CONTROL STRATEGY FOR SYMMETRIC CIRCULAR FORMATIONS OF MOBILE AGENTS WITH COLLISION AVOIDANCE

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Abstract

Collective motion is a promising field that studies how local interactions lead groups of individuals to global behaviors. Biologists try to understand how those subjects interplay in nature, and engineers are concerned with the application of interaction strategies to mobile vehicles, satellites, robots, etc. This paper introduces a collision avoidance mechanism to a model of particles with phase-coupled oscillators dynamics for symmetric circular formations. The mobile agents must establish the formation without colliding with each other.

Key words

Collective Motion, Synchronization, Phase-coupled oscillators, Symmetric circular formations.

1 Introduction

Models of phase-coupled oscillators are widely used in applications of collective motion [Sepulchre et al, 2007; Jain and Ghose, 2017]. One of the most proeminet paradigm of the field of phase-coupled oscillators is the Kuramoto Model [Kuramoto, 1984], in which the oscillators' interactions are mediated via sinusoidal coupling. This model opened possibilities for studies on dynamical systems' synchronization [Acebrón et al, 2005].

A well known model for collective motion was proposed by [Vicsek et al, 1995] in which depending on the density of the particles and the noise amplitude, the particles converge to an ordered motion.

[Sepulchre et al, 2007] developed a model to lead particles with coupled-oscillator dynamics to synchronized and balanced states, showing parallel and circular formations, with symmetric patterns for the latter case. [Jain and Ghose, 2017] employed what they called reference velocity to build a steering control law to drive the circular formation centroid to desired coordinates. These models can be used in data collection, surveillance and other applications of collective motion.

In this work we improve the control for symmetric circular formations proposed in [Sepulchre et al, 2007] by adding a collision avoidance mechanism, based on balanced states of Kuramoto model. The purpose is to guarantee that the agents do not collide with the neighbors in their vicinity.

2 Particles with coupled-oscillator dynamics

The original model aims to lead particles to collective behaviors, depending on its parameters.

Particles represent N identical individuals with unitary mass and velocity. They maneuver according to laws of interaction and at constant speed. Each particle position is given by $r_k = x_k + iy_k \in \mathbb{C}$, the direction of the velocity vector by $e^{i\theta_k} = \cos\theta_k + i\sin\theta_k$, and phase $\theta_k \in \mathbb{R}$ for $k = 1, \ldots, N$. The phase represents the heading angle of the particle.

Let $\boldsymbol{r} \doteq (r_1, \ldots, r_N)^T \in \mathbb{C}^N$ be the vector of particle positions, $\boldsymbol{\theta} \doteq (\theta_1, \ldots, \theta_N)^T \in \mathbb{T}^N$ the velocity vector, and $u_k(\boldsymbol{r}, \boldsymbol{\theta})$ the maneuver control (feedback control).

The particle model is the following

$$\dot{r_k} = e^{i\theta_k} \tag{1a}$$

$$\dot{\theta_k} = u_k(\boldsymbol{r}, \boldsymbol{\theta}) \tag{1b}$$

When $u_k = 0$ the particles move in a straight line towards their initial phase (heading angle) $\theta_k(0)$. Still, when $u_k = \omega_0$, i.e., if not coupled with others, particle k moves in circular trajectories centered at c_k with radius $|\omega_0|^{-1}$, as shown in Equation 2.

$$c_k \doteq r_k + \omega_0^{-1} i e^{i\theta_k} \tag{2}$$

The center of mass of the group is

$$R \doteq \frac{1}{N}\sum_{j=1}^{N} r_j \tag{3}$$

When $\theta_k = \theta_j$ for all pairs j and k, particles are synchronized. On the other hand, if the phases cancel each other with opposite values, they are said to be balanced. These two extreme states are measured with the Kuramoto order parameter [Strogatz, 2000], given by Equation 4.

$$p_\theta \doteq \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j} \tag{4}$$

with $e^{i\theta_k} = \cos\theta_j + i\sin\theta_j$, and $0 \leq p_\theta \leq 1$. The order parameter corresponds to the velocity of the particles center of mass, since $\dot{R} = p_\theta$. When $|p_\theta| = 0$, they are in a balanced state with the center of mass in a steady position. For $|p_\theta| = 1$ they are synchronized and the center of mass moves at unit velocity.

The rotation centroids of particles are represented by the vector $\boldsymbol{c} \doteq (c_1, \cdots, c_N) \in \mathbb{C}$. A circular formation is a relative equilibrium regime in which all particles travel around the same circle, i.e., $c_k = c_j$ for all pairs j and k.

Consider M a positive integer, divisor of N. A (M, N) pattern is a symmetric arrangement of N phases divided into M clusters. $M = 1$ corresponds to the synchronized state in which all particles have the same heading angle (phase), and the (N, N) pattern stands for phases uniformly spaced around the unitary circle (balanced state).

The control to achieve symmetric circular formations with all-to-all interaction [Sepulchre et al, 2007] is

$$u_k = \omega_0(1 + K_0\left\langle e^{i\theta_k}, P_k\boldsymbol{c}\right\rangle) - \frac{\partial U^{M,N}}{\partial\theta_k} \tag{5}$$

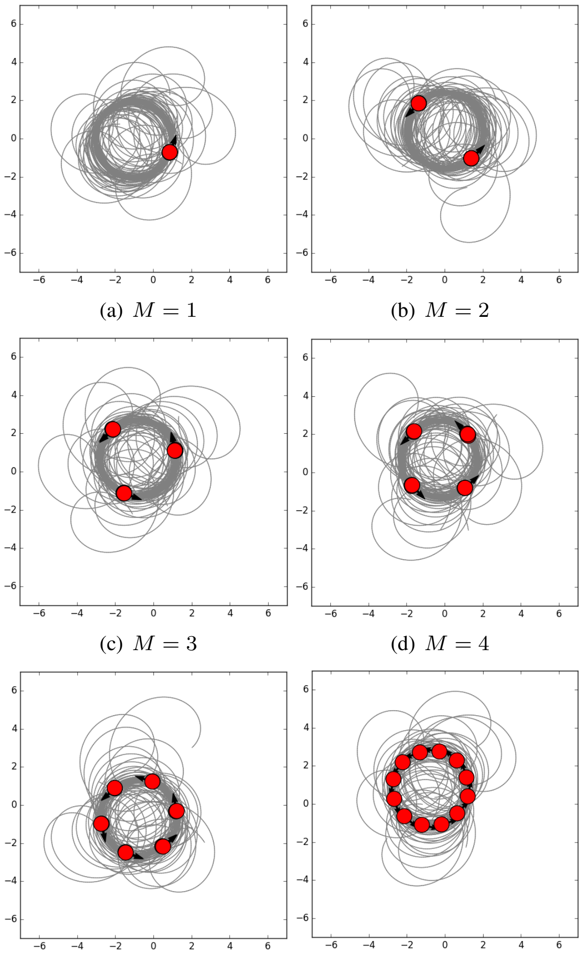
for $K_0 > 0$, and potential gradient

$$\frac{\partial U^{M,N}}{\partial\theta_k} = \frac{1}{N}\sum_{m=1}^{M}\sum_{j=1}^{N}\frac{K_m}{m}\sin(m(\theta_j - \theta_k)) \tag{6}$$

with P_k the projection matrix k-th row. This matrix is composed by $p_{ii} = \frac{N-1}{N}$ and $p_{ij} = \frac{-1}{N}$ for $i \neq j$, as shown in Equation 7.

$$P = \begin{pmatrix} \frac{N-1}{N} & \frac{-1}{N} & \cdots & \frac{-1}{N} \\ \frac{-1}{N} & \frac{N-1}{N} & \cdots & \frac{-1}{N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-1}{N} & \frac{-1}{N} & \cdots & \frac{N-1}{N} \end{pmatrix} \tag{7}$$

Figure 1 shows an example with $N = 12$ particles with random initial conditions, and all-to-all interaction.



(a) $M = 1$ (b) $M = 2$ (c) $M = 3$ (d) $M = 4$ (e) $M = 6$ (f) $M = 12$

Figure 1. Symmetric circular formation with control 5 for all-to-all interaction. Configuration: $N = 12$, $K = 0.1$ and $\omega_0 = 0.5$.

[Sepulchre et al, 2007] developed a simpler model, inspired in Kuramoto model, in which the particles phase are coupled with a sinusoidal function as shown in Equation 8.

$$\dot{\theta}_k = \omega_0 + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_k - \theta_j) \tag{8}$$

As in the symmetric circular control (Equation 5), the same natural frequency ω_0 is used for all oscillators. When $K > 0$ the particles achieve a balanced state, with the phases uniformly distributed around the unitary circle, and consequently leading the order parameter to zero. Motivated by this behavior, we propose the introduction of a repulsion term rep in Equation 9, so that the phases of close range particles are adjusted to avoid collision.

$$u_k = \omega_0(1 + K_0 \langle e^{i\theta_k}, P_k \boldsymbol{c}\rangle) - \frac{\partial U^{M,N}}{\partial \theta_k} + \text{rep} \quad (9)$$

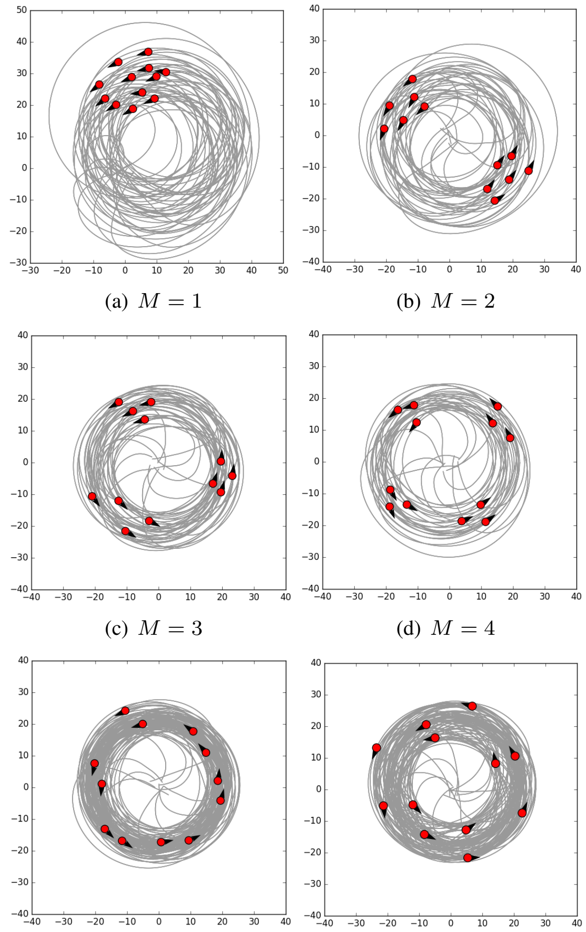
where K_0 must be positive, $K_r > 0$ is the strength of the repulsion, and rep is defined as follows

$$\text{rep} = \frac{K_r}{n(\mathcal{N}(r_k))} \sum_{j \in \mathcal{N}(r_k)} \sin(\theta_k - \theta_j) \qquad (10)$$

with $\mathcal{N}(r_k) \doteq \{j \in \mathbb{N} \mid \| r_k - r_j \| < d\}$ the set of neighbors of agent k, and $n(\mathcal{N}(r_k))$ the number of neighbors in the set $\mathcal{N}(r_k)$. An agent belongs to $\mathcal{N}(r_k)$ if it is within a predefined radius d, centered at r_k.

The idea behind the repulsion term of Equation 10 is that the agent k tries to balance its heading angle with its closest neighbors $\mathcal{N}(r_k)$. This results in an adjustment of agent's k heading angle to the opposite direction in relation to $\mathcal{N}(r_k)$.

Figure 2 shows simulation results with control 9, and parameters $N = 12$, $K = 0.1$, $K_r = 0.2$, $\omega_0 = 0.05$ and $d = 5$. The bigger the agents are the higher the radius d and/or the gain K_r must be, as they have to start to avoid the neighbors before they reach a critical distance. The parameters d and K_r state the initial distance considered for the usage of rep and also the maneuver intensity.



(a) $M = 1$ (b) $M = 2$

(c) $M = 3$ (d) $M = 4$

(e) $M = 6$ (f) $M = 12$

Figure 2. Symmetric circular formation with control 9 for all-to-all interaction with collision avoidance. Configuration: $N = 12$, $K = 0.1, K_1 = 0.2, \omega_0 = 0.05$ and $d = 5$.

3 Conclusions

For real applications, namely mobile robots, unmaned aerial vehicles, and others, one needs to define a collision avoidance mechanism. For this purpose, we added term, based on the Kuramoto model, in a model of phase-coupled oscillators for symmetric circular formations. When an agent has neighbors in its vicinity, this term tries to balance its phase with them in such a way that it goes to their opposite direction.

The concept of vicinity is defined by a radius around the agent and its size depends also on the agents size, because of the reaction time, before an imminent collision. The bigger the agents are the higher the radius must be, as they have to start to maneuver before they reach a critical distance.

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