DYNAMICS MODELING FOR SUBCRITICAL REACTOR CONTROLLED BY LINEAR ACCELERATOR

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Abstract
Some issues of subcritical reactor control by linear proton accelerator are considered. Also questions of dynamics modeling in subcritical reactor, taking into account the temperature feedbacks on the basis of point kinetics model are investigated. The resulting dynamics equations describe physical processes with characteristic times differing by orders of magnitude. Due to this feature some physical approximations were introduced to simplify the equations of dynamics based on the point kinetics model. This simplification allows to use standard methods for numerical integration of ODE.

Key words
Subcritical reactor, dynamics with feedbacks, ADS control, linear accelerator.

1 Introduction

Accelerator driven systems (ADS) is a new type of reactor which produces power even though it remains sub-critical throughout its life (Carminati and et al, 1993). The additional neutron supply, necessary to maintain nuclear reaction, comes from the interaction of an accelerated charged particle beam with a target. ADS can find a usage in nuclear power engineering for transmutation of long-lived radioactive waste, energy production and breeding new fissionable elements (Gerasimov and Kiselev, 2001), (Golovkina, Kudinovich, Ovsyannikov and Svistunov, 2016). The main advantage of ADS is high nuclear safety, because uncontrolled spontaneous chain fission reaction is eliminated.

In contrast to traditional critical reactors, where the control on reactor power rate is fulfilled with neutron absorbing rods, in ADS, subcritical reactor is controlled by charged particle accelerator. Reactivity coefficient changes in time due to feedbacks on temperature effects (fuel and coolant) and simultanious fuel burning and fission products accumulation. So problem of ADS power-level maintainance with accelerator as well as reactor dynamics investigation is arised.

The modeling processes progress in time with speeds different by orders of magnitude. In order to correctly take this speciality into account in numerical calculations simplified dynamics model based on point kinetics is proposed. Physically appropriate results were obtained using it.

2 Subcritical reactor control with accelerator

Dynamics of the ADS subcritical reactor depends on internal and external feedbacks. Internal feedbacks are defined by physical properties of the reactor core, whereas external feedbacks reflect reactor connection with power plant (coolant flow, coolant temperature at the reactor inlet). The structural scheme of ADS with feedbacks is presented in Figure 1 (Golovkina, Kudinovich and Svistunov, 2016).

For stable ADS operation at the constant power-level the reactor core must have a negative feedback on the fuel and coolant temperature as well as the average negative reactivity coefficient, which ensures reactor self-regulation and maintenance of the average temperature.

Thermal power-level of the reactor core is determined by the following expression:

$$N_T = \frac{E_f Q_f}{\nu}, \tag{1}$$

where E_f — energy released per one fuel nucleus fission, ν — average number of neutrons coming out in fission event, Q_f — intensity of fission neutrons generation, which in first approximation can be calculated by the following formula

$$Q_f = S\frac{1 - k_{\text{eff}}}{k_{\text{eff}}}. \tag{2}$$

Here S — external neutron source intensity.

As can be seen from equations (1) and (2), the reactor power depends on the intensity of the electronuclear

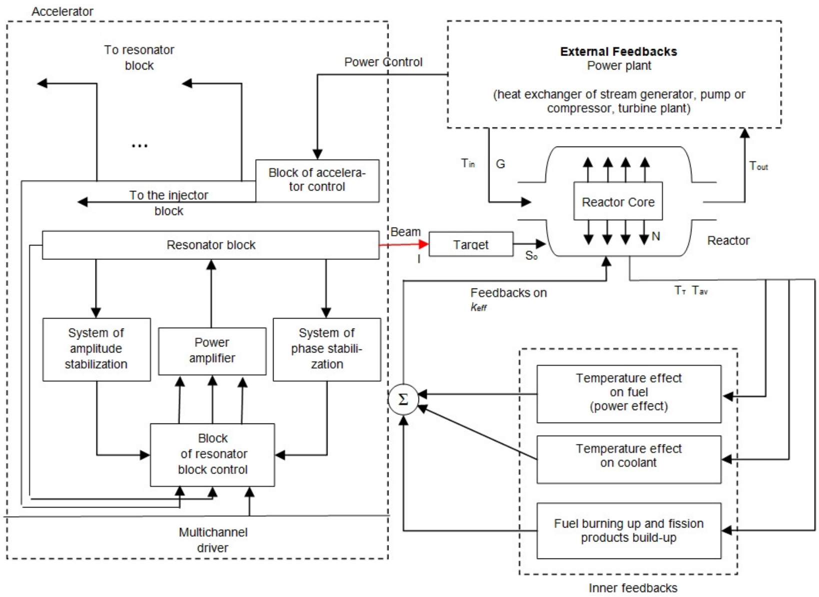

Figure 1. ADS structural scheme with feedbacks

neutron source and the value of effective multiplication factor k_{eff}, which is chosen to provide nuclear safety and nowadays for ADS it is admitted not to exceed value 0.98.

In traditional nuclear reactors ($k_{\text{eff}} = 1$), the reactor core is maintained in a critical state by a control system with operating elements in the form of rods made of neutron-absorbing materials that are mechanically introduced and withdrawn from the core. Effective multiplication factor is defined by physical characteristics of the reactor core and depends on temperature in the reactor and fuel burn-up. Reactivity reduction as a result of these processes is about 8% for thermal-neutron reactor and 1–3% for fast-neutron reactor. Therefore external neutron source intensity should be varied to compensate possible changes in k_{eff} value and consequently in the reactor power-rate.

3 Dynamics of subcritical reactor based on point kinetics model

Here and later instead of efficient multiplication factor k_{eff} we will consider reactivity of the reactor $\rho = \dfrac{k_{\text{eff}} - 1}{k_{\text{eff}}}$ which is a dimensionless quantity used to characterize reactor deviation from the critical state (Keepin, 1965).

Internal feedbacks cause the dependence of reactivity on the fuel elements and coolant temperature. The effect of the reactor temperature on its reactivity is called the fuel temperature effect, and the influence of the coolant temperature — coolant temperature effect (see Figure 1). Temperature effects are characterized by the respective temperature coefficients of reactivity α_T and α_{TH}. Usually the dependence of reactivity on temperature is represented by a linear function (Beckman, 2005)

$$\rho = \alpha_T \left(T_T - T_T^{\text{av}}\right) + \alpha_{TH} \left(T_{TH} - T_{TH}^{\text{av}}\right), \qquad (3)$$

where T_T and T_{TH} — current fuel and coolant temperature correspondently, T_T^{av} T_{TH}^{av} — temperature of fuel and coolant in the operating point.

The reactivity temperature effect is determined by two components: dependence of the core materials density on temperature and the Doppler effect (Usynin and Kusmartsev, 1985).

Taking into account the remarks made above, reactor core dynamics with thermal feedbacks is described by the following system of equations:

$$\begin{aligned}
&\frac{d\varphi(t)}{dt} = \frac{\left(\rho(t) - \beta_{\text{eff}}\right)\varphi(t)}{l} + \lambda C_{\text{eff}}(t) + q_{\text{eff}}(t),\\
&\frac{C_{\text{eff}}(t)}{dt} = \frac{\beta_{\text{eff}}\varphi(t)}{l} - \lambda C_{\text{eff}}(t), \qquad (4)\\
&\rho(t) = \rho_{\text{av}} + \alpha_T \left(\widehat{T}_T(t) - T_T^{\text{av}}\right) +\\
&\alpha_{TH} \left(T_{TH}(t) - T_{TH}^{\text{av}}\right),\\
&M_{TH} C_{TH} \frac{dT_{TH}(t)}{dt} = 2G C_{TH}(t) \left(T_{\text{in}} - T_{TH}(t)\right) +\\
&+hS \left(T_{\text{w}}(t) - T_{TH}(t)\right), \qquad (5)\\
&\rho_T(T_T, r) C_T(T_T, r) \frac{\partial T_T(r,t)}{\partial t} = \qquad (6)\\
&= \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_T(T_T, r)\frac{\partial T_T(r,t)}{\partial r}\right) + q_v(r,t),\\
&t > 0, \quad 0 < r < R,\\
&\varphi(0) = \varphi^{\text{ini}},\ C_{\text{eff}}(0) = C_{\text{eff}}^{\text{ini}},\\
&\rho(0) = \rho^{\text{ini}},\ T_{TH}(0) = T_{TH}^{\text{ini}},\ T_T(r,0) = T_T^{\text{ini}}(r).
\end{aligned}$$

Here t — time, r — fuel element radius coordinate,

M_{TH} — mass of coolant, $T_T(r,t)$ — fuel element temperature distribution, $\widehat{T}_T(t)$ — temperature averaged over the volume of the fuel element, T_{w} — temperature of the fuel element wall, T_{TH} — coolant temperature, G — coolant mass flow, S — area of heat delivery surface of the fuel elements in the reactor core, α_T — fuel temperature coefficient, α_{TH} — coolant temperature coefficient, h — coolant heat-transfer coefficient, λ_T — heat conductivity coefficient, C_T — specific heat capacity of the fuel element, ρ_T — density of the fuel element.

Taking into account equation (1) and assumption of time and spatial variables separation, made during point kinetics equations derivation (Usachev, 1955), change in time of energy release spatial distribution in the core is determined by the expression

$$N_0(\mathbf{r},t) = \frac{\varphi(t)E_f \int dE \int d\mathbf{\Omega}\mathbf{M_f}\tilde{F}(\mathbf{r},\mathbf{\Omega},E)}{\nu},$$

and change in time of the integral energy release:

$$N(t) = \frac{\varphi(t)E_f \left\langle \mathbf{M_f}\tilde{F}(\mathbf{r},\mathbf{\Omega},E)\right\rangle}{\nu}, \qquad (7)$$

then distribution of specific (by volume) energy release in equation (6) is defined as $q_v(r,t) = \dfrac{N(r,t)}{V_0}$, where V_0 — volume of the reactor core, $\tilde{F}(\mathbf{r},\mathbf{\Omega},E)$ — neutron flux spatial–angle–energy distribution, $\varphi(t)$ — shape factor of the neutron flux, $\mathbf{M_f}$ — linear fission operator, $\langle\cdot,\cdot\rangle$ — scalar product in l_2.

Simultaneous integration of equations (4), (5) and (6) with given initial and boundary conditions is rather difficult problem, since the physical processes described by them are characterized by time constant differing in orders of magnitude (Strakhovskaya and Fedorenko, 1998).

Four physical components with different characteristic time, can be separated:

1. *Prompt neutrons.* Average prompt neutrons lifetime l in the reactor depends on the neutron energy spectrum and changes from $5\cdot10^{-7}$ s (for fast reactors) to $5\cdot10^{-4}$ s (for thermal reactors).
2. *Delayed neutrons.* Average delayed neutrons lifetime $t_{\mathrm{del}} = 0.1 - 10$ s.
3. *Accelerator driver current.* Micro impulses period in the linear accelerator is $T = 5\cdot10^{-9}$ s, and macro impulses period is $T = 5\cdot10^{-3}$ s.
4. *Thermal feedbacks.* The time constant, characterizing the rate of the fuel elements temperature change with energy release change in time is not less than 0.01 s. The time constant characterizing the rate of coolant temperature change is determined by the time of its passage through the reactor core and is a few seconds.

Thus, the system of nonstationary equations (4)–(6) is characterized by significant variety of time constants defining the dynamics of simulated physical processes. So numerical solution of this system by standard methods (Hairer, Norsett and Wanner, 1993) requires the use of integration step, corresponding to the physical process with a minimum characteristic time (about 10^{-7} s). This approach is not appropriate, since the reactor Dynamics should be determined within a long period of time. In this regard, the influence of each of these physical components on the dynamics of subcritical reactor controlled by linear accelerator was analyzed and approximate models, which make it possible to use traditional methods for ODE numerical solution are obtained.

4 Calculation results

Under these physical assumptions: prompt neutrons approximation (Keepin, 1965), accelerator current continuity approximation and point approximation of the fuel element, the resulting system of differential–algebraic equations for longtime subcritical reactor dynamics calculation are obtained:

$$\begin{aligned}
&\varphi = \frac{\left(\lambda C_{\mathrm{eff}}(t) + q_{\mathrm{eff}}^{\mathrm{av}}\right) l}{\beta - \rho(t)},\\
&\frac{C_{\mathrm{eff}}(t)}{dt} = \frac{\beta_{\mathrm{eff}}\varphi(t)}{l} - \lambda C_{\mathrm{eff}}(t),\\
&\rho(t) = \rho_{\mathrm{av}} + \alpha_T\left(\widehat{T}_T(t) - T_T^{\mathrm{av}}\right), \qquad (8)\\
&M_{TH}C_{TH}\frac{dT_{TH}(t)}{dt} = 2GC_{TH}(t)\left(T_{\mathrm{in}} - T_{TH}(t)\right) +\\
&hS\left(\mu\widehat{T}_T(t) - T_{TH}(t)\right),\\
&M_T C_T\frac{dT_T(t)}{dt} = N(t) - hS\left(\mu\widehat{T}_T(t) - T_{TH}(t)\right).\\
&C_{\mathrm{eff}}(0) = C_{\mathrm{eff}}^{\mathrm{ini}},\ \rho(0) = \rho^{\mathrm{ini}},\ T_{TH}(0) = T_{TH}^{\mathrm{ini}},\\
&\widehat{T}_T(0) = T_{TH}^{\mathrm{ini}}.
\end{aligned}$$

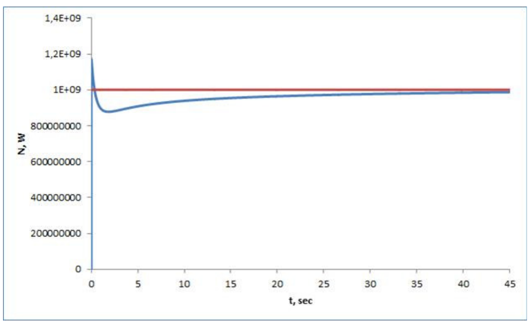

Figure 2. The ADS reactor power level change in time

Let us analyze dynamics of ADS with fast subcritical reactor and external pulsed neutron source after start-up from a cold state and coming up to the given power level. In Figure 2 and 3 as an example, calculation results of reactivity and power rate change are presented (Golovkina, Kudinovich, Ovsyannikov and Svistunov, 2014). As can be seen from Figure 2 in the initial moments there is a power excursion, which is suppressed by fuel temperature feedback (the Doppler effect). It also should be noted that fuel temperature remains constant after reactor start up due to fuel elements thermal inertia (Golovkina, Kudinovich and Svistunov, 2016).

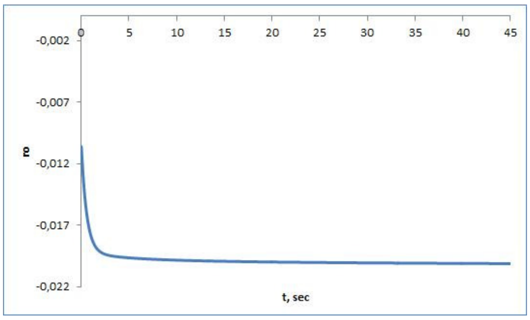

Figure 3. The reactivity coefficient change in time

5 Conclusion

Questions of dynamics modeling in subcritical reactor, taking into account the temperature feedbacks on the basis of point kinetics model are considered. Influence of modeling processes characteristic time influence on the longtime reactor core dynamics is investigated. After comprehensive analysis the initial dynamics equation were simplified in order to use for their numerical integration standard methods, particularly Runge-Kutta method of 4th order. As an example, subcritical reactor dynamics during start-up was calculated. As a result a short-time power surge higher the power rating level can be observed, wheres the fuel temperature doesn't exceed its rated value.

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