_{1 \le l \le L} \left|\delta_l\right|$.

5. Stability of System with Linear Low-Inertial Filter

Let's estimate the required output variable derivatives with the help of a low-inertia dynamic system such as

$$\mu_1 \dot{z} = A_z\left(y_l - z\right). \quad (12)$$

Here $z \in R^m$ is a state vector of the linear low-inertial filter, A_z is the coefficient matrix ($\left(-A_z\right)$ is Hurwitz's matrix), μ_1 is fast time constant; if $\mu_1 \to 0$, then $z \to y_l$ and $\dot{z} \to \dot{y}_l$.

Replace the vector-function $\operatorname{sgn}(g)$ in adaptive algorithm (10) with the vector-function $P = \{p_i\}$ because last function is determined at $\varepsilon = 0$ [Ambrasino, et al., 1984],

$$\operatorname{sgn}(g) \approx P(g),\ P^T = [p_1, p_2, ..., p_m], \tag{13}$$

where $p_i = \dfrac{g_i}{|g_i| + \varphi}$, φ=const, $0 < \varphi \le 1$, $g = F - \dot{z}$.

According to (12) and (13) the adaptive system on the interval $t_l \le t \le t_{l+1}$ is described

$$\begin{aligned} \dot{y}_l &= C^* A^* x + C^* B^* r + M_l - K_M, \\ \mu_1 \dot{z} &= A_z (y_l - z), \\ \mu_2 \dot{K}_M &= P(g),\ \mu_2 = 1/\gamma. \end{aligned} \tag{14}$$

Let $\mu_1 < 0.1\mu_2$, for simplicity and without loss of generality, assume $\mu_2 = 1$, enter the new time $t = \mu_1 \tau$, and denote $(a)' = da/d\tau$, $da/dt = \mu^{-1} da/d\tau$. Then in return for (14) we have the system equation in the new time

$$\begin{aligned} y_l' &= \mu_1 \left(C^* A^* x_l + C^* B^* r + M_l - K_M\right), \\ K_M' &= \mu_1 \nu P(g), \\ z' &= A_z (y_l - z). \end{aligned}$$

Decoupling motions and returning to usual time we have the subsystem of fast motions:

$$\mu \dot{z} = A_z (y_l - z).$$

It is stable for the reason that $(-A_z)$ is Hurwitz's matrix. The slow motions subsystem is

$$\begin{aligned} \dot{y}_l &= C^* A^* x + C^* B^* r + M_l - K_M, \\ \dot{K}_M &= \gamma P(g), \end{aligned}$$

where $g = \varepsilon_l$. Its asymptotically stable for all x, y, and M_l on the time interval $t_l < t < t_{l+1}$ is the result of Proposition 1. Our research has shown that the adaptive system (14) for $\mu_1 < 0.1\mu_2$ is stable.

If $\mu_1 < \mu_2$ and $\lim\limits_{\mu_{1,2} \to 0} (\mu_1/\mu_2) = 0$, then step-by-step degeneracy of the system (14) is carried out. In initial system it's possible to allocate three processes proceeding with different velocities. The processes in the observer are fast processes, therefore in the established regime, $z = y_l$. The adaptor processes are velocity-average processes, they will be too stable, because $\dot{z} = \dot{y}_l$ [Vostrikov and Shpilevaya, 2005, Shpilevaya, 2008]. The slow processes are described twice step-by-step degeneracy system, which coincides with the reference model. So, if $\mu_1 < \mu_2$, we have stability subsystems.

Now let $\mu_1 = \mu_2 = \mu$. The fast motions are described following equations ($y_l = const, x = const$)

$$\begin{aligned} \mu \dot{K}_M &= \nu P(g), \\ \mu \dot{z} &= A_z (y_l - z). \end{aligned} \tag{15}$$

If the control plant is stable, the reference model is formed depending on own properties of the plant, therefore usually rate of change F is commensurable with rate of change $\dot{y}_l$. When $y_l = const, x = const$, we have $\dot{y}_l \to 0$ and can suppose that $F = 0$. Write down the system (15) in variables ε_l and $s_l = z - y_l$,

$$\begin{aligned} \mu \dot{\varepsilon}_l &= \nu P(\varepsilon_l, s_l), \\ \mu \dot{s}_l &= -A_z s_l. \end{aligned} \tag{16}$$

The stability of the equilibrium state of this system is tested by a function

$$V = 0.5\, s_l^T s_l + 0.5 \varepsilon_l^T \varepsilon_l,$$

whose first derivative is equal

$$\dot{V} = -\mu^{-1} \varepsilon_l^T P - \mu^{-1} s_l^T A_z s_l.$$

According to $s_l^T A_z s_l > 0$ and properties of the function P (13) we have $\varepsilon_l^T P > 0$ for all s_l and ε_l, therefore $\dot{V} < 0$. Thus, at $s_l \to 0$ we have $\varepsilon_l \to 0$. So the equilibrium state of the system (16) with $\mu_1 = \mu_2 = \mu$ is stable. The slow motions have reference model dynamics

$$\dot{y}_l = C^* A^* x + C^* B^* r = F,$$

this is stable also.

According to the results of this subsection, Propositions 1 and 2**,** we can assert the following.

Proposition 3: The adaptive system (14) with piecewise parameter perturbations (2), (3) is stable on $t \in [t_0, t_f]$, if $(t_f - t_0) > \tau_l > t_n$; $\mu_1 < \mu_2$ or $\mu_1 = \mu_2 = \mu$, and $\gamma > \delta_{\max}$.

Our study has shown that the adaptive system is stable with "fast" and "slow" adaptation loops. In the adaptive systems with the linear low-inertial filter the small parameters must be $\mu_1 \approx \mu_2$ or $\mu_1 < \mu_2$. However it's necessary to note, that at $\mu_1 \approx \mu_2$ an opportunity of adaptor completely are not realized, as during transient time of the linear low inertial filter the regulator coefficients are formed depending on s_l instead of ε_l. If $\mu_1 << \mu_2$, the adaptive system becomes sensitive to the noise of measurements of the output variables. In [Vostrikov and Shpilevaya, 2005] it is shown that a variant $\mu_1 > \mu_2$ and $\lim_{\mu_{1,2}} (\mu_2 / \mu_1) = 0$ is not acceptable, because fast movements do not provide stability of the adaptive system.

6. Example

In this section we consider the adaptive control of a plant in two work regimes. Its dynamics is

$$\begin{cases} \dot{x}_1 = a_{l11}x_1 + a_{l12}x_2, \\ \dot{x}_2 = a_{l21}x_1 + a_{l22}x_2 + a_{l23}x_3 + u_1, \\ \dot{x}_3 = a_{l31}x_1 + a_{l32}x_2 + a_{l33}x_3 + u_2, \end{cases}$$

$y_1 = x_2$, $y_2 = x_3$.

Denote $m_{li} = a_{l(i+1)1}x_1$. Nominal parameters of the plant are

$$A^0 = \begin{bmatrix} -1 & -0.5 & 0 \\ 2.5 & -4 & 0.5 \\ 10 & -3.5 & -3 \end{bmatrix},$$

the first subsystem has parameters $A_1 = 1.5A^0$ and the second subsystem parameters are $A_2 = 2A^0$. Here $\tau_l = \tau_{l+1} = \tau = 5$. The system outputs must be monotonous processes with $t_n = 3.5$ and an allowable static error: $e_s \le 5\%$; reference inputs are $r_1 = 2(t)$, $r_2 = 1(t)$. According to these conditions the reference dynamics for each output is $F_i = -y_i + r_i$, $i = 1,2$. Using the technique given in Section 3, we have

$$u_i = -a^0_{(i+1)2}y_1 - a^0_{(i+1)3}y_2 - y_i + r_i + M_{li},$$

where $M_{li} = -\tilde{a}_{l(i+1)2}y_1 - \tilde{a}_{l(i+1)2}y_2 - m_{li}$. In this case, (9) has a following kind:

$$u_i = -a^0_{(i+1)2}y_1 - a^0_{(i+1)3}y_2 - y_i + r_i + k_i,$$

the adaptive algorithm is (10). The adaptor gains are $\gamma_1 = \gamma_2 = 100$, the fast time constants are $\mu_1 = \mu_2 = 0.01$.

System trajectories look as shown in Figures 1 and 2. We can see the adaptive system reveals stability under the piecewise parameter perturbations. In this system we can influence the transition time by reference model parameters and the adaptor gain. Note in this example we have $\dim x > \dim u$ and $M_l \in \Re_{2\times1}$ for $A_l \in \Im_{3\times3}$. It demonstrates that the considered approach of the adaptive control design can be extended to a more general case of systems with piecewise parameter perturbations.

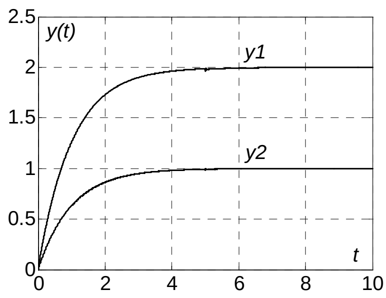

Fig. 1. Output processes

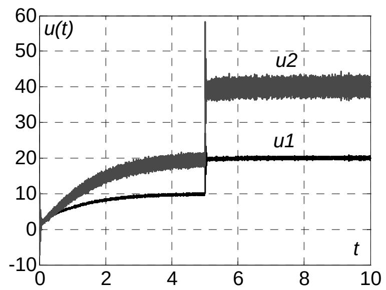

Fig. 2. Control processes

7. Conclusion

We have developed an adaptive control algorithm for plants, which have piecewise parameter perturbations, using the velocity vector method. However it may be synthesized with the help of the speed-gradient law too [Fradkov, 1991]. For stability study we applied the techniques developed for switched systems and method used for continuous system analysis. There were the common Lyapunov function technique and the singular perturbation method. In our design approach we use the perturbation velocity, and as a result we get the "fast" adaptive algorithm. Due to the "fast" adaptor we can stabilize the system with the help of one controller with additive adjustment loop as opposed to other systems where a few controllers are used. It is expected that proposed design method can be extended to a more general case of plants with piecewise parameter perturbations and of plants than considered in Section 2.

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