DISCRETE-TIME ADAPTIVE CONTROL FOR CONTINUOUS-TIME SYSTEMS USING N-DELAY LIMITING-ZERO MODEL AND ITS APPLICATIN TO A DD SERVO SYSTEM

Naoki Mizuno*, Takao Minamihama* and Takatomo Asai**

**Nagoya Institute of Technology, Dept. of Computer Science and Engineering,*
Graduate School of Engineering,
Gokiso-cho, Showa-ku, Nagoya, 466-8555 Japan
Tel &Fax:+82-52-735-5339, E-mail:nmizuno@nitech.ac.jp
*** Nagoya Institute of Technology, Dept. of Mechanical Engineering*

Abstract: A new method for designing a discrete-time adaptive control system for continuous-time system is presented. Using a new discrete-time model called the *n*-delay limiting-zero model, this method avoids the problem that the non-minimum phase zeros which arise in the case of rapid sampling of the continuous-time systems having the relative degree greater than one. In this paper, first, the motivation and principle of the *n*-delay limiting-zero model of the continuous-time system is discussed. Next, the procedure of constructing the adaptive control system based on the *n*-delay limiting-zero model is presented. Moreover, in order to illustrate the effectiveness of the proposed method, computer simulations and on-line experiments for DD servo system are carried out.

Keywords: adaptive control, discrete-time control, continuous time system, *n*-delay input model, limiting zero, non-minimum phase system., DD servo system

1. INTRODUCTION

For tracking control of time varying trajectories, the control system based on the equivalent inversion of the controlled plant is one of the simple control strategy and has extended to an adaptive control technique as model following or model reference adaptive control system (MRACS). However, unstable zeros (zeros on or outside the unit circle) arose from fast sampling of a continuous-time plant (Astrom et al., 1984; Hara et al., 1987) make the discrete-time MRAC impossible to apply many of real systems. In order to circumvent this problem, we have proposed a new MRACS based on the 2-delay limiting-zero model of the plant (Mizuno and Sato, 2005). This method is an extension of 2D-MRACS proposed by Miyasato (1992).

In this paper, we extend the above method by utilizing a new discrete-time model, called *n*-delay limiting-zero model, which is a combination of the 2-delay limiting-zero model (Mizuno and Sato, 2005) and *n*-delay input model (Miyasato, 1991). If a *n*-delay limiting-zero model is adopted as the discrete-time model of the continuous-time system, the discrete-time zeros can be easily assigned by fixed controller and the model reference adaptive controller is designed for the zero assigned discrete-time system, since the unstable zeros of the *n*-delay limiting-zero model are known beforehand. Therefore, the model reference control is achieved with simple adaptive controller even for continuous-time system having the relative degree greater than one at a fast sampling.

First, the relation between the continuous-time system and its *n*-delay discrete-time form is given. Next, we give the brief description of the *n*-delay input model and zero assignment by feed forward compensation. Then, we describe the limiting zeros of the *n*-delay input model and the motivation of recommending the *n*-delay limiting-zero model as a discrete-time model of the continuous-time system. In the main part of this paper, we propose a new design method of the MRACS based on the *n*-delay limiting zero model. Finally, the results of computer simulations and on-line experiments for DD servo system are presented.

2. ZERO ASSIGNMENT BY N-DELAY INPUT CONTROL

In this section, we give the brief description of the zero assignment by *n*-delay input control (Miyasato, 1991). Consider a continuous-time, single-input, single-output linear system, (called the controlled system) described by:

$$A_c(s)y(t) = B_c(s)u(t-\tau) \tag{1}$$

or

$$y(t) = G(s)u(t-\tau), \quad G(s) = B_c(s)/A_c(s) \tag{2}$$

where *G(s)* is a strictly proper rational transfer function and τ is a time delay. *y(t)* and *u(t)* denote the continuous-time system input and output, respectively. $A_c(s)$ and $B_c(s)$ are the polynomials in the differential operator, $s = d/dt$, of order n_c and *m* as follows.

$$A_c(s) = s^{n_c} + a_{Cn_c-1}s^{n_c-1} + .. + a_{C1}s + a_{C0} \quad (3)$$

$$B_c(s) = b_{Cm}s^m + b_{Cm-1}s^{m-1} + .. + b_{C1}s + b_{C0}, \; b_{Cm} \neq 0, \quad m < n_c \quad (4)$$

The control problem considered here is to determine a suitable control input $u(t)$ such that the discrete-time asymptotic model-following is achieved for any controlled system with arbitrary zeros. In order to solve this problem, the *n*-delay sampling method is adopted. Let (*A,b,c,d*) ($A \in R^{n_c \times n_c}$, $b, c \in R^{n_c}$, *d*(integer)>0) be a discrete-time model of (1),(2) based on the usual sampling method with the sampling period T/n, that is,

$$G_{T/n}(z) = z^{-d}c^T(zI - A)^{-1}b \quad (5)$$

Where d is a time delay in the discrete-time representation ($d = [\tau/(T/n)]$), and $G_{T/n}(z)$ is the pulse transfer function of the controlled system with sampling period T/n. For this controlled system, if the degree n_c and the time delay *d* are known, then the *n*-delay sampling representation of the controlled system can be seen as the *n*-inputs $[u_1(i), u_2(i), \cdots, u_n(i)]$, single-output $[y(i)]$ system described as follows:

$$A(z^{-1})y(i) = z^{-d}(B_1(z^{-1})u_1(i) + B_2(z^{-1})u_2(i) + \cdots + B_n(z^{-1})u_n(i)) \quad (6)$$

$$A(z^{-1}) = 1 + \sum_{k=1}^{n_c} a_k z^{-k} = z^{-n_c}\det(zI - A^n) \quad (7)$$

$$B_j(z^{-1}) = \sum_{k=1}^{n_c} b_{jk}z^{-(k-1)} = z^{-(n_c-1)}c^T adj(zI - A^n)A^{n-j}b \quad (j = 1,2,\cdots,n) \quad (8)$$

Where z is a shift operator, such as $zy(i) = y(i+1)$ and the next assumption is introduced for the controlled system.

[Assumption 1]

One of the b_{1j} (*j=1,…,n*) is non-zero and $B_j(z^{-1})$ are relatively prime.

If the above assumption holds and when the controlled system is known, the equivalent zeros of the controlled system can be arbitrary assigned by the following *n*-delay inputs control.

[Basic zero assign scheme]

$$u_j(i) = G_j(z^{-1})v(i) \quad (j=1,\ldots,n) \quad (9)$$

Where

$$G_j(z^{-1}) = g_{j1} + \cdots + g_{jng}z^{-(n_g-1)} \quad (k = 1,\cdots,n) \quad (10)$$

are the solutions of the following polynomial identity .

$$G_1(z^{-1})B_1(z^{-1}) + G_2(z^{-1})B_2(z^{-1}) + \cdots + G_n(z^{-1})B_n(z^{-1}) = B^*(z^{-1}) \quad (11)$$

Where,

$$B^*(z^{-1}) = b_0^* + b_1^*z^{-1} \cdots + b_{n_{b^*}}^* z^{-n_{b^*}} \quad (12)$$

is any Hurwits polynomial. From the assumption 1, $G_j(z^{-1})(j = 1,2,\cdots,n)$ satisfying (11), always exist. By using the *n*-delay inputs (9) and (10), all equivalent zeros from the signal $v(i)$ to the output $y(i)$ are assigned to the zeros of the polynomial $B^*(z^{-1})$. Moreover, if *n* is assigned equal to $n_c - 1$, the feed forward input compensator $G_j(z^{-1})(j = 1,2,\cdots,n)$ becomes to the simple feedforward gain g_j $(j = 1,2,\cdots,n)$.

However, in case of adaptive control, all parameters in polynomials $B_1(z^{-1}), B_2(z^{-1}), \cdots B_n(z^{-1})$ should be estimated and the polynomial identity should also be solved based on the estimated parameters to assign equivalent zeros. In this case, the numerical difficulty may occur when the estimated polynomials has common factors. If the polynomials $B_1(z^{-1}), B_2(z^{-1}), \cdots B_n(z^{-1})$ of the *n*-delay input model have some special properties, the procedure of assigning equivalent zeros may become more easy to implement. From this point of view, we investigate the limiting zeros of the *n*-delay input model of the continuous-time system.

Based on the limiting-zeros of the 2-delay discrete-time model of the continuous-time system (Mizuno and Sato, 2005), we have obtained the following result about limiting-zeros of the *n*-delay discrete-time model.

[Limiting-Zeros of *n*-delay input model]

If $B_c(s)$ has the zeros $z_1,\ldots,z_m$ then the zeros of $B_1(z^{-1}), B_2(z^{-1}), \cdots B_n(z^{-1})$ converge to the zeros of the following polynomials as *T* tends to 0 respectively.

$$B_i(z^{-1}) = B_{iN}(z^{-1})B_{iL}(z^{-1}) \; (i = 1,2,\cdots,n) \quad (13)$$

$B_{iN}(z^{-1})$ $(i = 1,2,\cdots,n)$: the unknown zeros of this *m*th order polynomial are $e^{z_1T'},\ldots,e^{z_mT'}$ (T': function of T)

$B_{iL}^r(z^{-1})$ $(i = 1,2,\cdots,n)$: the known zeros (*n*-delay limiting-zeros) of this (*r-1*)th order polynomials only depend on the relative degree (sometimes unstable).

$$B_{iL}\left(z^{-1}\right)=\frac{1}{r!}\left(\frac{T}{n}\right)^{r}\sum_{k=0}^{r-1} b_{ik}\, z^{-k} \tag{14}$$

$$b_{ik} = \sum_{l=0}^{k}\left\{\left(nl+n+1-i\right)^{r}-\left(nl+n-i\right)^{r}\right\}\times\frac{\left(-1\right)^{k-l}r!}{\left(k-l\right)!\left(r+l-k\right)!} \tag{15}$$

The fact that the limiting-zeros of the system only depend the relative degrees of the continuous-time system, is the key property of introducing the *n*-delay limiting-zero models (Astrom et al., 1984; Hara et al., 1987).

If the convergence speed of the excess zeros to the limiting-zeros is faster than the convergence speed of the other poles and zeros to corresponding discrete-time ones, then $B_1(z^{-1}), B_2(z^{-1}), \cdots B_n(z^{-1})$ can be approximated for the rapid sampling as;

$$B_i(z^{-1}) \approx B_{iN}(z^{-1})B_{iL}(z^{-1})\ (i=1,2,\cdots,n) \tag{16}$$

The convergence property is confirmed by numerical computation for many different types of continuous-time system and is theoretically proven for 1-delay simple case (Mizuno and Fujii,1988). If the n-delay model is adopted instead of the 2-delay input model, the input sampling period becomes relatively short for the same output sampling period. From this point of view, *n*-delay limiting-zero model can more accurately model the continuous-time system compared with the 2-delay limiting-zero model. This is the motivation of proposing the *n*-delay limiting-zero model of the plant.

3. CONTROLLER DESIGN USING N-DELAY LIMITING-ZERO MODEL

The idea mentioned above leads to the following new representation of the discrete-time model.

The control problem considered here is to determine a suitable control input $u(t)$ such that the discrete-time asymptotic model-following is achieved for any continuous-time system with stable continuous-time zeros.

Assume that the relative degree of the continuous-time system is known and the sampling interval is short compared with the time constant of the plant, then the plant can be described as follows.

$$\begin{aligned} A\left(z^{-1}\right)y(i) = z^{-d}&\left(B_{1N}\left(z^{-1}\right)B_{1L}\left(z^{-1}\right)u_1(i)+\cdots\right.\\ &\left.+B_{nN}\left(z^{-1}\right)B_{nL}\left(z^{-1}\right)u_n(i)\right)\\ &+z^{-d}\left(B_{1e}(z^{-1})u_1(i)+\cdots+B_{ne}(z^{-1})u_n(i)\right)\end{aligned} \tag{17}$$

$$B_{ie}(z^{-1}) = B_i(z^{-1}) - B_{iN}(z^{-1})B_{iL}(z^{-1})\ (i=1,2,\cdots,n) \tag{18}$$

where the polynomial $B_{ie}(z^{-1})\ (i=1,2,\cdots,n)$ is a residual polynomial which converges to zero as the sampling period T tends to 0. Based on this property, we will ignore the polynomial $B_{ie}(z^{-1})$ and modelled the controlled system by the approximate model called "*n*-delay limiting-zero model" of the form:

$$\begin{aligned} A\left(z^{-1}\right)y(i) = z^{-d}&\left(B_{1N}\left(z^{-1}\right)B_{1L}\left(z^{-1}\right)u_1(i)+\cdots\right.\\ &\left.+B_{nN}\left(z^{-1}\right)B_{nL}\left(z^{-1}\right)u_n(i)\right)\end{aligned} \tag{19}$$

If all continuous-time zeros lie in left half of the s-plane, this model has stable zeros $B_{iN}(z^{-1})$ and stable/unstable zeros of $B_{iL}(z^{-1})$.

In almost all cases, the *n*-delay limiting-zero model of the system has unstable zeros of $B_{iL}(z^{-1})$, the equivalent zeros of the controlled system should be assigned to stable ones using the following feed forward compensators.

$$B_{jN}(z^{-1})u_j(i) = G_j(z^{-1})v(i) \quad (j=1,\ldots,n) \tag{20}$$

where $G_i(z^{-1})\ (i=1,2,\cdots,n)$ is the solution of the following polynomial identity similar to Eq.(11).

$$G_1\left(z^{-1}\right)B_{1L}\left(z^{-1}\right)+\cdots+G_n\left(z^{-1}\right)B_{nL}\left(z^{-1}\right)=B^*\left(z^{-1}\right) \tag{21}$$

The above identity can be solved without numerical difficulty, because the polynomial $B_{iL}(z^{-1})$ is known and has simple parameters as shown in Eqs. (14),(15). Moreover, if the number of the input delay n is equal to the degree of the $B_{iL}(z^{-1})\ (i=1,2,\cdots,n)$, the polynomial solution $G_i(z^{-1})\ (i=1,2,\cdots,n)$ reduces to the scalar gain $g_i\ (i=1,2,\cdots,n)$. When the polynomial $B^*(z^{-1})$ is chosen as a Hurwits one, the model reference control law of the signal $v(i)$ can be synthesized as follows:

$$L(z^{-1})B^*(z^{-1})v(i) = A^*(z^{-1})y_m(i+d) - D(z^{-1})y(i) \tag{22}$$

where $y_m(i)$ is a desired output to be followed and

$$L(z^{-1}) = 1+\cdots+l_{d-1}z^{-(d-1)} \tag{23}$$

$$D(z^{-1}) = d_0+\cdots+d_{n-1}z^{-(n-1)} \tag{24}$$

are the solutions of the following Diophantine equation.

$$A^*(z^{-1}) = A(z^{-1})L(z^{-1}) + z^{-d}D(z^{-1}) \tag{25}$$

and

$$A^*(z^{-1}) = 1 + a_1^* z^{-1} + .. + a_{n_{a^*}}^* z^{-n_{a^*}} \tag{26}$$

is any Hurwits polynomial.

4. IMPROVEMENT OF INPUT PROPERTY IN N-DELAY INPUT CONTROL SYSTEM

From the theoretical analysis of the n-delay input control system, the real input to the controlled system $u(t)$

$$\begin{aligned} u(t) &= u_1(iT)(iT \le t < iT + T/n) \\ &= u_2(iT)(iT + T/n \le t < iT + 2T/n) \\ &\vdots \\ &= u_n(iT)(iT + (n-1)T/n \le t < (i+1)T) \end{aligned} \tag{27}$$

is uniformly bounded even for the controlled system with unstable zeros (non-minimum phase system). However, the input difference $\Delta u_{j-1}(i)$ $(j = 2, \cdots, n)$ between the n-delay inputs $u_j(i)$ and $u_{j-1}(i)$;

$$\begin{aligned} \Delta u_{j-1}(i) &= u_{j-1}(i) - u_j(i) \\ &= \frac{G_{j-1}(z^{-1})}{B_{j-1N}(z^{-1})} v(i) - \frac{G_j(z^{-1})}{B_{jN}(z^{-1})} v(i) \end{aligned} \tag{28}$$

does not always converge to the same sequence in steady state. To achieve that $\Delta u_{j-1}(i)$=0 as $i \to \infty$, we redefine the feed forward compensators $G_i(z^{-1})$ $(i = 1,2,\cdots,n)$ as follows.

$$\begin{aligned} G_i(z^{-1}) = G_{i0}(z^{-1}) + B_{1L}(z^{-1}) \cdots \\ B_{(i-1)L}(z^{-1}) B_{(i+1)L}(z^{-1}) \cdots \quad (i = 3, \cdots, n) \\ B_{(n-1)L}(z^{-1}) B_{nL}(z^{-1}) T_i(z^{-1}) \end{aligned} \tag{29}$$

Where $G_{i0}(z^{-1})$ $(i = 1,2,\cdots,n)$ are the one set of the solution of Eq.(21). In this case, if $T_1(z^{-1}) + \cdots + T_n(z^{-1}) = 0$, the following relations hold.

$$\begin{aligned} G_{10}(z^{-1}) B_{1L}(z^{-1}) + \cdots + G_{n0}(z^{-1}) B_{nL}(z^{-1}) \\ + B_{1L}(z^{-1}) \cdots B_{nL}(z^{-1}) (T_1(z^{-1}) + \cdots + T_n(z^{-1})) = B^*(z^{-1}) \end{aligned} \tag{30}$$

Equation (30) means that the re-defined feed forward compensators $G_i(z^{-1})$ $(i = 1,2,\cdots,n)$ also assigns the same zeros as Eq.(21) and have more degrees of freedom in its selection. When using the re-defined $G_i(z^{-1})$ $(i = 1,2,\cdots,n)$ as the feed forward compensator, the input differences of the n-delay inputs become to

$$\begin{aligned} \Delta u_j(i) = [&\frac{G_{j-10}(z^{-1}) + B_{1L}(z^{-1}) \cdots B_{j-2L}(z^{-1}) B_{jN}(z^{-1}) \cdots B_{nN}(z^{-1}) T_{j-1}(z^{-1})}{B_{j-1N}(z^{-1})} \\ &- \frac{G_{j0}(z^{-1}) + B_{1L}(z^{-1}) \cdots B_{j-1L}(z^{-1}) B_{j+1N}(z^{-1}) \cdots B_{nN}(z^{-1}) T_j(z^{-1})}{B_{jN}(z^{-1})}] v(i) \end{aligned} \tag{31}$$

To achieve that $\Delta u_{j-1}(i)$=0 as i $\to \infty$,

$$\frac{G_{j-10}(z^{-1}) + B_{1L}(z^{-1}) \cdots B_{j-2L}(z^{-1}) B_{jN}(z^{-1}) \cdots B_{nN}(z^{-1}) T_{j-1}(z^{-1})}{B_{j-1N}(z^{-1})} - \frac{G_{j0}(z^{-1}) + B_{1L}(z^{-1}) \cdots B_{j-1L}(z^{-1}) B_{j+1N}(z^{-1}) \cdots B_{nN}(z^{-1}) T_j(z^{-1})}{B_{jN}(z^{-1})}$$

should contain the model of the desired input (that is the same model of the desired output) in its factor (Mizuno and Sato, 1998;1999). Assume that the model of the desired input or output can be described as $\Omega(z^{-1})$ such that:

$$\Omega(z^{-1}) y_m(i) = 0 \quad (\Omega(z^{-1}) \bar{v}(i) = 0) \tag{32}$$

The arbitrary polynomials $T_1(z^{-1}), \cdots, T_n(z^{-1})$ should be the solution of the following equation.

$$\begin{aligned} &\frac{G_{j-10}(z^{-1}) + B_{1L}(z^{-1}) \cdots B_{j-2L}(z^{-1}) B_{jN}(z^{-1}) \cdots B_{nN}(z^{-1}) T_{j-1}(z^{-1})}{B_{j-1N}(z^{-1})} \\ &- \frac{G_{j0}(z^{-1}) + B_{1L}(z^{-1}) \cdots B_{j-1L}(z^{-1}) B_{j+1N}(z^{-1}) \cdots B_{nN}(z^{-1}) T_j(z^{-1})}{B_{jN}(z^{-1})} = \Omega(z^{-1}) R_j(z^{-1}) \end{aligned} \tag{33}$$

By using this polynomials $T_1(z^{-1}), \cdots, T_n(z^{-1})$, the differences between the n-delay inputs becomes zero as follows.

$$\Delta u_j(i) = \Omega(z^{-1}) R_j(z^{-1}) v(i) = 0 \tag{34}$$

It is noted that this modification does not affect the location of the assigned zeros and the tracking property of the control system.

5. DESIGN OF ADAPTIVE CONTROL SYSTEM

When the parameters of the controlled system are unknown, first, we estimate the unknown parameters then construct the adaptive controller based on the estimated parameters. For simplicity, we describe the controlled system in the following vector form.

[Controlled system]

$$y(i) = \theta^T \xi(i-1) \tag{35}$$

$$\theta = (a_1, \cdots, a_{n_d}, b_{10}, \cdots b_{1m}, \cdots b_{n0}, \cdots b_{nm})^T \tag{36}$$

$$\begin{aligned} \xi(i-1) = (&y(i-1), \cdots, y(i-n_d), \\ &B_{1L}(z^{-1}) u_1(i-d), \cdots, B_{1L}(z^{-1}) u_1(i-d-m), \\ &B_{2L}(z^{-1}) u_2(i-d), \cdots, B_{2L}(z^{-1}) u_2(i-d-m), \\ &\vdots \\ &B_{nL}(z^{-1}) u_n(i-d), \cdots, B_{nL}(z^{-1}) u_n(i-d-m))^T \end{aligned} \tag{37}$$

We consider the adaptive identifier, the adaptive law, and the control law as follows.

[Adaptive identifier]

$$\hat{y}(i) = \hat{\theta}^T(i-1) \xi(i-1) \tag{38}$$

$\hat{\theta}(i-1)$ is a current estimate of θ at the time instant $t=(i-1)T$ and is updated by the following adaptive law.

[Adaptive law]

$$\hat{\theta}(i)=\hat{\theta}(i-1)+\frac{P(i-1)\xi(i-1)}{1+\xi^T(i-1)P(i-1)\xi(i-1)}\varepsilon(i) \tag{39}$$

$$\varepsilon(i)=y(i)-\hat{y}(i) \tag{40}$$

$$P(i)=\frac{1}{\lambda_1(i)}\left(P(i-1)-\frac{\lambda_2(i)P(i-1)\xi(i-1)\xi^T(i-1)P(i-1)}{\lambda_1(i)+\lambda_2(i)\xi^T(i-1)P(i-1)\xi(i-1)}\right) \tag{41}$$

where , $P(0)=P(0)^T>0, 0<\lambda_1(i)\le 1$, $0\le\lambda_2(i)<2$.
If $|\varepsilon(i)|\le\delta$ $(\delta>0)$ then $\varepsilon(i)\leftarrow 0$ for robustness.

[Control law]

$$\hat{B}_{kn}\left(i-d_c,z^{-1}\right)u_k(i)=G_k\left(z^{-1}\right)v(i) \tag{42}$$

$$\hat{L}\left(i-d_c,z^{-1}\right)B^*\left(z^{-1}\right)v(i)=A^*\left(z^{-1}\right)y_m(i+d)-\hat{D}\left(i-d_c,z^{-1}\right)y(i) \tag{43}$$

where $d_c(\ge 0)$ is a computational delay.
$\hat{L}(i-d,z^{-1})$ and $\hat{D}(i-d,z^{-1})$ are determined such that

$$\hat{A}\left(i,z^{-1}\right)\hat{L}\left(i,z^{-1}\right)+z^{-d}\hat{D}\left(i,z^{-1}\right)=A^*\left(z^{-1}\right) \tag{44}$$

$$\hat{L}\left(i,z^{-1}\right)=1+\cdots+\hat{l}_{d-1}(i)z^{-(d-1)} \tag{45}$$

$$\hat{D}\left(i,z^{-1}\right)=\hat{d}_0(i)+\cdots+\hat{d}_{n_d-1}(i)z^{-(n_d-1)} \tag{46}$$

where $\hat{A}(i,z^{-1})$ and $\hat{B}_j(i,z^{-1})(j=1,2,\cdots,n)$ are

$$\hat{A}\left(i,z^{-1}\right)=1+\sum_{j=1}^{n_d}\hat{a}_j(i)z^{-j} \tag{47}$$

$$\hat{B}_{kn}\left(i,z^{-1}\right)=\sum_{j=1}^{m}\hat{b}_{kj}(i)z^{-(j-1)} \tag{48}$$

Fig 1 shows the structure of the proposed adaptive control system based on *n*-delay limiting-zero model.

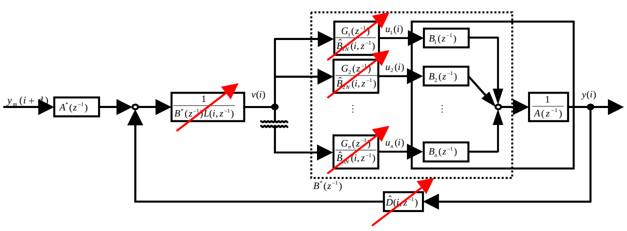

Fig.1 Structure of proposed adaptive control system

6. EVALUATION OF PROPOSED METHOD

In order to investigate the effectiveness of the proposed methods, first, computer simulations are performed for different types of continuous-time plants having the relative degree greater than one. Only typical results are shown by limitation space. For example, when the transfer function of the controlled system is described as follows:

$$G(s)=\frac{(s+5)}{(s+1)(s+2)(s+3)(s+4)} \tag{49}$$

The 3-delay limiting-zero model of the system is of the form as.

$$A(z^{-1})y(i)=z^{-1}(B_{1N}(z^{-1})B_{1L}(z^{-1})u_1(i)+B_{2N}(z^{-1})B_{2L}(z^{-1})u_2(i)+B_{3N}(z^{-1})B_{3L}(z^{-1})u_3(i)) \tag{50}$$

$$B_{1L}=\frac{T^3}{162}\left(19+34z^{-1}+z^{-2}\right)\ B_{2L}=\frac{T^3}{162}\left(7+40z^{-1}+7z^{-2}\right)\ B_{3L}=\frac{T^3}{162}\left(1+34z^{-1}+19z^{-2}\right), \tag{51}$$

In this case, although the desired output is the combination of the step and sinusoidal signal, the model of the input signal is set at $\Omega(z^{-1})=1-z^{-1}$ (model of step signal) for simplicity. The other design parameters in the adaptive schemes are assigned as follows.

Sampling period: T=0.03, *d*=1
Input delay *n: 2 or 3.*
Design polynomials: $A^*\left(z^{-1}\right)=\left(1-0.85z^{-1}\right)^4$, $B^*(z^{-1})=1$
Adaptive algorithm: $\lambda_1=1, \lambda_2=0$

Only typical simulation results are shown in Fig.2. In this figure, the upper traces show the outputs for *n*=2 (red line) and *n*=3(blue line), the middle and the lower show tracking errors (red line for *n*=2 and blue line for *n*=3) and control input for *n*=3 respectively. From these figures, it can be seen that the proposed adaptive controller works well and 3-delay input control has smaller tracking error than 2-delay input control.

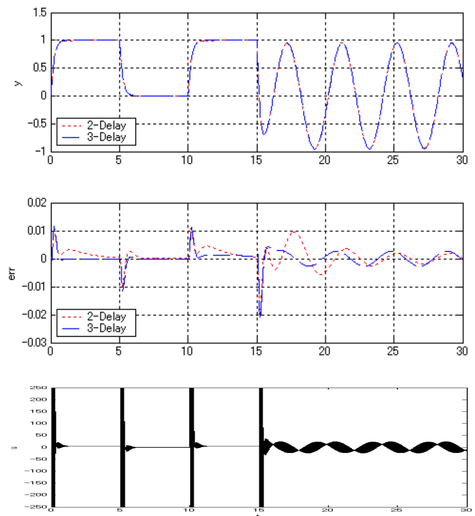

Fig.2 Simulation results of proposed adaptive control system

7. APPLICATION TO A REAL SYSTEM

Next, we apply the proposed schemes to the DD motor (Yokokawa Precision Corporation, Type: DMB1045). Table 1 shows the specifications of the DD motor.

The experimental setup consists of a host computer (AMD-K6:300MHz) with a WinCon2.0 (Real-time Controller; Quancer Consulting Inc.) interface board and a DD servomotor as shown in Fig.3.

Table 1 Specification of DD motor

Rated Output	W	380
Rated Torque	N·m	30
Rated Speed	rps	2
Encoder Resolution	p/rev	655,360
Rotor Inertia	$kg \cdot m^2$	23×10^{-3}

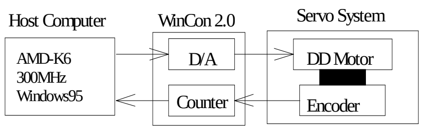

Fig. 3 Experimental setup

In the design of adaptive controllers, the model of the DD motor is assumed to be 2nd order system. The motor dynamics has been estimated using the following 2-delay limiting-zero model.

$$A(z^{-1})y(i) = \frac{T^2}{8} z^{-1}\Big(B_{1N}(z^{-1})(3+z^{-1})u_1(i) + B_{2N}(z^{-1})(1+3z^{-1})u_2(i)\Big) \quad (51)$$

In the experiments, the sampling interval of output T is set at 2ms. The other design parameters in the controller are assigned as follows.

Design polynomials: $A^*(z^{-1}) = (1-0.7z^{-1})^2$, $B^*(z^{-1}) = 1$

Adaptive algorithm: $P(0) = 0.1$, $\lambda_1 = \lambda_2 = 1$, $\delta = 0.001$

Only typical experimental results are shown in Fig.4.

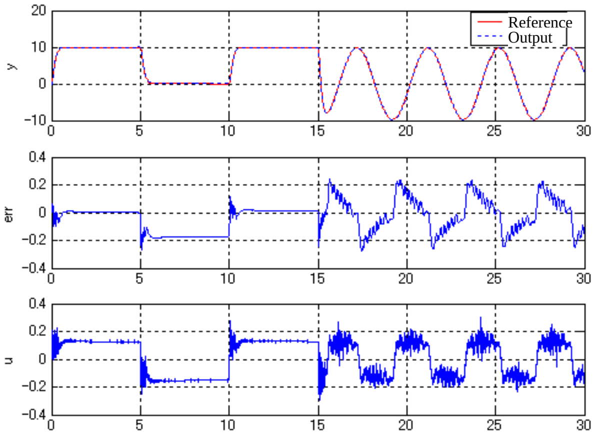

Fig. 4 Experimental result of tracking control by using proposed adaptive controller

In this figure, the upper traces show the reference signal (red line) and the output(blue line), the middle and the lower show tracking error and control input respectively. From these results, although the modeling error exists, the adaptive control system using n-delay limiting-zero model is effective to control the DD servo system with fast sampling rate.

8. CONCLUSIONS

In this paper, we have presented the discrete-time adaptive control system using *n*-delay limiting-zero model, especially to handle the continuous-time system having the relative degree greater than one. The control performance is evaluated by computer simulations and extensive experiments for DD servo system. From the experimental results, it is shown that the proposed scheme can be successfully applied to a real system. Moreover, the reduction of the input amplitude can be attained even when the model structure of the controlled system is not adequate.

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