CLOSED-LOOP IDENTIFICATION PROPERTIES IN A GENERIC TWO-DEGREE OF FREEDOM CONTROL SYSTEM

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Abstract: The paper analyses the asymptotic limit variances of a closed-loop identification scheme based on a method introduced by the authors and named *KB*-parametrization. The limit variances reachable in closed-loop identification based on this parametrization are analyzed and their convergence is shown to the open-loop case.

Keywords: two-degree of freedom controller, closed-loop identification

1. INTRODUCTION

Many adaptive control methods are based on some kind of closed-loop identification scheme. A *generic two-degree of freedom* (*G2DOF*) scheme was introduced in Keviczky (1995), when the process is open-loop stable and it is allowed to cancel the stable process poles, which case occurs at many practical tasks. This framework and topology are based on the *Youla-parametrization* (Maciejowski, 1989). providing all realizable stabilizing regulators (*ARS*) for open-loop stable plants and capable to handle the plant time-delay, too.

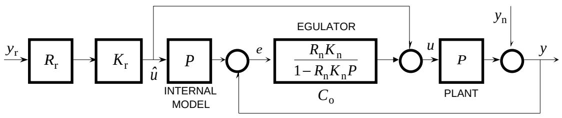

Fig. 1. The *generic 2DOF* (*G2DOF*) control system

A *G2DOF* control system is shown in Fig. 1, where y_r is the reference and y_n is the output noise (or disturbance) signal. The optimal *ARS* regulator of the *G2DOF* scheme can be given by an explicit form

$$C_o = \frac{R_n K_n}{1 - R_n K_n P} = \frac{Q_o}{1 - Q_o P} = \frac{R_n G_n P_+^{-1}}{1 - R_n G_n P_- z^{-d}} \quad (1)$$

where

$$Q_o = Q_n = R_n K_n = R_n G_n P_+^{-1} \quad \text{with} \quad K_n = G_n P_+^{-1} \quad (2)$$

is the associated optimal *Y-parameter* furthermore

$$Q_r = R_r K_r = R_r G_r P_+^{-1} \; ; \; K_r = G_r P_+^{-1} \quad (3)$$

assuming that the process is factorable as

$$P = P_+ \overline{P}_- = P_+ P_- z^{-d} \quad (4)$$

where P_+ means the *IS* and P_- does the *IU* factors, respectively. z^{-d} corresponds to the discrete time-delay, where d is the integer multiple of the sampling time. It is interesting to see how the transfer characteristics of the closed-loop look like:

$$\begin{aligned} y &= R_r K_r P \, y_r + \left(1 - R_n K_n P\right) y_n = \\ &= R_r G_r P_- z^{-d} y_r + \left(1 - R_n G_n P_- z^{-d}\right) y_n = y_t + y_d \end{aligned} \quad (5)$$

where y_t is the tracking (servo) and y_d is the regulating (or disturbance rejection) independent behaviors of the closed-loop response, respectively.

For the above *G2DOF* scheme closed-loop identification (*ID*) is needed. The scheme in Fig. 1 suggests a special way for combined *ID* and control. Introduce the internal auxiliary signal $\hat{u}$ from Fig. 1

$$\hat{u} = P_r K_r y_r = P_r G_r P_+^{-1} y_r \quad (6)$$

Observe that it is possible to use $\hat{u}(k)$ as an input signal and $y(k)$ as output signal generated by the apriori known part of the controller in a closed-loop to the identification procedure.

Besides the above two signal pairs $\left[\hat{u}, \Delta\varepsilon^{F}\right]$ and $\left[\hat{u}, y\right]$ it is possible to find further pairs: $\left[u, y\right]$, $\left[\hat{u}, x\right]$ or $\left[\hat{u}, e\right]$, which can also be used for closed-loop *ID* of the model M (see Fig. 2, which is another form of Fig. 1). These possible cases differ by the resulting modeling errors, strongly influencing the model accuracy in certain frequency domains.

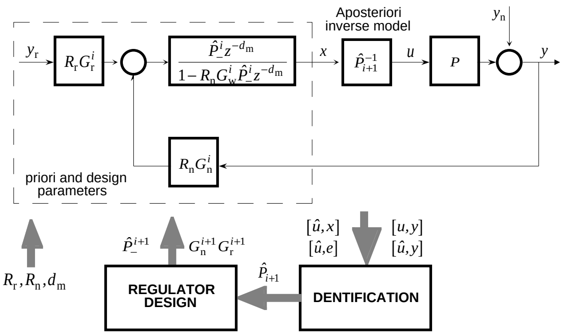

Fig. 2. Special scheme formulating combined *ID* and control strategy

2. COMPARISON OF CLOSED-LOOP *ID* ERRORS

Introduce the additive

$$\Delta = P - \hat{P} \quad ; \quad \Delta_+ = P_+ - \hat{P}_+ \quad ; \quad \Delta_- = \overline{P}_- - \hat{\overline{P}}_- \tag{7}$$

and relative model errors

$$\ell = \frac{\Delta}{\hat{P}} = \frac{P - \hat{P}}{\hat{P}} \quad ; \quad \ell_+ = \frac{\Delta_+}{\hat{P}_+} \quad ; \quad \ell_- = \frac{\Delta_-}{\hat{\overline{P}}_-} \tag{8}$$

In the sequel it is shown how the modeling errors of different *ID* methods depend on the relative model error ℓ.

Open-loop ID

$$\varepsilon_{\mathrm{ol}} = y - \hat{P}\,u' = \hat{P}\,\ell\,u'\Big|_{\ell\to 0;\,u'=y_\mathrm{r}} \approx P\,\ell\,y_\mathrm{r} \tag{9}$$

where u' used in open-loop is assumed equal to y_r.

Parallel-in-loop ID

$$\varepsilon_{\mathrm{pil}} = y - \hat{P}\,u = \frac{R_\mathrm{r} G_\mathrm{r} \hat{P}_- z^{-d}}{1 + R_\mathrm{n} G_\mathrm{n} \hat{P}_- z^{-d}\,\ell}\,\ell\,y_\mathrm{r}\Bigg|_{\ell\to 0} \approx$$
$$\approx R_\mathrm{r} G_\mathrm{r} P_- z^{-d}\,\ell\,y_\mathrm{r}\Big|_{\overline{P}_-=1} = R_\mathrm{r}\,\ell\,y_\mathrm{r} \tag{10}$$

where the *ID* is performed in the closed-loop between u and y. It is interesting to note that in this case u depends also on the output noise y_n, which makes the input correlated (caused by the so-called "circulating noise"), therefore special further conditions are to be fulfilled.

ID based on KB parametrization

There is a natural possibility to perform *ID* avoiding the above "circulating noise" issue, namely to perform the *ID* between $\hat{u}$ (see Fig. 1) and y. In this approach (called *KB-parametrization* (Keviczky, Bányász, 1994; 1995)) $\hat{u}$ depends on the apriori model estimate $\hat{P}_i$, so only iterative scheme can be constructed.

$$\varepsilon_{\mathrm{KB}} = \frac{\left(R_\mathrm{r} G_\mathrm{r} \hat{P}_- z^{-d}\right)\left(1 - R_\mathrm{n} G_\mathrm{n} \hat{P}_- z^{-d}\right)}{1 + R_\mathrm{n} G_\mathrm{n} \hat{P}_- z^{-d}\,\ell}\,\ell\,y_\mathrm{r}\Bigg|_{\ell\to 0} \approx$$
$$\approx \left(R_\mathrm{r} G_\mathrm{r} P_- z^{-d}\right)\left(1 - R_\mathrm{n} G_\mathrm{n} P_- z^{-d}\right)\ell\,y_\mathrm{r}\Big|_{\overline{P}_-=1} = \tag{11}$$
$$= R_\mathrm{r}\left(1 - R_\mathrm{n}\right)\ell\,y_\mathrm{r}$$

where $\varepsilon_{\mathrm{KB}} = y - \hat{P}\,\hat{u}$ and

$$\hat{u} = R_\mathrm{r} K_\mathrm{r}\, y_\mathrm{r} = R_\mathrm{r} G_\mathrm{r} \hat{P}_+^{-1}\, y_\mathrm{r} \tag{12}$$

(If an independent output noise is present with a usual linear noise model, then it is possible to prove that the output noise in the *G2DOF* closed-loop scheme asymptotically becomes also independent similarly to the open-loop case (Keviczky and Bányász, 1998).)

j	Type	ε	W_j
1	$\varepsilon_{\mathrm{ol}}$	$y - \hat{P}\,u'$	$\lvert P \rvert$
2	$\varepsilon_{\mathrm{pil}}$	$y - \hat{P}\,u$	$\lvert P_\mathrm{r} \rvert$
3	$\varepsilon_{\mathrm{KB}}$	$y - \hat{P}\,\hat{u}$	$\lvert P_\mathrm{r}(1 - P_\mathrm{n}) \rvert$

The three cases are summarized in the above Table, where the different weighting factors W_j are shown for the different cases.

Note that the accuracy of the estimated model at a given frequency is inverse proportional to the weight in the modeling error at that frequency. Observe that W_1, W_2 are low-pass filters. So "good" model estimation can not be expected around the vital cross-over frequency ω_c using these cases. W_3 gives the best weighting, because its maximum is the geometrical mean of the tracking and regulating bandwidths.

3. EQUIVALENT NOISE-MODEL STRUCTURES

Based on Fig. 1. and 2 one can easily draw the final *ID* scheme for the *KB*-parametrization as Fig. 3 shows. Here the noise-model H_o is introduced generating the output noise y_n from the independent source noise w and input r.

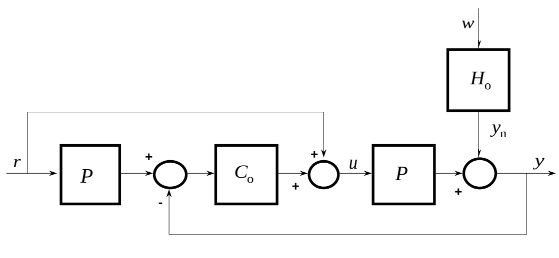

Fig. 3. The closed-loop *ID* scheme for *KB*-parametrization

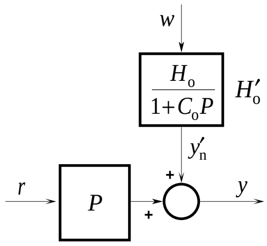

Fig. 4. The virtually opened loop with the "augmented" noise model

Assuming that C_o is according to (1), i.e. an ARS Youla parametrized regulator, it is easy to check that the KB-parametrization opens the closed-loop for reference model excitation in the exact model matching case, when $\hat{P}=P$. However, the original output noise model H_o is changed ("augmented"). Based on this observation an equivalent "open-loop" scheme can be derived shown in Fig. 4.

The KB-scheme with the Y-parameter gives the sensitivity function

$$\frac{1}{1+C_oP}=\left.\frac{1}{1+\frac{Q_oP}{1-Q_oP}}\right|_{\Delta=0}=1-Q_oP=1-R_nG_nP_-z^{-d} \tag{13}$$

so the augmented modified noise model of the scheme (see Fig. 5) is

$$H_o'=H_o(1-Q_oP)=H_o\left(1-R_nG_nP_-z^{-d}\right) \tag{14}$$

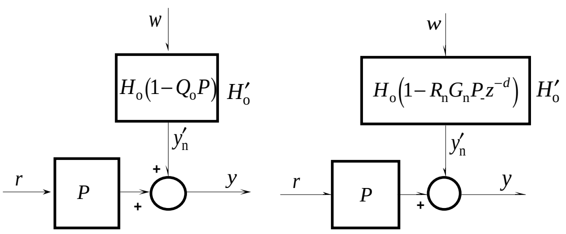

Fig. 5. Equivalent noise-model schemes

The KB-parametrization, when P is not known (Fig. 6) and based on the model $\hat{P}$ of the process, gives the equivalent closed-loop transfer function form as

$$y=\frac{1+C\hat{P}}{1+CP}Pr+\frac{1}{1+CP}y_n=\left(\frac{1+C\hat{P}}{1+CP}P\right)r+\frac{H_o}{1+CP}w=$$
$$=P'r+H'w=P'\left(\hat{P},P,C\right)r+H'(H_o,P,C)w \tag{15}$$

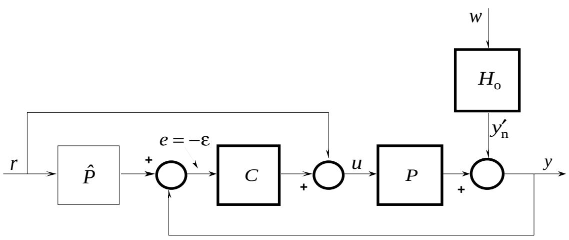

Fig. 6. Model-based KB-parametrized closed-loop

The equivalent virtually opened closed-loop is now shown in Fig. 7, where $P'\left(\hat{P}=P\right)=P$ and $H'\left(\hat{P}=P\right)= \quad =H_o(1-Q_oP)=H_o'$. This figure presents the possible identification scheme via ε.

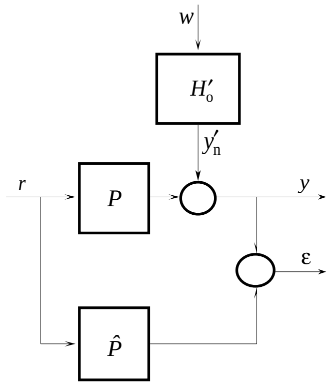

Fig. 7. Equivalent model-based noise-model scheme for the KB-parametrized closed-loop

Let us investigate P' and H' in the $K\ B$ parametrization. These transfer functions P' and H' are the projected version of the original P and H_o for $\Delta,\ell\neq 0$. Thus

$$P'=\frac{1+C\hat{P}}{1+CP}P=\left(1-\frac{C\Delta}{1+CP}\right)P=\left(1-\hat{Q}\Delta\right)P \tag{16}$$

and assuming $C=\frac{Q}{1-Q\hat{P}}$ and $\hat{Q}=\frac{C}{1+CP}$

$$P'=P-\frac{CP\Delta}{1+CP}=P-\frac{QP\Delta}{1+Q\Delta}=\frac{1}{1+Q\Delta}P \tag{17}$$

where the complementary sensitivity function

$$T=\frac{CP}{1+CP}=\frac{\frac{Q}{1-Q\hat{P}}P}{1+\frac{Q}{1-Q\hat{P}}P}=\frac{QP}{1+\Delta Q} \tag{18}$$

was used. Similarly the sensitivity function

$$S=\frac{1}{1+CP}=\frac{1}{1+\frac{Q}{1-Q\hat{P}}P}=1-\frac{QP}{1+Q\Delta} \tag{19}$$

Observe the limiting properties at $\Delta,\ell\to 0$, i.e. $\hat{P}=P$

$$Q_o=Q(\Delta=0)=R_nG_n\hat{P}_+^{-1}\Big|_{\Delta=0}=R_nG_nP_+^{-1} \tag{20}$$

$$P'(\Delta=0)=P \tag{21}$$

$$\left.\frac{CP}{1+CP}\right|_{\Delta=0}=Q_oP=R_nG_nP_+^{-1}P_+P_-z^{-d}=R_nG_nP_-z^{-d} \tag{22}$$

$$S=\left.\frac{1}{1+CP}\right|_{\Delta=0}=(1-Q_oP)=1-R_nG_nP_-z^{-d} \tag{23}$$

The projection of the original noise-model H_o is more complicated:

$$\begin{aligned} H'_o &= (1-QP)H_o\big|_{\Delta=0} = \left(1-P_n G_n P_- z^{-d}\right)H_o \\ &= (1-Q_o P)H_o = \left[1-P_n G_n \hat{P}_- z^{-d}(1+\ell)\right]H_o \end{aligned} \quad (24)$$

so

$$H'(\Delta=0) = H_o(1-Q_o P) = H'_o \quad (25)$$

Thus

$$y = \underbrace{\frac{1}{1+Q\Delta}P}_{P'}\, r + \underbrace{H_o\left(1-\frac{QP}{1+Q\Delta}\right)}_{H'} w = P' r + H' w \quad (26)$$

and represented by Fig. 8.

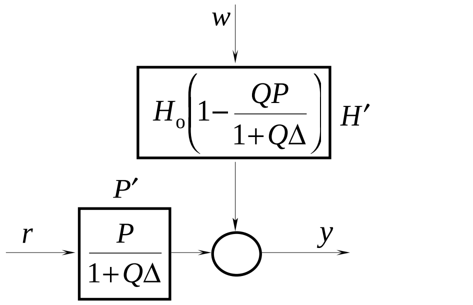

Fig. 8. Simplified equivalent model-based noise-model scheme for the *KB*-parametrized closed-loop

It is interesting to see that the input of the process at the *KB* parametrization is

$$\begin{aligned} u &= \frac{1+C\hat{P}}{1+CP} r - \frac{C}{1+CP} y_n \Big|_{\hat{P}=P} = r - H_o Q_o y_n \\ &= r - \left(H_o R_n G_n P_- z^{-d}\right) w \end{aligned} \quad (27)$$

$$\frac{C}{1+CP} = \frac{\dfrac{Q}{1-Q\hat{P}}}{1+\dfrac{Q}{1-Q\hat{P}}P} = \frac{Q}{1+Q\Delta}\Big|_{\Delta=0} = Q_o = R_n G_n P_- z^{-d} \quad (28)$$

This means that in case of convergence the reference signal r will act directly on the process.

4. PREDICTION ERROR IDENTIFICATION

The equivalent closed-loop system equation using the *KB* parametrization is

$$y = P' r + H' w \quad (29)$$

and it is easy to show that the prediction error equation is

$$\hat{\varepsilon}_{pr} = \frac{1}{\hat{H}'}\left\{\frac{1-\hat{P}Q}{1+Q\Delta}\Delta r + \left[H_o(1-QP)\frac{1+\hat{C}\Delta}{1+Q\Delta} - \hat{H}'\right] w\right\} + w \quad (30)$$

Investigate the possible convergence points, where $\hat{P}$ and $\hat{H}'$ will converge to

$$P' - \hat{P} = \frac{P}{1+Q\Delta} - \hat{P} = \frac{P-\hat{P}-\hat{P}Q\Delta}{1+Q\Delta} = \frac{1-\hat{P}Q}{1+Q\Delta}\Delta \quad (31)$$

and similarly

$$\begin{aligned} H' - \hat{H}' &= H_o\left(1-\frac{QP}{1+Q\Delta}\right) - \hat{H}' = H_o(1-QP)\frac{1+\hat{C}\Delta}{1+Q\Delta} - \hat{H}' = \\ &= \underbrace{H_o(1-Q_o P)}_{H'_o} - \hat{H}' = H'_o - \hat{H}' \end{aligned} \quad (32)$$

Using (32) finally the prediction error is

$$\hat{\varepsilon}_{pr} = \frac{1}{\hat{H}'}\left\{\frac{1-\hat{P}Q}{1+Q\Delta}\Delta r + \left[H_o(1-QP)\frac{1+\hat{C}\Delta}{1+Q\Delta} - \hat{H}'\right] w\right\} + w \quad (33)$$

where

$$\hat{H}' = H_o(1-QP)\frac{1+\hat{C}\Delta}{1+Q\Delta}\Big|_{\Delta=0} = H_o(1-Q_o P) = H'_o \quad (34)$$

and

$$\hat{C} = \frac{C}{1+C\Delta} \quad (35)$$

The possible convergence point $\Delta \to 0$ and $\hat{H}' \to H'(\Delta=0) = H'_o$ when P is in the model set and $\hat{H}'$ is in the augmented H'_o noise model set.

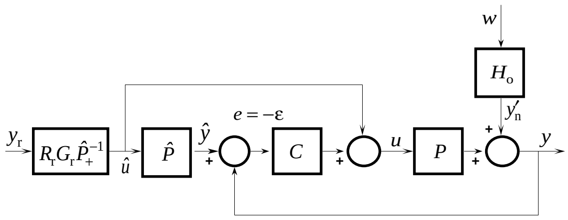

Fig. 9. The *G2DOF* system with noise-model

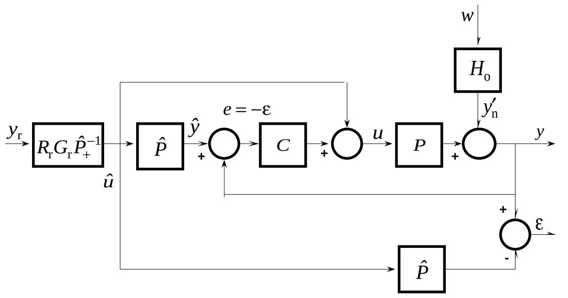

Fig. 10. *G2DOF* system with noise-model and *ID* scheme

4. GENERIC SCHEME AND *KB*-PARAMETRIZATION

Try to investigate the relationship between the *G2DOF* system and the *KB*-parametrization. The equivalent forms (Fig. 9 and 10) show the parallel relation to the *KB*-parametrized closed-loop. Because $\hat{P}$ is equivalent to a *KB*-parametrized auxiliary closed-loop, the above scheme can be again rewritten into a parallel special closed loop form given on Fig. 11, which is self-explanatory.

The related uncertainty model is shown on Fig. 12, where

$$-e=\varepsilon=\frac{R_{\rm r}G_{\rm r}\hat{P}_{-}z^{-d}\ell}{1+CP}y_{\rm r}+\frac{1}{1+CP}y_{\rm n}\quad ;\; y_{\rm n}=H_{\rm o}w \qquad (36)$$

Similarly to the previous sections one can write

$$-e=\varepsilon=\left(1-\frac{QP}{1+Q\Delta}\right)R_{\rm r}G_{\rm r}P_{-}z^{-d}\ell y_{\rm r}+H_{\rm o}\left(1-\frac{QP}{1+Q\Delta}\right)w=$$
$$=(1-QP)\frac{1+\hat{C}\Delta}{1+Q\Delta}\left\{\ell R_{\rm r}G_{\rm r}P_{-}z^{-d}y_{\rm r}+H_{\rm o}w\right\} \qquad (37)$$

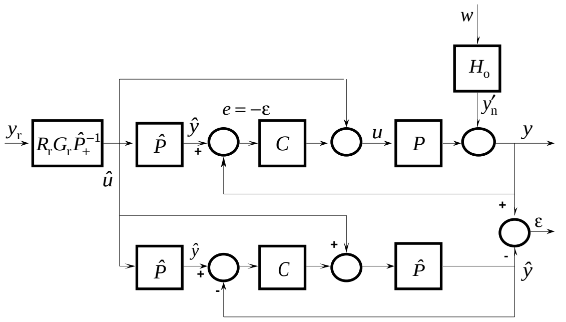

Fig. 11. The equivalent parallel closed-loop *ID* scheme for the *G2DOF* system

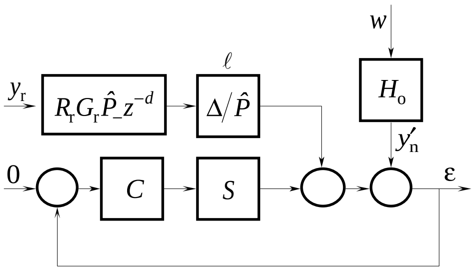

Fig. 12. Uncertainty scheme of the *G2DOF* system

Summarize the different formulas for Q

$$Q_{\rm o}=\frac{C_{\rm o}}{1+R_{\rm o}P}\quad;\quad C_{\rm o}=\frac{Q_{\rm o}}{1-Q_{\rm o}P}$$
$$Q=\frac{C}{1+C\hat{P}}\quad;\quad C=\frac{Q}{1-Q\hat{P}} \qquad (38)$$

Let us introduce

$$\hat{Q}=\frac{C}{1+CP}=\frac{\dfrac{Q}{1-Q\hat{P}}}{1+\dfrac{Q}{1-Q\hat{P}}P}=\left.\frac{Q}{1+Q\Delta}\right|_{\Delta=0}=Q=Q_{\rm o} \qquad (39)$$

Then it follows

$$\hat{Q}+\hat{Q}CP=C\rightarrow\hat{Q}=C\left(1-\hat{Q}P\right)\rightarrow C=\frac{\hat{Q}}{1-\hat{Q}P} \qquad (40)$$

by definition and similarly

$$\frac{Q}{1-QP}=\frac{C}{1-C\Delta}=\hat{C}\,, \qquad (41)$$

furthermore it is obvious that

$$\frac{\hat{Q}}{1-\hat{Q}S}=R\neq\hat{R}=\frac{Q}{1-QS} \qquad (42)$$

Note that in the *G2DOF* scheme $\hat{u}$ corresponds to r.

5. VARIANCE ESTIMATION

Here we follow the procedure and basic approach of Ljung (1987). The prediction error for $\Delta\rightarrow 0$ is

$$\hat{\varepsilon}_{\rm pr}(\Delta=0)\approx\frac{1}{\hat{H}'}\left\{S_{\rm o}\,\Delta\,r+\left(H'_{\rm o}-\hat{H}'\right)w\right\}+w$$
$$=\frac{1}{\hat{H}'}\left\{S_{\rm o}\,\Delta\,r+\tilde{H}'\,w\right\}+w \qquad (43)$$

where

$$\tilde{H}'=H'_{\rm o}-\hat{H}'$$
$$S_{\rm o}=1/(1+C_{\rm o}P)=1-Q_{\rm o}P \qquad (44)$$

and the corresponding frequency spectra

$$\Phi_{\hat{\varepsilon}_{\rm pr}}=\frac{1}{\left|\hat{H}'\right|^2}\left\{\left|S_{\rm o}\right|^2\left|\Delta\right|^2\Phi_r-\tilde{H}'\lambda_{\rm o}\right\}+\lambda_{\rm o}$$
$$\lambda_{\rm o}={\rm E}\left\{w^2\right\}=\Phi_w\;\;;\;\;H'_{\rm o}=H_{\rm o}S_{\rm o} \qquad (45)$$

So virtually $\Phi'_u=\left|S_{\rm o}\right|^2\Phi_r$ is the acting input and $\Phi'_{y_{\rm n}}=\left|S_{\rm o}\right|^2\left|H_{\rm o}\right|^2\lambda_{\rm o}$ is the output noise spectra. Let us follow the notations of Ljung (1987)

$${\rm cov}\begin{bmatrix}\hat{\rm G}_{\rm N}\\ \hat{\rm H}_{\rm N}\end{bmatrix}\cong\frac{n}{N}\Phi'_{y_{\rm n}}\begin{bmatrix}\Phi_{u'} & \Phi_{u'e}\\ \Phi_{eu'} & \lambda_{\rm o}\end{bmatrix}^{-1}=$$
$$=\frac{n}{N}\Phi'_{y_{\rm n}}\begin{bmatrix}\lambda_{\rm o} & -\Phi_{eu'}\\ -\Phi_{u'e} & \Phi'_u\end{bmatrix}\frac{1}{\rm det} \qquad (46)$$

where $\Phi_{u'e}=\Phi_{eu'}=0$ and

$$\Phi'_{y_{\rm n}}=\left|S_{\rm o}\right|^2\Phi_{y_{\rm n}}=\left|S_{\rm o}\right|^2\left|H_{\rm o}\right|^2\lambda_{\rm o} \qquad (47)$$

$${\rm det}=\lambda_{\rm o}\Phi'_u-\left|\Phi_{u'e}\right|^2=\lambda_{\rm o}\Phi'_u \qquad (48)$$

Finally the following covariances are obtained

$${\rm cov}\begin{bmatrix}\hat{\rm G}_{\rm N}\\ \hat{\rm H}_{\rm N}\end{bmatrix}\cong\frac{n}{N}\Phi'_{y_{\rm n}}\begin{bmatrix}\lambda_{\rm o} & -\Phi_{eu'}\\ -\Phi_{u'e} & \Phi'_u\end{bmatrix}\frac{1}{\lambda_{\rm o}\Phi'_u}=$$
$$=\frac{n}{N}\Phi'_{y_{\rm n}}\begin{bmatrix}\dfrac{1}{\Phi'_u} & 0\\ 0 & \dfrac{1}{\lambda_{\rm o}}\end{bmatrix} \qquad (49)$$

The partial covariances are

$${\rm cov}\left[\hat{\rm G}_{\rm N}\right]=\frac{n}{N}\frac{\Phi'_{y_{\rm n}}}{\Phi'_u}=\frac{n}{N}\frac{\left|S_{\rm o}\right|^2\Phi_{y_{\rm n}}}{\left|S_{\rm o}\right|^2\Phi_r}=\frac{n}{N}\frac{\Phi_{y_{\rm n}}}{\Phi_r} \qquad (50)$$

$$\mathrm{cov}\left[\hat{H}'_N\right] = \frac{n}{N}\frac{\Phi'_{y_n}}{\lambda_o} = \frac{n}{N}\frac{\Phi_{y_n}}{\lambda_o}|S_o|^2 \tag{51}$$

Shortly summarizing

$$\mathrm{cov}\left[\hat{G}_N\right] = \frac{n}{N}\frac{\Phi_{y_n}}{\Phi_r} \quad ; \quad \mathrm{cov}\left[\hat{H}'_N\right] = \frac{n}{N}\frac{\Phi_{y_n}}{\lambda_o}|S_o|^2 \tag{52}$$

which means that we obtained the classical results available for open-loop *ID* for the process parameters. The variance estimate of the noise parameters are weighted by $|S_o|^2$. This weight is generally smaller than one except the special medium frequency range, as Fig. 13 shows.

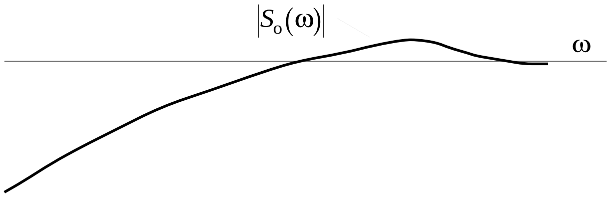

Fig. 13. Uncertainty scheme of the *G2DOF* system

The above result is quite understandable because instead of H_o the augmented noise-model $H_o S_o \approx H'_o$ is identified, because

$$H_o = \frac{1}{S_o} H'_o \tag{53}$$

So finally the elimination of the influence of $|S_o|$ is obtained. It is interesting to compare the variance estimate classical results for open-loop and closed-loop ID:

	open-loop	*K-B*	closed-loop
$\hat{G}_N$	$\frac{n}{N}\frac{\Phi_{y_n}}{\Phi_u}$	$\frac{n}{N}\frac{\Phi_{y_n}}{\Phi_r}$	$\frac{n}{N}\frac{\Phi_{y_n}}{\Phi_u}\frac{1}{1-\frac{\Phi_u^e}{\Phi_u}}$
$\hat{H}_N$	$\frac{n}{N}\frac{\Phi_{y_n}}{\lambda_o}$	$\frac{n}{N}\frac{\Phi'_{y_n}}{\lambda_o} = \frac{n}{N}\frac{\Phi_{y_n}}{\lambda_o}\lvert S_o\rvert^2$	$\frac{n}{N}\frac{\Phi_{y_n}}{\lambda_o}\left(1+\frac{\Phi_u^e}{\Phi_u^r}\right)$

Here

$$\frac{1}{1-\frac{\Phi_u^e}{\Phi_u}} > 1 \tag{54}$$

because

$$\Phi_u^e < \Phi_u \quad \text{and} \quad 1+\frac{\Phi_u^e}{\Phi_u^r} > 0 \tag{55}$$

This means that the closed-loop *ID* variances are always greater than the open-loop ones and *KB* scheme based *ID*. It is easy to check that

$$\frac{\Phi_u^r+\Phi_u^e}{\Phi_u^r} = \frac{|S_o|^2\Phi_r + |C|^2|S_o|^2|S_o|^2\lambda_0}{|S_o|^2\Phi_r} = 1+\frac{|C|^2|S_o|^2\lambda_o}{\Phi_r} = $$
$$= 1+\frac{|P|^2|C|^2|S_o|^2\lambda_o}{|P|^2\Phi_r} = 1+\frac{|T|^2\lambda_o}{|P|^2\Phi_r} \ge 1 \tag{56}$$

where

$$T = \frac{CP}{1+CP} \tag{57}$$

is the complementary sensitivity function, thus the second term is a special noise/signal ratio.

6. CONCLUSIONS

The paper analyses the asymptotic limit variances of a closed-loop identification scheme based on a method introduced by the authors and named *KB*-parametrization. This parametrization is a possible competitor of the Youla-Kucera parametrization, because it is much simpler. Adaptive and/or iterative control algorithms can be easily constructed using this approach. Generally parameter variances obtained from closed-loop identification are greater than the open-loop ones. Using the *KB*-parametrization the proper closed-loop *ID* can asymptotically reach the open-loop variances.

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