ADAPTIVE CONTROL FOR ATTENUATING VIBRATIONS USING NONUNIFORM SAMPLING

J. Jugo * **I. Arredondo** *

* *D. Electricidad y Electrónica, F. Ciencia y Tecnología (UPV/EHU) Campus Leioa, apdo 48940 , Bizkaia, SPAIN*
E-mail: josu.jugo@ehu.es, Fax:6013071

Abstract: Different adaptive control algorithms have been developed for attenuating vibrations on systems such as electromechanical devices. In this work it is introduced the use of nonuniform sampling methods reducing the mean sampling rate, with the aim of minimizing the execution time of such algorithms. In particular, the proposed algorithm is a feedforward adaptive vibration controller which generates sinusoidal signals whose amplitudes and phases are updated by the adaptation algorithm at randomized sampling times. Such signals are applied at the system obtaining a minimization of the vibrations. Simulation results show the effectiveness of the adaptive control approach, allowing an important increment in the mean sampling period.

Keywords: Randomized sampling, adaptive control, vibration attenuation

1. INTRODUCTION

A large number of electromechanical systems are affected by repetitive disturbances with sinusoidal form as is the case of rotating machines. The reduction of the effects of such vibrations can be crucial since those vibrations can damage internal parts of the devices or lead to the reduction of the mechanical strength, not allowing a proper operation, for instance in relation to the precision. In order to minimize those sinusoidal perturbations different techniques have been developed using passive and active elements, being remarkable the active noise control techniques based on digital signal processing, (Kuo and Morgan, 1996; Elliot, 2001).

In particular, adaptive filtering methods, (Widrow and Stearns, 1985; Goodwin and Sin, 1984; Elliot, 2001), are able to compensate not only large vibrations near critical resonance points, but also during non stationary states. Generally, the adaptive vibration controller has to generate signals at the synchronous frequency, which are related with the perturbation signal, in order to compensate its effects. With this purpose, different filtering strategies have been developed in the time and harmonic domains. For instance, LMS-type adaptive algorithms have proven them effectiveness in different real situations as can be rotating machines, (Nonami and Liu, 1999; J. Shi and Qin, 2004; Zhang and Markert, 2001).

On the other hand, the use of digital signal processing techniques for control purposes is increasing thanks to the higher calculation power and bandwidth of the modern digital signal processors and related hardware as reconfigurable hardware (FPGA). However, the increasing complexity of the task performed for the different control systems, as communication or fault detection, makes interesting the reduction of the sampling rate for allowing real-time performance of the overall system. In this sense, the use of randomized sampling techniques has led to increasing the working bandwidth without decrementing the sampling time, since its main advantage is the elimination of the aliasing.

These techniques are known as digital alias-free signal processing (DASP), and their advantages have shown its special interest in the analysis of high frequency signals. In particular, the use of pseudorandom sampling techniques can improve the useful bandwidth with high sampling times in the mean, (I. Bilinskis, 1992). On the other hand, these techniques add, logically, some drawbacks related directly to the stochastic nature of the resulting dynamics.

The main purpose of this work is the introduction of an adaptive vibration control scheme which applies DASP techniques, showing that it is possible to improve the calculation time maintaining the reduction of the vibrations. In this paper, the proposed scheme is a modified version of an adaptive feedforward controller of least-squares type, (Widrow and Stearns, 1985), which generates periodic compensating signals ideally having adequate frequency, magnitudes and phases respect to the synchronous disturbances in order to minimize the resulting vibrations. In such scheme, the main characteristic is the application of a randomized adaptive algorithm for obtaining the estimated parameter vector.

The paper is organized as follows. The basic properties of the randomized signal processing are briefly described in section 2. Section 3 describes the problem statement and the scheme of the modified adaptive feedforward control strategy to reduce the sinusoidal perturbations. Simulation results which illustrate the effectiveness of the presented approach to reduce the vibrations in the system are presented in section 4. Finally, conclusions and future research perspectives end the paper in section 5.

2. RANDOMIZED SIGNAL PROCESSING

The importance of digital signal processing (DSP) today is out of discussion, being the fast Fourier transform (FFT) one of the fundamental tools. DSP techniques are based, in general, on the uniform and periodic sampling of signals, leading to the well-known problem of aliasing. This phenomenon appears similarly in multirate systems which are today profusely used. In contrast, the aliasing problem disappears applying randomized sampling times, for any finite sampling-time in the mean, leading to DASP techniques, (I. Bilinskis, 1992; *http://www.edi.lv/dasp-web/dasp-papers/dasp-papers.html*, n.d.).

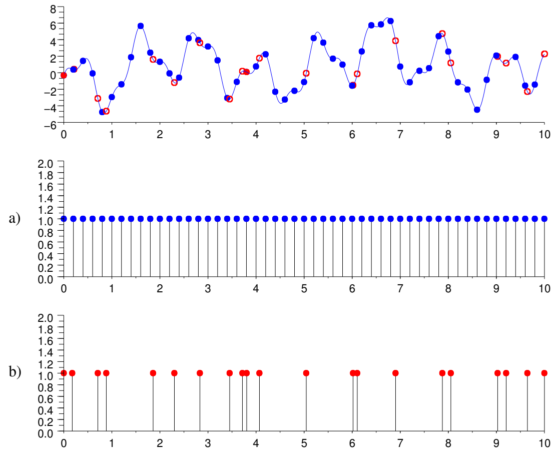

Figure 1. Different sampling techniques. a) Regular sampling b) Randomized sampling.

However, the aliasing-free property is due to the theoretically possible zero (infinitesimally small) distance between two random sampling instants. In practice, this is not possible and a minimum distance must be defined. Although the alias-free condition is not fullfilled, this minimum time distance is directly related to the equivalent Nyquist frequency using DASP for any mean sampling-time, (I. Bilinskis, 1992; *http://www.edi.lv/dasp-web/dasp-papers/dasp-papers.html, n.d.)*. For example, in Fig. 1, the uniform sampling and randomized sampling of a signal is represented. In the first case the sampling frequency is 5Hz and then the maximum frequency to be consider 2.5Hz. In the second case, the mean sampling frequency is 2Hz while the maximum frequency to be consider is 11.5/2 Hz since the minimum sampling distance is 1/11.5 s. That is, using a nonregular sampling technique with a higher sampling period in the mean the applicable frequency bandwidth is higher than using a regular sampling time.

One of the main drawbacks of the DASP technique is that very extended DSP techniques, as the FFT algorithm, are not valid and, in general, specific algorithms must be developed for using nonuniform sampling in any application. The discrete implementation of the Fourier transform for a signal $x(t)$ with randomized sampling times can be written as

$$\mathcal{X}(f) = \sum_{k=1}^{n} x(t_k) e^{-2\pi f t_k j} \qquad (1)$$

with n the number of samples and t_k the randomized sampling instants, which presents a very similar implementation of an ordinary regular DFT, but there is not a fast version like the FFT algorithm. In order to show the validity of the DFT transform (1), an example is considered applying randomized sampling times of mean $E[T_k] = 1$s., with $T_k = t_k - t_{k-1}$, to a pure sinusoid signal of frequency $f = 10Hz$. The resultant signal representation in the frequency domain can be observed

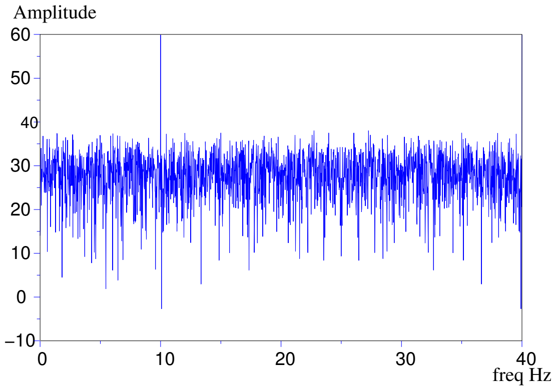

Figure 2. Spectrum of a one tone signal at 10Hz sampled at 1Hz (mean) using DASP

in Fig. 2. The minimum distance between samples is $\Delta_k = 1/50$s. and, then the equivalent Nyquist frequency is $25Hz$ (the first frequency alias is $40Hz$, not shown in the figure). Note that, if a regular sampling is utilized, the equivalent frequency spectrum is obtained by mean of a sampling time $E[T_k] = T = 0.05$s, that is, twenty times lower.

In Fig. 2, another characteristic derived from the randomized sampling can be observed: the increment of the noise floor at non existent frequency components in the original signal under analysis. Although this property can be a serious drawback in different applications, the use of specific reconstruction techniques can avoid this problem.

In the following sections, the application of randomized sampling for attenuating vibrations is analyzed.

3. PROBLEM STATEMENT AND ESTIMATION ALGORITHM

In this section it is presented an adaptive filter based on the LMS algorithm for attenuating vibrations which uses nonuniform sampling techniques. In Fig. 3, the basic block diagram of an adaptive filter is displayed. The objective is to develop a particular version of the adaptive scheme of such figure with a nonregular sampling time in order to minimize the calculation power required, taking into account the previously presented theoretical considerations about the aliasing effect. Note that the main objective is not the evaluation of an adaptive scheme in particular but the application of DASP techniques for vibration reduction.

The scheme selected for randomizing is an adaptive feedforward vibration controller (AFVC), which has been successfully applied using standard periodic sampling techniques, with an adequate sampling frequency, (J. Shi and Qin, 2004).

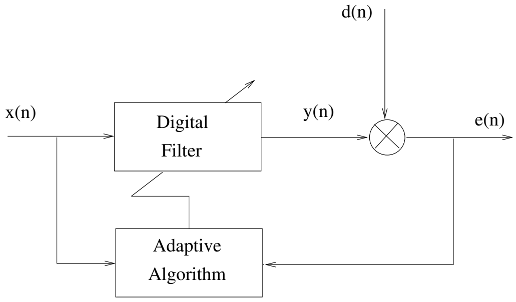

Figure 3. Block diagram of an adaptive filter

The scheme of the randomized version of the AFVC (RAFVC) is shown in Fig. 4. In such scheme, there are two different inputs for the controller: the signal $y(t)$ where the perturbation effect acts and the frequency of the sinusoidal perturbation Ω, which is considered known in this paper. The perturbation signal (continuous) may be described as

$$d(t) = A_d \sin(\Omega t + \varphi_d) = A_{d_1} \sin(\Omega t) + A_{d_2} \cos(\Omega t)$$

and the output of the RAFVC will be also a sinusoidal signal, defined by the next discrete expression

$$\begin{aligned} \vartheta(kT) &= A_\vartheta(j) \sin(\Omega kT + \varphi_\vartheta(j)) \\ &= A_s(j) \sin(\Omega kT) + A_c(j) \cos(\Omega kT) \end{aligned} \quad (2)$$

Analyzing this signal, two different discrete times are observed: a regular and periodic discrete time denoted by kT and a second discrete time represented by the sequence index j. The first regular kT time represent the actualization time of the perturbation correction signal $\varphi(kT)$ while the indice j represents the randomized actualization of the adaptive parameters of the controller at time t_j where $A_c(j)$ and $A_s(j)$ are adjusted to minimize the effect of the synchronous disturbance. Note that in the original adaptive scheme, the adaptation instants t_j and the uniform sampling-instants $t_k = kT$ are coincident.

In this case, the desired output signal is considered null and, then, $r(j) = 0$ and the performance measure is given by

$$\xi(j) = E\left\{\epsilon^2(j)\right\} = E\left\{y^2(j)\right\} \quad (3)$$

This scheme is considered as a direct adaptive method, (J. Shi and Qin, 2004).

3.1 Adaptive algorithm

The system outputs denoted by means of $y(t) = \begin{bmatrix} y_1 \; y_2 \; \dots \; y_n \end{bmatrix}^T$ are the input signals of the adaptive controller, being n the number of system

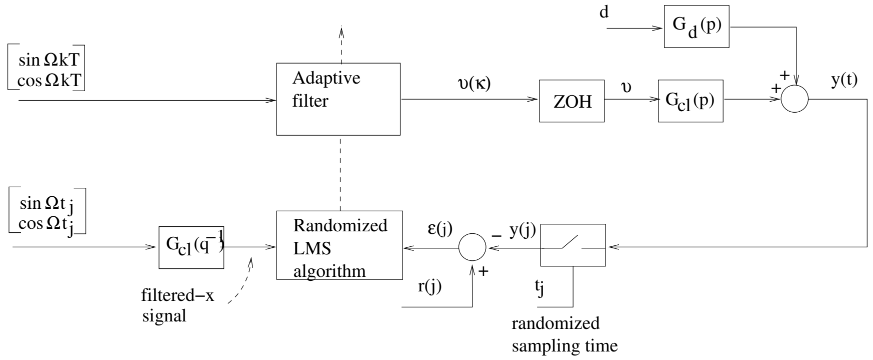

Figure 4. Block diagram of the vibration controller including the RAFVC, with an output perturbation.

outputs. These signals are sampled in randomized sampling instants t_j, obtaining $y(j)$ in discrete time. The adaptive controller provides a discrete signal vector $\vartheta(k) = \left[\, \vartheta_1 \;\, \vartheta_2 \;\, \ldots \;\, \vartheta_n \,\right]^T$ which tries to compensate the vibrations and each of the signals ϑ_i follows the expression given in eq. (2). In this case, a one-to-one relation between inputs and outputs is considered. Note that the controller output is defined for regular sampling instant $t_k = kT$.

In addition, the adaptive parameters in eq. (2) are grouped in the vector

$$\begin{aligned}\hat{\theta}_c(j) = \big[\, & A_{1s}(j) \;\, A_{1c}(j) \;\, A_{2s}(j) \;\, A_{2c}(j) \\ & A_{ns}(j) \;\, A_{nc}(j) \,\big]^T\end{aligned} \tag{4}$$

From Fig. 4 the output signal can be represented as

$$y(t) = G_d(s)d(t) + G_{cl}(s)\vartheta(t) \tag{5}$$

where the transfer functions $G_d(s)$ and $G_{cl}(s)$ describe the influence of the disturbance and the input over the output $y(t)$, respectively, and $\vartheta(t)$ is the continuous version of the RAFVC controller output, obtained with a zero order hold.

The design objective is the obtaining of the RAFVC parameter vector θ_c which minimizes the performance measure of eq. (3). As is shown in the block diagram, the filtered-x variant of the LMS algorithm is considered, (Widrow and Stearns, 1985). This objective is fulfilled adjusting the vector θ_c along the gradient direction Δ. As this gradient is unavailable, the next unbiased estimate of the gradient is used, (Widrow and Stearns, 1985)

$$\hat{\Delta}(j) = 2y(j)G_{cl}(q_j^{-1}) \begin{bmatrix} \sin \Omega t_j \\ \cos \Omega t_j \end{bmatrix} \tag{6}$$

where $G_{cl}(q_j^{-1})$ represents the discrete version of $G_{cl}(s)$, being a transfer function whose parameters changes stochastically with t_j. Thus, the resulting system can be considered as a jump discrete system, (Mariton, 1990).

However, since the parameter adaptation rate will be chosen to be much lower than the closed-loop dynamics described by $G_{cl}(s)$, it makes sense to replace this transfer function by a constant, for example, $\lambda/2$. Consequently, the eq. (6) can be rewritten as

$$\hat{\Delta}(j) = \lambda y(j) \begin{bmatrix} \sin \Omega t_j \\ \cos \Omega t_j \end{bmatrix} \tag{7}$$

Defining

$$\varphi(j) = \begin{bmatrix} s_1 & c_1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & s_1 & c_1 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & s_1 & c_1 \end{bmatrix}$$

with $s_1 = \sin \Omega t_j$ and $c_1 = \cos \Omega j$ and $\varphi(j)$ is a $n \times 2n$ matrix, the adaptive algorithm becomes

$$\hat{\theta}_c(j) = \hat{\theta}_c(j-1) + \gamma \varphi(j)^T y(j)$$

where γ is a real constant which must be chosen such that the estimation process is much slower that the closed-loop dynamics described by $G_{cl}(s)$, (J. Shi and Qin, 2004).

An important characteristic of the proposed scheme observed in Fig. 4 derives from the nonuniform nature of the adaptation process and the uniform nature of the compensation signal itself.

Note that the adaptive signals (and the below presented estimation algorithm) depend on the knowledge of the frequency Ω and noise and variations in the measured speed can lead to low effectiveness in the vibration cancellation and stability problems, (Nonami and Liu, 1999). Those problems must be considered in future works.

3.2 Stability

The performance of the standard LMS algorithm is well-known, (Widrow and Stearns, 1985; Elliot, 2001). However, the stochastic nature of the actualization instants in the proposed adaptive controller must be considered. The requisites for stability can be resumed into two conditions: the different random processes are stationary and independent and the adaptation process is sufficiently slow. The first condition leads to a constant input correlation matrix $\mathbf{R}$ and the second, to fulfill the condition

$$0 < \gamma < \frac{1}{\lambda_{max}}$$

where λ_{max} is the largest eigenvalue of the matrix $\mathbf{R}$.

From eq. (5) the discrete output $y(j)$ can be interpreted as a deterministic sinusoidal signal sampled randomly plus a discrete stochastic process. Then, the randomized LMS algorithm considered fulfills the required conditions if the random process t_j and any other random process actuating in the system are stationary and independent, with a sufficiently low γ.

4. SIMULATION RESULTS

This section presents simulation results which show the effectiveness of the RAFVC adaptive controller developed to reduce the vibrations caused by sinusoidal perturbations. These simulations have been performed using the java-based Ptolemy II software developed in the University of California at Berkeley. The system used as testbed is described by the next equations:

$$y(t) = G_r(s)(r(t) + d(t))$$

where the transfer matrix is

$$G_r(s) = \begin{pmatrix} \frac{10}{1+1e-3s} & \frac{0.9}{1+5e-4s} \\ \frac{-2}{1+2e-3s} & \frac{7}{1+8e-3s} \end{pmatrix}$$

As can be observed in Fig. 5, where the scheme of the proposed example is shown, the perturbation signals are first considered in the input and, being the reference signal zero, the objective is to reduce the vibrations around the equilibrium point. In particular, the block implementing the randomized sampling process is shown in Fig. 6. The mean sampling period is $2e-2$ s. while the minimum time distance between samples is $1e-4$s., and the same regular period is used to calculate the compensating sine and cosine signals. In addition, the frequency of the perturbation signals is 600Hz. Note that the adaptive estimation of the coefficients is performed with a mean frequency of $50Hz$ which is much lower that the required using a regular sampling period.

Fig. 7 shows the important reduction of the distortion observed in this example for the selected randomized adaptive algorithm, validating the proposed scheme.

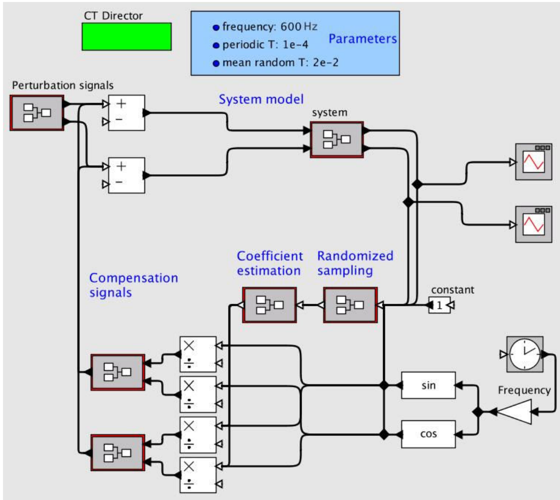

Figure 5. Implementation of the RAFVC adaptive algorithm using Ptolemy II software.

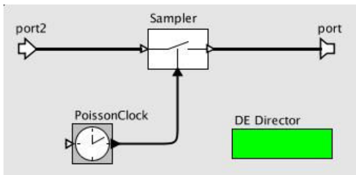

Figure 6. Implementation of the randomized sampling using PtolemyII.

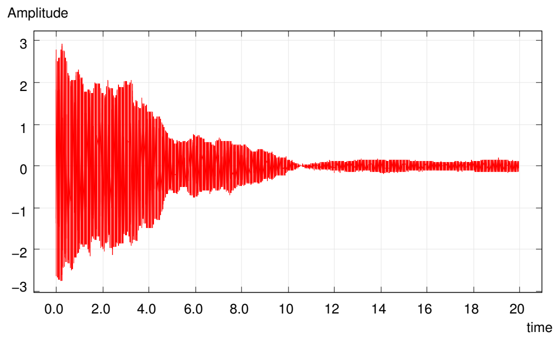

Figure 7. Elimination of the distortion in the proposed system (Perturbation in the input).

The result of the randomized adaptation process is shown in Fig. 8, where it is displayed the time evolution of the parameters which define the amplitudes and phases of the compensation signals generated by the adaptive vibration controller. Such a figure shows the convergence of the parameters. In particular, the convergence rate which appears in these simulations is obtained using a constant $\gamma = 0.04$ in the adaptive algorithm.

An interesting feature of the scheme implemented in Fig. 5 using Ptolemy II is the possibility of remote access by mean of java applets. In particular, this example is accessible in the web page http:/www.ehu.es/gaudee/jjugo/applets/rafvc/rafvc.html.

Similar considerations are valid for the elimination of distortion when the perturbation signals act in the output, as in the scheme proposed in Fig.

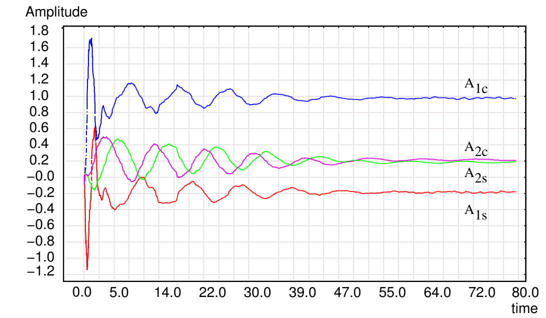

Figure 8. Evolution of the adaptive vibration controller parameters

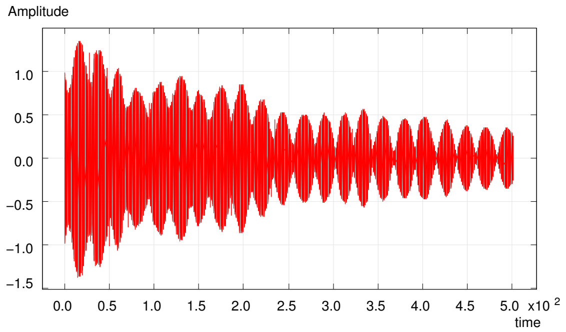

Figure 9. Elimination of the distortion in the proposed system (Perturbation in the output).

4. Fig. 9 shows the elimination of the distortion for sinusoidal perturbation signals in the output, which is slower than in the previous case for this particular example.

Finally, it should be noted that these results have been obtained for a constant speed. In real systems speed variations are very usual and their effect reduces the effectiveness of the vibration elimination and can lead to stability problems. Such difficulties must be considered in future works.

5. CONCLUSIONS

In this work the scheme of an adaptive vibration controller using randomized sampling-time in the estimation algorithm is presented. The use of such a controller results in a reduction of the periodical perturbations observed in the system outputs thanks to the compensation signals generated by the proposed adaptive control scheme RAFVC, while the calculation power is optimized. This improvement is obtained since the controller updates the parameters which define the amplitudes and phases of the compensation signals in randomized time instants and those resulting signals effectively counteract the effects of the perturbations. On the other hand, the compensation signals are generated regularly in periodic time instants.

The effectiveness of the developed randomized adaptive controller has been shown by means of simulation results, allowing an important reduction of the calculation time in the mean.

In conclusion, such a control strategy could provide an interesting solution, for instance, in high-speed mechanical systems.

In future research works, the convergence of the parameter vector should be compared with equivalent schemes using regular sampling techniques. In addition, different scheme alternatives can be analyzed and possible real-time implementation problems must be considered.

6. ACKNOWLEDGMENTS

The authors are very grateful to the CICYT, the university of the Basque Country and the Basque Government for the support of this work, through projects DPI2002-04155-C02-01, PTR95-0897.OP.CT and 9/UPV00224.310-15254/2003, and the Predoctoral grant BF104.466, respectively.

REFERENCES

Elliot, S. (2001). *Signal processing for active control*. Academic Press. London.

Goodwin, G. C. and K. S. Sin (1984). *Adaptive Filtering, Prediction and Control*. Prentice-Hall Inc. New Jersey.

http://www.edi.lv/dasp-web/dasp-papers/dasp-papers.html (n.d.).

I. Bilinskis, A. Mikelsons (1992). *Randomized signal processing*. Prentice-Hall International.

J. Shi, R. Zmood and L. Qin (2004). Synchronous disturbance attenuation in magnetic bearing systems using adaptive compensating signals. *Control Engineering Practice* **12**, 283–290.

Kuo, S.M and D.R Morgan (1996). *Active noise control systems*. Wiley-Interscience. NewYork.

Mariton, M. (1990). *Jump Linear Systems in Automatic Control*. Marcel- Dekker. NewYork.

Nonami, K. and Z. Liu (1999). Adaptive unbalance vibration control of magnetic bearing system using frequency estimation for multiple periodic disturbances with noise. In: *Proceedings 1999 IEEE Conf. Control Applications*. pp. 576–581. Hawai,Usa.

Widrow, B. and S.D. Stearns (1985). *Adaptive signal processing*. Prentice-Hall International.

Zhang, X. T. and R. Markert (2001). Adaptive control methods to minimize the unbalance response of magnetically suspended elastic rotors during run-up and run-down. In: *Proceedings of the Sixth International Symposium on Magnetic Suspension Technology*. pp. 13–18. Turin, Italy.