A generalized model of active media with a set of interacting pacemakers: application to the heart beat analysis

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I. INTRODUCTION

Representation of active media by ensembles of coupled excitable and oscillatory elements is very useful method of analysis because it allows to deeply understand main dynamical processes. As is known this approach goes back to the model of Wiener and Rosenblueth [7], according to which a medium consists of single elements being in one of three possible states: excited, refractory or rest. Many useful concepts like phase-locked patterns, synchronization and spatio-temporal chaos have become popular due to detailed studies of similar nonlinear models [5, 8].

Investigations of such an example of an active medium as cardiac tissue are of significant scientific interest owing to vital importance of its stability. Real heart cells demonstrate oscillatory properties (can be reset and entrained), are excitable, have refractory time, during which they do not respond to external stimulation, are heterogeneous and fatigue (are less excitable following rapid stimulation), and hence the heart can be considered as consisting of oscillatory and excitable elements.

Some models treat the cardiac tissue as an active conductive system, taking into account oscillatory properties of heart cells. In this case the cardiac rhythms can be described on the basis of the dynamical system theory (see e.g. [1–4, 6] and refs. therein).

The rhythm of autonomous biological oscillators can also undergo an external periodic perturbation (e.g. activity of cells of the AV junction is subjected to sinus rhythm), depending on both the stimulus magnitude and its phase within the cycle. It is known that when the frequency and the amplitude of the external periodic stimulation are varied, a diversity of phase diagrams can be established between the stimulus and the self-sustained oscillator (see e.g. [6]). In some situations the rhythm of the biological oscillator is entrained (or phase-locked) to the external stimulation so that for each M cycles of the stimulation there are N cycles of the autonomous oscillator rhythm. This occurs at a fixed phase (or phases) of the stimulus and is called $M : N$ phase-locking or entrainment, which appears as a time–periodic sequence. In particular, entrainment of $1 : 1$, within which rhythms of the oscillator and external stimulus are matched, is defined as synchronization.

In this work we developed a general simplified model describing a network of oscillatory elements coupled by their response to internal depolarization of mutual stimulations. Our primary aim was to keep the model as simple as possible and to introduce a minimal number of parameters. Therefore, in our model the pacemakers are fully characterized by their intrinsic cycle length T. Their interaction is described by PRCs. At first, we considered two interacting pacemakers to demonstrate the basic concepts of the model. Then we applied our approach to construct a pacemaker network model with global coupling. As a next step, this PRC based model of coupled pulse oscillators was applied to derive an additional, useful for controlling, model of three pacemakers of the cardiac conductive system. Our further intention was to go on to the next level and represent each pacemaker as an ensemble of interacting oscillatory elements. Extrapolation of the approach to the one- and two-dimensional matrices (or lattices) of pacemaker cells allows to construct active media with a set of oscillators coupled to nearest neighbors.

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