SINGLE IMPULSIVE STABILIZATION OF COMPLEX DYNAMICAL NETWORKS

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Abstract

A single impulsive controller is designed for the efficient stabilization of complex dynamical networks with undirected strongly connected topology. In the modeling of the dynamical network, impulsive effects are considered in the information exchanging process of two connected nodes. The convergence rate of the stabilization process is meanwhile estimated together with the convergence analysis. A numerical example with small-world coupling is given to illustrate the derived theoretical results.

Key words

Stabilization; Complex dynamical network; Single impulsive controller.

1 Introduction

In the past decade, there has been an increasing interest in understanding the collective behavior of networked systems which are formed by local interconnections of small subsystems [Arenas et al., 2008; Wang and Chen, 2002; Strogatz, 2001]. Synchronization of complex dynamical network, which means that all of the network nodes agree upon certain dynamical trajectory depending on their initial conditions, intrinsic system dynamics and network structure, has been one of the most interesting collective behavior for networked systems [Zhou and Kurths, 2006; Lu et al., 2010; Cao et al., 2008; Lu and Chen, 2006; Belykh et al., 2006]. This is partly because synchronization behavior arises ubiquitously in biological systems and physical systems, and also its wide applications in the fields of parallel image processing [Krinsky et al., 1991], pattern storage and retrieval [Hoppensteadt and Izhikevich, 2000] and secure communication [VanWiggeren and Roy, 1998].

For many real-world networks, the transmitted signal between two connected nodes is often subject to instantaneous perturbations and experience abrupt change at certain instants which may be caused by switching phenomenon, frequency change or other sudden noise, that is, do exhibit impulsive effects [Guan et al., 2005; Bainov and Simeonov, 1989; Chen and Zheng, 2009]. Such systems can be described by impulsive differential equations which have been used successfully in modeling many practical problems that arise in the fields of natural sciences and technology [Bainov and Simeonov, 1989]. Some interesting results about synchronization of impulsive networks and stability of impulsive systems have been obtained in [Guan et al., 2005; Liu et al., 2005; Zhou et al., 2007; Zhang et al., 2006]. Synchronization of the dynamical networks without external force is realized by utilizing the interconnections between the nodes [Lu et al., 2010; Lu and Ho, 2010]. The final self-synchronized state, which depends not only on the network structure but also on the initial values, and the intrinsic dynamical behavior of each single node, is very difficult to predict. However, there is a strong requirement for many physical and biological dynamical networks to regulate the behavior of large ensembles of interacting units by external forces [Mazenc et al., 2008]. Hence, it is desirable and important to investigate the problem of stabilization of dynamical networks via controllers.

Impulsive controllers have shown its efficiency for the stabilization of dynamical networks [Guan et al., 2010; Liu et al., 2005]. However, in most of the previous results, impulsive controllers should be added to each of the nodes in the networks, which would make the implementation of the controllers very expensive and difficult. By utilizing the impulsive coupling between the nodes, a single impulsive controller, which is easy and cheap to implement in practice, is designed to stabilize the dynamical networks. As usual, strongly connected dynamical network with undirected coupling is studied. The concept of "average impulsive interval" [Lu et al., 2010] is used to describe wider range of impulsive signals and to make the obtained results less conservative. The convergence rate of the impulsively controlled dynamical network is estimated. Finally, a numerical example with small-world structure is given to illustrate the efficiency of the designed contoller.

$Notations$: The standard notations will be used in this paper. Throughout this paper, for real symmetric matrices X and Y, the notation $X \leq Y$ (respectively, $X < Y$) means that the matrix $X - Y$ is negative semi-definite (respectively, negative definite); I_n is the identity matrix of order n, and I is an identity matrix with compatible dimensions; we use $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ to denote the minimum and maximum eigenvalue of a real symmetric matrix, respectively; the notation $\mathbb{R}$ denotes the set of real numbers; $\mathbb{R}^{n\times n}$ are $n \times n$ real matrices; the superscript "T" represents the transpose; diag$\{\cdots\}$ stands for a block-diagonal matrix; matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 Model description and some preliminaries

We consider the following impulsive dynamical network [Lu et al., 2010]:

$$\begin{cases} \dot{x}_i(t) = Cx_i(t) + Bf(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), \\ \qquad t \geq 0,\ t \neq t_k,\ k \in \mathbb{N}, \\ x_j(t_k^+) - x_i(t_k^+) = \mu \cdot (x_j(t_k^-) - x_i(t_k^-)), \\ \qquad \text{for } (i,j) \text{ satisfying } a_{ij} > 0, \end{cases} \quad (1)$$

where $x_i(t) = \mathbb{R}^n$ is the state vector of the i-th node at time t; $C \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$; $f(x_i(t)) = [f_1(x_{i1}(t)), f_2(x_{i2}(t)), ..., f_n(x_{in}(t))]^T$; The configuration coupling matrix $A = (a_{ij})_{N\times N}$ is defined as follows: if there is a connection between node i and node j ($i \neq j$), then $a_{ij} = a_{ji} > 0$; otherwise, $a_{ij} = a_{ji} = 0$; and the diagonal elements are defined as $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$. According to the above definition, the configuration coupling matrix is symmetric, which implies that the corresponding network is undirected. $\Gamma = \text{diag}\{\gamma_1, \gamma_2, ..., \gamma_n\} > 0$ is the inner coupling positive definite matrix between two connected nodes i and j; c is the coupling strength; The time series $\{t_1, t_2, t_3, ...\}$ is a sequence of strictly increasing impulsive moments. Throughout this paper, μ is assumed to satisfy $|\mu| < 1$ which means that corresponding impulsive effects are synchronizing [Lu et al., 2010]. We always assume that $x_i(t)$ is left-hand continuous at $t = t_k$, i.e., $x(t_k) = x(t_k^-)$. Hence, the solutions of (1) are piecewise left-hand continuous functions with discontinuities at $t = t_k$ for $k \in \mathbb{N}$.

Assumption 1. *The nonlinear function $f(x(t)) = [f_1(x_1(t)), ..., f_n(x_n(t))]^T$ is assumed to satisfy a Lipschitz condition, that is, there exist constants $l_k > 0$ ($k = 1, 2, \ldots, n$) such that $|f_k(x_1) - f_k(x_2)| \leq l_k|x_1 - x_2|$ ($k = 1, 2, \ldots, n$) holds for any $x_1, x_2 \in \mathbb{R}$. Denote $L = diag\{l_1, l_2, ..., l_n\}$.*

Remark 1. *For complex dynamical networks, it is realistic to consider impulsive effects (sudden changing) in the process of signal exchanging instead of in each node. The constant impulsive gain considered in this paper is only for the sake of analytical simplification, and it does not cause any loss of generality in the sense of stabilization analysis.*

Remark 2. *From the second equation of* (1)*, we can derive that $x_i(t_k^+) = \mu \cdot x_i(t_k^-) + \tilde{c}_k(\forall\ \tilde{c}_k \in \mathbb{R}^n)$, which further implies that $x_i(t_k^+) - c_k = \mu \cdot (x_i(t_k^-) - c_k)(\forall\ c_k \in \mathbb{R}^n)$. Then, the impulses occurring in the process of coupling can be essentially regarded such that identical impulsive controllers are operating on the nodes of the network. Hence, a corresponding impulsive control strategy can be developed to determine the value of $x_i(t_k^+)$.*

Let x^* be an equilibrium point of the isolated dynamical system: $\dot{z}(t) = Cz(t) + Bf(z(t))$. In this section, a single impulsive controller will be used to stabilize the dynamical network (1) to the objective state x^*. Without loss of generality, the first node is selected to be controlled, and the impulsive controller is designed as follows:

$$x_1(t_k^+) - x^* = \mu \cdot (x_1(t_k^-) - x^*). \quad (2)$$

Let $e_i(t) = x_i(t) - x^*$ for $i = 1, 2, ..., N$. By adding the single impulsive controller (2) to the dynamical network (1), we obtain the following impulsive dynamical network:

$$\begin{cases} \dot{e}_i(t) = Ce_i(t) + Bg(e_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t), \\ \qquad t \geq 0,\ t \neq t_k,\ k \in \mathbb{N}, \\ e_j(t_k^+) - e_i(t_k^+) = \mu \cdot (e_j(t_k^-) - e_i(t_k^-)), \\ \qquad \text{for } (i,j) \text{ satisfying } a_{ij} > 0, \\ e_1(t_k^+) = \mu \cdot e_1(t_k^-), \end{cases} \quad (3)$$

where $g(e_i(t)) = f(e_i(t) + x^*) - f(x^*)$ and $c\sum_{j=1}^{N} a_{ij}\Gamma x^* = 0$ are utilized.

Since the impulsive effects are synchronizing and the impulses will be used for the stabilization of dynamical networks, we do not need long intervals between impulses. Hence, the following Definition 1 is given to guarantee that the occurrence frequency of the impulses is not too low.

Definition 1. *[Lu et al., 2010] The average impulsive interval of the impulsive sequence $\zeta = \{t_1, t_2, \cdots\}$ is* less than T_a, *if there exist positive integer N_0 and positive number T_a, such that*

$$N_\zeta(T, t) \geq \frac{T - t}{T_a} - N_0, \quad \forall T \geq t \geq 0, \quad (4)$$

where $N_\zeta(T, t)$ denotes the number of impulsive times of the impulsive sequence ζ in the time interval (t, T).

Definition 2. *The impulsive dynamical network (1) are said to be globally exponentially stabilized if there exist* $\eta > 0$*,* $T > 0$ *and* $M_0 > 0$*, such that for any initial values,* $\|e_i(t)\| \leq M_0 e^{-\eta t}$ *hold for all* $t > T$*, and for any* $i = 1, 2, ..., N$*.*

Lemma 1. *[Horn and Johnson, 1990] For an irreducible matrix* A *with non-negative off-diagonal elements, which satisfies the zero-row-sum condition, we have the following propositions:*

(1). If λ *is an eigenvalue of* A *and* $\lambda \neq 0$*, then* $\mathrm{Re}(\lambda) < 0$*;*

(2). A *has an eigenvalue* 0 *with multiplicity 1 and its corresponding right eigenvector is* $[1, 1, ..., 1]^T$*;*

Lemma 2. *For any vectors* $x, y \in \mathbb{R}^n$*, scalar* $\epsilon > 0$*, and positive definite matrix* $Q \in \mathbb{R}^{n\times n}$*, the following inequality holds:* $2x^T y \leq \epsilon x^T Q x + \epsilon^{-1} y^T Q^{-1} y$*.*

3 Stabilization of strongly connected networks

In this section, a criterion will be established to verify whether the single impulsive controller (2) is effective for the globally exponential stabilization of the undirected dynamical network (1) with a strongly connected structure. That is, globally exponential stability of the controlled dynamical network (3) will be studied. Undirected and strongly connectivity of the network means that the corresponding coupling matrix A is symmetric and irreducible.

Theorem 1. *Consider the controlled dynamical network (3) with a symmetric irreducible coupling matrix* A*. Suppose that Assumption 1 holds, and the average impulsive interval of the impulsive sequence* $\zeta = \{t_1, t_2, ...\}$ *is less than* T_a*. Then, the controlled dynamical network (3) is globally exponentially stable with the convergence rate* η *if*

$$\eta \triangleq \frac{2ln(|\mu|)}{T_a} + \delta < 0, \tag{5}$$

where $\delta = \lambda_{\max}(C + C^T + BB^T + L^T L)$*.*

Proof. Construct a Lyapunov function in the form of

$$V(t) = \sum_{i=1}^{N} e_i^T(t) e_i(t). \tag{6}$$

Thus, for $t \in (t_{k-1}, t_k]$, taking the time derivative of the Lyapunov function (6) along the trajectories of (3), we get

$$\begin{aligned}
&\dot{V}(t) \\
&= 2\sum_{i=1}^{N} e_i^T(t)[Ce_i(t) + Bg(e_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t)] \\
&= 2\sum_{i=1}^{N}[e_i^T(t)Ce_i(t) + e_i^T(t)Bg(e_i(t))] \\
&\quad +2c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_i^T(t)\Gamma e_j(t).
\end{aligned} \tag{7}$$

By Assumption 1 and Lemma 2, the following inequality can be derived:

$$\begin{aligned}
&2(x_i - x_j)^T B(f(x_i) - f(x_j)) \\
&\leq (x_i - x_j)^T (BB^T + L^T L)(x_i - x_j).
\end{aligned} \tag{8}$$

According to the facts that A is an irreducible and symmetric Laplacian matrix with zero-row-sum and non-negative off-diagonal elements, we can conclude from Lemma 1 that $\lambda_{\max}(A) = 0$. Hence, we obtain the following inequality:

$$\begin{aligned}
&2c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_i^T(t)\Gamma e_j(t) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N} 2ca_{ij}[\sum_{k=1}^{n} e_{ik}^T(t)\gamma_k e_{jk}(t)] \\
&= \sum_{k=1}^{n} 2c\gamma_k (e^k(t))^T A e^k(t) \\
&\leq \lambda_{\max}(A)\sum_{k=1}^{n} 2c\gamma_k (e^k(t))^T e^k(t) \\
&= 0,
\end{aligned} \tag{9}$$

where $e^k(t) = [e_{1k}(t), e_{2k}(t), ..., e_{Nk}(t)]^T$.

By referring to the inequalities (8) and (9), it follows from (7) that, for $t \in (t_{k-1}, t_k]$,

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^{N} e_i^T(t)(C + C^T + BB^T + L^T L)e_i(t) \\
&\leq \lambda_{\max}(C + C^T + BB^T + L^T L)\sum_{i=1}^{N_i} e_i^T(t) e_i(t) \\
&= \delta \cdot V(t).
\end{aligned} \tag{10}$$

Hence, we yield

$$V(t) \leq e^{\delta(t - t_{k-1})} V(t_{k-1}^+), t \in (t_{k-1}, t_k], k \in \mathbb{N} \tag{11}$$

It follows from (3) that $x_j(t_k^+) - x_i(t_k^+) = \mu \cdot (x_j(t_k^-) - x_i(t_k^-))$, for each pair of (i, j) satisfying

$a_{ij} > 0$. Since A is irreducible, for any suffix j ($j \neq 1$), there exist suffixes $s_1, s_2, ..., s_m$, such that $a_{1,s_m} > 0$, $a_{s_m,s_{m-1}} > 0, \cdots$ and $a_{s_1,j} > 0$. Thus, for the pair of suffixes 1 and j ($\forall j = 2, 3, ..., N$), we have

$$\begin{aligned} & x_j(t_k^+) - x_1(t_k^+) \\ = & (x_j(t_k^+) - x_{s_1}(t_k^+)) + (x_{s_1}(t_k^+) - x_{s_2}(t_k^+)) + \cdots \\ & + (x_{s_m}(t_k^+) - x_1(t_k^+)) \\ = & \mu \cdot (x_j(t_k^-) - x_{s_1}(t_k^-)) + \mu \cdot (x_{s_1}(t_k^-) - x_{s_2}(t_k^-)) \\ & + \cdots + \mu \cdot (x_{s_m}(t_k^-) - x_1(t_k^-)) \\ = & \mu \cdot (x_j(t_k^-) - x_1(t_k^-)). \end{aligned} \tag{12}$$

Combining (12) with (2), we obtain $x_j(t_k^+) - x^* = \mu \cdot (x_j(t_k^-) - x^*)$, that is,

$$e_j(t_k^+) = \mu \cdot e_j(t_k^-). \tag{13}$$

Hence, for $t = t_k, k \in \mathbb{N}$, one gets

$$\begin{aligned} V(t_k^+) &= \sum_{i=1}^{N} e_i^T(t_k^+) e_i(t_k^+) \\ &= \mu^2 \sum_{i=1}^{N} e_i^T(t_k^-) e_i(t_k^-) \\ &= \mu^2 V(t_k^-). \end{aligned} \tag{14}$$

The following results come from (11) and (14).

For $t \in (t_0, t_1]$, $V(t) \leq e^{\delta(t-t_0)} V(t_0^+)$, which leads to $V(t_1) \leq e^{\delta(t_1-t_0)} V(t_0^+)$ and $V(t_1^+) \leq \mu^2 e^{\delta(t_1-t_0)} V(t_0^+)$.

Similarly, for $t \in (t_1, t_2]$, $V(t) \leq e^{\delta(t-t_1)} V(t_1^+) \leq \mu^2 e^{\delta(t_1-t_0)} V(t_0^+)$.

In general, for $t \in (t_k, t_{k+1}]$, $V(t) \leq \mu^{2k} e^{\delta(t-t_0)} V(t_0^+)$.

Let $N_\zeta(t, t_0)$ be the number of impulsive times of the impulsive sequence ζ on the interval (t_0, t). Hence, for any $t \in \mathbb{R}$, we obtain

$$V(t) \leq \mu^{2N_\zeta(t,t_0)} \cdot e^{\delta(t-t_0)} \cdot V(t_0^+). \tag{15}$$

Since the average impulsive interval of the impulsive sequence $\zeta = \{t_1, t_2, \cdots\}$ is less than T_a, we have

$$N_\zeta(t, t_0) \geq \frac{t - t_0}{T_a} - N_0, \quad \forall T \geq t \geq 0. \tag{16}$$

Since $|\mu| < 1$, it follows from (15) and (16) that

$$\begin{aligned} & V(t) \\ \leq & \mu^{2N_\zeta(t,t_0)} \cdot e^{\delta(t-t_0)} \cdot V(t_0^+) \\ \leq & \mu^{2(\frac{t-t_0}{T_a} - N_0)} \cdot e^{\delta(t-t_0)} \cdot V(t_0^+) \\ \leq & \mu^{-2N_0} \cdot e^{\frac{2ln(|\mu|)}{T_a}(t-t_0)} \cdot e^{\delta(t-t_0)} \cdot V(t_0^+) \\ \leq & \mu^{-2N_0} \cdot e^{(\frac{2ln(|\mu|)}{T_a} + \delta)(t-t_0)} \cdot V(t_0^+). \end{aligned} \tag{17}$$

It can be concluded from (17) that there exists constant $M_0 = \mu^{-2N_0}$, such that

$$V(t) \leq M_0 \cdot e^{\eta(t-t_0)} \cdot V(t_0^+), \tag{18}$$

which further implies that

$$\|e_i(t)\|^2 \leq V(t) = O(e^{\eta(t-t_0)}). \tag{19}$$

If follows from $\eta < 0$ that the whole dynamical network (1) can be globally exponentially stabilized to the equilibrium point x^* by the single impulsive controller (2). Theorem 1 is proved completely. □

Remark 3. *In Theorem 1, the effects of single impulsive controller and a single node dynamic concerning stabilization are respectively expressed by* $\frac{2ln(|\mu|)}{T_a}$ *and* δ. *The smaller* $\frac{2ln(|\mu|)}{T_a}$ *is, the higher the speed of the stabilizing process will be.*

Remark 4. *It should be noted that the equilibrium point* x^* *can be replaced by any trajectory* $s(t)$ *satisfying* $\dot{s}(t) = Cs(t) + Bf(s(t))$. *The whole dynamical network* (1) *can also be globally exponentially stabilized to the trajectory* $s(t)$ *by slightly modifying the single impulsive controller* (2) *to be* $x_1(t_k^+) - s(t) = \mu \cdot (x_1(t_k^-) - s(t))$.

4 Numerical example

In this section, a numerical example is given for the illustration of our theoretical results. A chaotic system is considered as an isolated node of the dynamical network, which is described by the following equation [Zou and Nossek, 1993]:

$$\dot{x}(t) = Cx(t) + Bf(x(t)), \tag{20}$$

where $x(t) = (x_1(t), x_2(t), x_3(t))^T \in \mathbb{R}^3$ is the state vector, and the parameters are $C = -\text{diag}\{1, 1, 1\}$, $B = \begin{pmatrix} 1.16 & -1.5 & -1.5 \\ -1.5 & 1.16 & -2.0 \\ -1.2 & 2.0 & 1.16 \end{pmatrix}$, and $f_i(s) = (|s+1| - |s-1|)/2$. Thus the Lipschitz condition is fulfilled with constants $l_1 = l_2 = l_3 = 1$. The above choice of the parameters is purposely to design a chaotic system shown in Figure 1. The single neural network model (20) has a chaotic attractor with the initial values $x_0 = (0.3, 0.2, 0.1)^T$. With the given parameters, the equilibria of (20) are, respectively, $x_1^* = (-1.22259936, -0.655802861, 0.697535771)^T$, $x_2^* = (0, 0, 0)^T$, and $x_3^* = (1.22259936, 0.655802861, -0.697535771)^T$. In this example, x_3^* is selected to be the objective state.

We consider a Watts-Strogatz small-world network [Watts and Strogatz, 1998] with 300 interconnected nodes. In this example, the parameters are set as $N =$

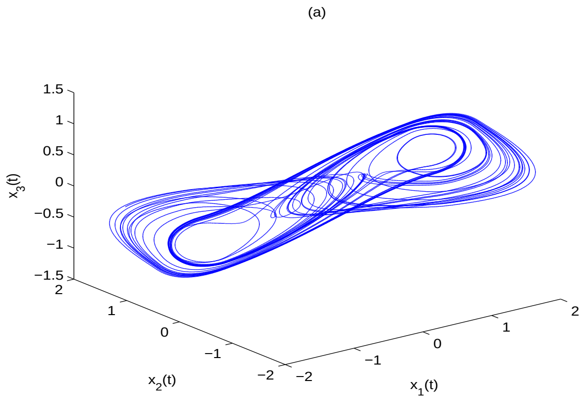

Figure 1. (a). Chaotic attractor; (b). Time series.

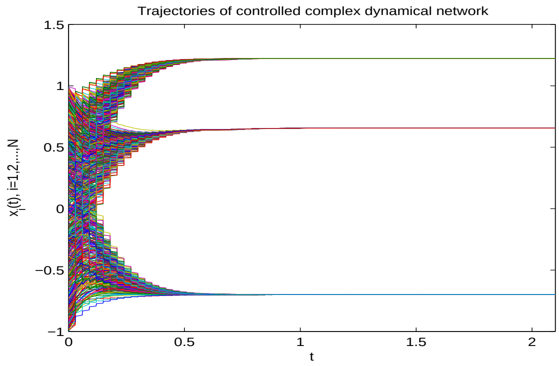

Figure 3. State variables of the small-world coupled dynamical network (1) under the single impulsive controller (2).

300, $K = 4$ and $p = 0.1$ to generate a small-world network. When the small-world network is generated, the coupling strength a_{ij} for each edge is defined as follows: if there is a connection between nodes i and j ($i \neq j$), then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0$. Suppose that the average impulsive interval T_a of the impulsive sequence is less than 0.037, and the impulsive strength $\mu = 0.8$. By simple computation, we obtain that $\frac{2ln(|\mu|)}{T_a} = -12.7511$, $\delta = 11.9753$ and $\eta = -0.7758$. According to Theorem 1, it can be concluded that the complex dynamical network (1) can be stabilized to the equilibrium point x_3^* via the single impulsive controller (2). Figure 2 represents an impulsive sequence $\bar{\zeta}$ with the average impulsive interval $T_a = 0.035$ and $N_0 = 20$. The stabilization process of the small-world coupled dynamical network is plotted in Figure 3, in which the initial conditions of the nodes are randomly chosen from $[-1, 1]$. The effective of the single impulsive controller has been illustrated by this numerical example.

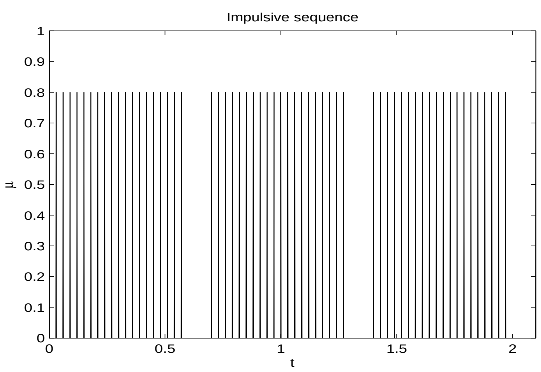

Figure 2. Impulsive sequence.

5 Conclusion

A single impulsive controller is designed to stabilize a complex dynamical network with impulsively interconnected nodes. There is no requirement on the lower bound and upper bound of impulsive intervals. The convergence rate of the stabilizing process is also obtained. A numerical example is finally given to illustrate the efficiency of the single impulsive controller.

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References

Arenas, A., íaz-Guilera, A. D., Kurths, J., Moreno, Y. and Zhou, C. (2008). Synchronization in complex networks. *Physics Reports*, 469(3):93–153.

Bainov, D. and Simeonov, P. (1989). *Systems with Impulsive Effect: Stability Theory and Applications*. Ellis Horwood Limited, Chichester, UK.

Belykh, I., Belykh, V. and Hasler, M. (2006). Synchronization in asymmetrically coupled networks with node balance. *Chaos*, 16:015102.

Cao, J., Chen, G. and Li, P. (2008). Global synchronization in an array of delayed neural networks with hybrid coupling. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 38(2):488–498.

Chen, W. and Zheng, W. (2009). Input-to-state stability and integral input-to-state stability of nonlinear impulsive systems with delays. *Automatica*, 45(6): 1481–1488.

Guan, Z., Hill, D. and Shen, X. (2005). On hybrid impulsive and switching systems and application to nonlinear control. *IEEE Transactions on Automatic Control*, 50(7):1058–1062.

Guan, Z., Liu, Z., Feng, G. and Wang, Y. (2010). Synchronization of complex dynamical networks with time-varying delays via impulsive distributed control. *IEEE Transactions on Circuits and Systems-I*, 57(8): 2182–2195.

Hoppensteadt, F. and Izhikevich, E. (2000). Pattern recognition via synchronization in phase-locked loop neural networks. *IEEE Transactions on Neural Networks*, 11(3):734–738.

Horn, R. and Johnson, C. (1990). *Matrix Analysis*. Cambridge University Press.

Krinsky, V., Biktashev, V. and Efimov, I. (1991). Autowave principles for parallel image processing. *Physica D*, 49(1-2):247–253.

Liu, B., Liu, X., Chen, G. and Wang, H. (2005). Robust impulsive synchronization of uncertain dynamical networks. *IEEE Transactions on Circuits and Systems-I*, 52(7):1431–1441.

Lu, J. and Ho, D. (2010). Globally exponential synchronization and synchronizability for general dynamical networks . *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 40(2):350–361.

Lu, J., Ho, D. and Cao, J. (2010). A unified synchronization criterion for impulsive dynamical networks. *Automatica*, 46:1215–1221.

Lu, W. and Chen, T. (2006). New approach to synchronization analysis of linearly coupled ordinary differential systems. *Physica D*, 213(2):214–230.

Mazenc, F., Malisoff, M. and Harmand, J. (2008). Further results on stabilization of periodic trajectories for a chemostat with two species. *IEEE Transactions on Automatic Control*, 53(1):66–74.

Strogatz, S. (2001). Exploring complex networks. *Nature*, 410:268–276.

VanWiggeren, G. and Roy, R. (1998). Communication with chaotic lasers. *Science*, 279(5354):1198–1200.

Wang, X. and Chen, G. (2002). Synchronization in scale-free dynamical networks: robustness and fragility. *IEEE Transactions on Circuits and Systems-I*, 49(1):54–62.

Watts, D. and Strogatz, S. (1998). Collective dynamics of 'small-world' networks. *Nature*, 393(6684): 440–442.

Zhang, H., Guan, Z. and Ho, D.(2006). On synchronization of hybrid switching and impulsive networks. *45th Proc. IEEE Conf. Decision Control*, pages 2765–2770.

Zhou, C. and Kurths, J. (2006). Dynamical weights and enhanced synchronization in adaptive complex networks. *Physical Review Letters*, 96(16):164102.

Zhou, J., Xiang, L. and Liu, Z. (2007). Synchronization in complex delayed dynamical networks with impulsive effects. *Physica A*, 384(2):684–692.

Zou, F. and Nossek, J. (1993). Bifurcation and chaos in cellular neural networks. *IEEE Transactions on Circuits and Systems-I*, 40(3):166–173.