STABILITY OF SOLUTION OF A HYDROELASTICITY PROBLEM FOR VISCOUS LIQUID

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Abstract

The problem about stability of the movement of the elastic plate, which is a part of the border dividing the areas filled with viscous incompressible liquid, is considered. The research method based on creation of Lyapunov's functionals for the related nonlinear system of the partial differential equations for the unknown aerohydrodynamic functions and deformations of plate is used. The sufficient stability conditions of the movement of an elastic plate imposing restrictions on the parameters of mechanical system are received.

Key words

Hydroelasticity; Stability; Elastic plate; Viscous liquid.

1 Introduction

At the design and exploitation of structures, devices, mechanisms for various applications, interacting with liquid, an important problem is to ensure the reliability of their functioning and longer life. Similar problems are common to many branches of engineering. In particular, such problems arise in missilery, aircraft construction, instrumentation, and so on. The essential value in the calculation of structures that interact with the liquid has a stability study of the deformable elements, as the impact of the liquid may lead to its loss.

Thus, at designing of the structures and devices interacting with the liquid, it is necessary to solve problems related to the investigation of stability required for their functioning and operational reliability.

Many theoretical and experimental studies is devoted to the stability of elastic bodies interacting with the gas and liquid. Among the studies we should be noted studies [Ageev, Kuznetsova, Kulikov, Mogilevich and Popov, 2014; Kheiri and Paidoussis, 2015; Kontzialis, Moditis and Paidoussis, 2017; Moditis, Paidoussis and Ratigan, 2016; Mogilevich, Popov, Popova and Christoforova, 2016; Mogilevich, Popov, Rabinsky and Kuznetsova, 2016; Naumova, Ivanov, Voloshinova and Ershov, 2015; Sokolov and Razov, 2014; Zvyagin and Gur'ev, 2017] and many others. Among the works of the authors of this article, note the articles and monographs [Ankilov and Velmisov, 2013, 2015, 2016; Velmisov, Ankilov and Semenova, 2016].

Taken in the work determination of stability of elastic body correspond to the Lyapunov concept of stability of dynamical systems. The problem can be formulated as follows: for any values of the parameters characterizing the system «liquid-solid» to the small deformations of bodies at the initial time $t=0$ (i.e., a small initial deviations from the equilibrium position) will correspond to small deformations and at any time $t>0$.

2 Mathematical model

We investigate stability of the movement (by Lyapunov) an elastic plate which is a part $(x=a, y_0<y<y_*)$ of the border L_0 dividing two areas S_1 and S_2, filled with viscous incompressible liquid. Areas S_1, S_2 have border L_1, L_2 and L_0 any form.

We enter designations: $u(y,t)$ and $w(y,t)$, $y\in(y_0,y_*)$ are deformations of an elastic plate in the direction of axes of *Oy* and *Ox* respectively;

$$v_1(x,y,t)=\begin{cases} v_{11}(x,y,t),\ (x,y)\in S_1,\\ v_{12}(x,y,t),\ (x,y)\in S_2,\end{cases}$$

$$v_2(x,y,t)=\begin{cases} v_{21}(x,y,t),\ (x,y)\in S_1,\\ v_{22}(x,y,t),\ (x,y)\in S_2\end{cases}$$

are liquid velosity vector projections;

$P(x,y,t)=\begin{cases} P_1(x,y,t),\ (x,y)\in S_1,\\ P_2(x,y,t),\ (x,y)\in S_2\end{cases}$ is pressure in liquid.

A function $w(y,t)\in C^{4,2}\left\{[y_0,y_*]\times R^+\right\}$, i.e. it belongs to four times continuously differentiable functions with respect to the variable y on the interval (y_0,y_*) and twice continuously differentiable with respect to the variable t at $t\ge 0$ and takes real values. A function $u(y,t)\in C^{2,2}\left\{[y_0,y_*]\times R^+\right\}$, i.e. it belongs to twice continuously differentiable functions with respect to the variable y on the interval (y_0,y_*) and twice continuously differentiable with respect to the variable t at $t\geq 0$ and takes real values.

The functions $v_{1i}(x,y,t)$, $v_{2i}(x,y,t)$, $P_i(x,y,t)\in$ $\in C^{2,1}\left\{S_i\times R^+\right\}$, i.e. it belongs to twice continuously differentiable functions with respect to the variables x,y in the area S_i and continuously differentiable with respect to the variable t at $t\geq 0$ and takes real values.

The mathematical definition of the problem has an appearance

$$\rho(v_{1t}+v_1v_{1x}+v_2v_{1y})=-P_x+\mu(v_{1xx}+v_{1yy}),\quad (x,y)\in S_1\bigcup S_2; \tag{1}$$

$$\rho(v_{2t}+v_1v_{2x}+v_2v_{2y})=-P_y+\mu(v_{2xx}+v_{2yy}),\quad (x,y)\in S_1\bigcup S_2; \tag{2}$$

$$v_{1x}+v_{2y}=0,\quad (x,y)\in S_1\bigcup S_2; \tag{3}$$

$$v_1(L_k)=v_2(L_k)=0,\quad k=1,2; \tag{4}$$

$$v_1\left(L_0\setminus(y_0,y_*)\right)=v_2\left(L_0\setminus(y_0,y_*)\right)=0; \tag{5}$$

$$v_1(a,y,t)=\dot{w}(y,t),\ v_2(a,y,t)=0,\ y\in(y_0,y_*); \tag{6}$$

$$\begin{cases}-EF\left(u'(y,t)+\frac{1}{2}w'^2(y,t)\right)'+M\ddot{u}(y,t)=0,\\ -EF\left[w'(y,t)\left(u'(y,t)+\frac{1}{2}w'^2(y,t)\right)\right]'+Dw''''(y,t)+\\ +M\ddot{w}(y,t)+N(t)w''(y,t)+\beta_2\dot{w}''''(y,t)+\beta_1\dot{w}(y,t)+\\ +\beta_0w(y,t)=P_1(a,y,t)-P_2(a,y,t),\ y\in(y_0,y_*).\end{cases} \tag{7}$$

The indices x, y, t below denote partial derivatives with respect to x, y, t; the bar and the point denote the partial derivatives with respect to y and t, respectively; ρ, μ are density and dynamic coefficient of viscosity of liquid; $D=Eh^3/(12(1-\nu^2))$ is flexural stiffness of plate; h is thickness of plate; $M=h\rho_n$ is linear mass of plate; $F=h/(1-\nu^2)$; E, ρ_n are elasticity modulus and the linear density of the plate; ν is Poisson coefficient; $N(t)$ is compressing $(N>0)$ or tensile $(N<0)$ forces of the plate; β_2,β_1 are coefficients of internal and external damping; β_0 is stiffness coefficient of the base (bed).

Compressive (tensile) force $N(t)$ element may depend on time. For example, at a non-stationary heat exposure on the plate the $N(t)$ is as follows:

$$N(t)=N_0+N_T(t),$$

$$N_T(t)=-\frac{T_0(t)}{1-\nu},\ \ T_0(t)=E\alpha_T\int_{-h/2}^{h/2}T(z,t)\,dz,$$

where α_T – the temperature coefficient of linear expansion, $T(z,t)$ – the law of temperature change over the thickness of the plate, N_0 – the constant component of force generated at fixing of the plate.

The equations (1)–(3) describe the movement of liquid in areas S_1,S_2, the equation (7) describe the dynamic of a plate; conditions (4)-(6) are conditions of sticking of viscous liquid.

The boundary conditions at the ends of the plate at $y=y_0$ and $y=y_*$ can take the form:

1) rigid clamping (fig. 1a):

$$w(y,t)=w'(y,t)=u(y,t)=0; \tag{8a}$$

2) hinge securely fastened (fig. 1b):

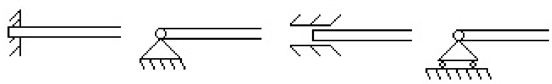
$$w(y,t)=w''(y,t)=u(y,t)=0; \tag{8b}$$

3) rigid mobile jamming (fig. 1c):

$$w(y,t)=w'(y,t)=u'(y,t)=0; \tag{8c}$$

4) hinged movable anchorage (fig. 1d):

$$w(y,t)=w''(y,t)=u'(y,t)+\frac{1}{2}w'^2(y,t)=0. \tag{8d}$$



a) b) c) d)

Figure 1. The method of fixing

We will notice that for the description of the movement of liquid the nonlinear equations of Navier-Stokes are used, and boundary conditions (6), as well as the right part of the equation (7), are written down in the assumption that deformations of a plate are small.

We give the initial conditions:

$$w(y,0)=f_1(y),\quad \dot{w}(y,0)=f_2(y),\qquad u(y,0)=f_3(y),\quad \dot{u}(y,0)=f_4(y), \tag{9}$$

which must be agreed with the boundary conditions (8). According to the definition of functions $w(y,t)$, $u(y,t)$: $f_1(y),\ f_2(y)\in C^4[y_0,y_*]$, $f_3(y),\ f_4(y)\in$ $\in C^2[y_0,y_*]$. The norms in the spaces $C^4[y_0,y_*]$ and $C^2[y_0,y_*]$ are defined by the equalities

$$\|f_i(y)\|=\sup_{0\leq m\leq 4}\max_{y\in[y_0,y_*]}\left|\frac{d^mf_i(y)}{dy^m}\right|,\ i=1,2\,,$$

$$\|f_i(y)\|=\sup_{0\leq m\leq 2}\max_{y\in[y_0,y_*]}\left|\frac{d^mf_i(y)}{dy^m}\right|,\ i=3,4\,.$$

Give also the initial conditions:

$$v_1(x,y,0)=f_5(x,y),\quad v_2(x,y,0)=f_6(x,y), \tag{10}$$

which must be agreed with the boundary conditions (4), (5) and (6). According to the definition of functions $v_1(x,y,t),v_2(x,y,t)$: $f_5(x,y),\ f_6(x,y)\in$ $\in C^2\{S_1\bigcup S_2\}$. The norm in the space $C^2\{G\}$ is defined by the equality

$$\|f_i\|=\sup_{0\leq n+m\leq 2}\max_{(x,y)\in S_1\bigcup S_2}\left|\frac{\partial^{m+n}f_i(x,y)}{\partial x^n\partial y^m}\right|,\ i=5,6.$$

3 Stability investigation

Definition 3.1 The solution of the problem (1)–(8) for five unknown functions $u(y,t)\in C^{2,2}\left\{[y_0,y_*]\times R^+\right\}$, $w(y,t)\in C^{4,2}\left\{[y_0,y_*]\times R^+\right\}$, $v_1(x,y,t)$, $v_2(x,y,t)$, $P(x,y,t)\in C^{2,2}\left\{S_1\cup S_2\times R^+\right\}$ is called stable with respect to perturbations of the initial data (9), (10), if for any arbitrarily small positive number $\delta>0$ exist number $\varepsilon=\varepsilon(\delta)>0$, such that for any functions $f_1(y),\ f_2(y)\in C^4[y_0,y_*]$, $f_3(y),\ f_4(y)\in C^2[y_0,y_*]$ and $f_5(x,y),\ f_6(x,y)\in C^2\{S_1\cup S_2\}$, satisfying the boundary conditions and the conditions of the smallness by the norm $\|f_1(y)\|<\varepsilon, \|f_2(y)\|<\varepsilon$, $\|f_3(y)\|<\varepsilon, \|f_4(y)\|<\varepsilon$, $\|f_5(x,y)\|<\varepsilon$, $\|f_6(x,y)\|<\varepsilon$, the inequalities $|w(y,t)|<\delta, |u(y,t)|<\delta,\ y\in[y_0,y_*]$ and $|v_1(x,y,t)|<\delta$, $|v_2(x,y,t)|<\delta$, $|P(x,y,t)|<\delta$, $(x,y)\in S_1\cup S_2$ will be performed for any time $t>0$.

The similar definitions of stability with respect to perturbations of the initial data can be given separately for the functions themselves $u(y,t)$, $w(y,t)$, $v_1(x,y,t)$, $v_2(x,y,t)$, $P(x,y,t)$ and its partial derivatives.

We enter the designations: λ_1, η_1 are the smallest eigenvalues of the boundary value problems for the equations $\psi''''=-\lambda\psi''$, $\psi''''=\eta\psi$, $y\in(y_0,y_*)$ with boundary conditions corresponding (8) for the function $w(y,t)$.

Theorem 3.1. Let the conditions

$$\beta_2\eta_1+\beta_1\geq 0,\quad \dot N(t)>0, \tag{11}$$

$$N(t)<\lambda_1 D \tag{12}$$

be satisfied. Then the solution $w(y,t)$, $v_1(x,y,t), v_2(x,y,t)$ of problem (1)-(8) and the derivatives $\dot u(y,t),\dot w(y,t)$ are stable with respect to perturbations of the initial data $v_1(x,y,0)$, $v_2(x,y,0), \dot u(y,0), u'(y,0),\ \dot w(y,0), w'(y,0),\ w''(y,0)$.

Proof. We will write down the equations (1),(2) as

$$\rho(v_{1t}+v_2v_{1y}-v_2v_{2x})=-\left(P+\frac{1}{2}\rho V^2\right)_x+\mu\Delta v_1, \tag{13}$$

$$\rho(v_{2t}+v_1v_{2x}-v_1v_{1y})=-\left(P+\frac{1}{2}\rho V^2\right)_y+\mu\Delta v_2, \tag{14}$$

where $V^2=v_1^2+v_2^2$ is liquid velosity square, Δ is laplacian.

Multiplying the equation (13) on $v_1(x,y,t)$, the equation (14) on $v_2(x,y,t)$, and adding the received expressions, with the accounting of the equation of continuity (3), we will obtain

$$\left(\frac{1}{2}\rho V^2\right)_t=-\left[v_1\left(P+\frac{1}{2}\rho V^2\right)\right]_x-\left[v_2\left(P+\frac{1}{2}\rho V^2\right)\right]_y+\mu\left[(v_1v_{1x}+v_2v_{2x})_x+\right. \tag{15}$$

$$\left.+(v_1v_{1y}+v_2v_{2y})_y-v_{1x}^2-v_{1y}^2-v_{2x}^2-v_{2y}^2\right].$$

Considering the boundary conditions (8), we will obtain equalities

$$-\int_{y_0}^{y_*}\dot w\left[w'\left(u'+\frac{1}{2}w'^2\right)\right]'dy-\int_{y_0}^{y_*}\dot u\left(u'+\frac{1}{2}w'^2\right)'dy=$$

$$=\int_{y_0}^{y_*}\dot w'w'\left(u'+\frac{1}{2}w'^2\right)dy+\int_{y_0}^{y_*}\dot u'\left(u'+\frac{1}{2}w'^2\right)dy=$$

$$=\frac{1}{2}\left(\int_{y_0}^{y_*}\left(u'+\frac{1}{2}w'^2\right)^2dy\right)_t, \tag{16}$$

$$\int_{y_0}^{y_*}\dot w\ddot w dy=\frac{1}{2}\left(\int_{y_0}^{y_*}\dot w^2dy\right)_t,\quad \int_{y_0}^{y_*}\dot u\ddot u dy=\frac{1}{2}\left(\int_{y_0}^{y_*}\dot u^2dy\right)_t,$$

$$\int_{y_0}^{y_*}\dot w w''''dy=\int_{y_0}^{y_*}\dot w''w''dy=\frac{1}{2}\left(\int_{y_0}^{y_*}w''^2dy\right)_t,$$

$$N(t)\int_{y_0}^{y_*}\dot w w''dy=-N(t)\int_{y_0}^{y_*}\dot w'w'dy=-\frac{1}{2}\left(N(t)\int_{y_0}^{y_*}w'^2dy\right)_t+$$

$$+\frac{1}{2}\dot N(t)\int_{y_0}^{y_*}w'^2dy,\quad \int_{y_0}^{y_*}\dot w\dot w''''dy=\int_{y_0}^{y_*}\dot w''^2dy.$$

Multiplying the first equation of system (7) on $\dot u(y,t)$, the second equation of system (7) on $\dot w(y,t)$, and adding the received expressions and integrating from y_0 to y_*, with the accounting of the equalities (16), we will obtain

$$\left(\frac{1}{2}\int_{y_0}^{y_*}\left(EF\left(u'(y,t)+\frac{1}{2}w'^2(y,t)\right)^2+M(\dot u^2(y,t)+\right.\right. \tag{17}$$

$$+\dot w^2(y,t))+Dw''^2(y,t)-N(t)w'^2(y,t)+$$

$$\left.\left.+\beta_0w^2(y,t)\right)dy\right)_t=\int_{y_0}^{y_*}\left(\beta_2\dot w''^2(y,t)+\beta_1\dot w^2(y,t)-\right.$$

$$\left.-\frac{1}{2}\dot N(t)w'^2(y,t)-\left(P_1(a,y,t)-P_2(a,y,t)\right)\dot w(y,t)\right)dy.$$

We will enter the functional into consideration

$$J(t)=\frac{1}{2}\iint_S\rho V^2dS+\frac{1}{2}\int_{y_0}^{y_*}\left(EF\left(u'(y,t)+\frac{1}{2}w'^2(y,t)\right)^2+\right.$$

$$+M\left(\dot u^2(y,t)+\dot w^2(y,t)\right)+Dw''^2(y,t)- \tag{18}$$

$$\left.-N(t)w'^2(y,t)+\beta_0w^2(y,t)\right)dy,$$

where $S=S_1\cup S_2$.

For a derivative $\frac{\partial J}{\partial t}$ of this functional on time, using expressions (15), (17) and applying Green's formula, we find

$$\frac{\partial J}{\partial t}=\oint_{L_1\cup L_0}\left[-v_{11}\left(P_1+\frac{1}{2}\rho V_1^2\right)+\mu(v_{11}v_{11x}+v_{21}v_{21x})\right]dy+$$

$$+\left[v_{11}\left(P_1+\frac{1}{2}\rho V_1^2\right)-\mu(v_{11}v_{11y}+v_{21}v_{21y})\right]dx+$$

$$+\oint_{L_2\cup L_0}\left[-v_{12}\left(P_2+\frac{1}{2}\rho V_2^2\right)+\mu(v_{12}v_{12x}+v_{22}v_{22x})\right]dy+$$

$$+\left[v_{22}\left(P_2+\frac{1}{2}\rho V_2^2\right)-\mu(v_{12}v_{12y}+v_{22}v_{22y})\right]dx- \quad (19)$$

$$-\mu\iint_S\left(v_{1x}^2+v_{1y}^2+v_{2x}^2+v_{2y}^2\right)dS-\int_{y_0}^{y_*}\Big(\beta_2\dot{w}''^2(y,t)+$$

$$+\beta_1\dot{w}^2(y,t)-\frac{1}{2}\dot{N}(t)w'^2(y,t)-$$

$$-\left(P_1(a,y,t)-P_2(a,y,t)\right)\dot{w}(y,t)\Big)dy.$$

Considering the boundary conditions (4)-(6) and the equations (7), we have

$$\frac{\partial J}{\partial t}=\int_{y_0}^{y_*}\left[-v_{11}\left(P_1+\frac{1}{2}\rho v_{11}^2\right)\right]dy+$$

$$+\int_{y_*}^{y_0}\left[-v_{12}\left(P_2+\frac{1}{2}\rho v_{12}^2\right)\right]dy-\mu\iint_S\Big(v_{1x}^2+v_{1y}^2+v_{2x}^2+$$

$$+v_{2y}^2\Big)dS-\int_{y_0}^{y_*}\Big(\beta_2\dot{w}''^2(y,t)+\beta_1\dot{w}^2(y,t)- \quad (20)$$

$$-\frac{1}{2}\dot{N}(t)w'^2(y,t)-\left(P_1(a,y,t)-P_2(a,y,t)\right)\dot{w}(y,t)\Big)dy=$$

$$=-\int_{y_0}^{y_*}\left(\beta_2\dot{w}''^2(y,t)+\beta_1\dot{w}^2(y,t)-\frac{1}{2}\dot{N}(t)w'^2(y,t)\right)dy-$$

$$-\mu\iint_S\left(v_{1x}^2+v_{1y}^2+v_{2x}^2+v_{2y}^2\right)dS.$$

We consider the boundary value problems for the equations $\psi''''=-\lambda\psi''$, $\psi''''=\eta\psi$, $y\in(y_0,y_*)$ with boundary conditions (8) for the function $w(y,t)$. These problems are self-adjoint and completely defined. For the function $w(y,t)$, using Rayleigh's inequality [Kollatc, 1968], we obtain the estimates

$$\int_{y_0}^{y_*}w(y,t)w''''(y,t)dy\ge-\lambda_1\int_{y_0}^{y_*}w(y,t)w''(y,t)dy,$$
$$\int_{y_0}^{y_*}w(y,t)w''''(y,t)dy\ge\eta_1\int_{y_0}^{y_*}w(y,t)w(y,t)dy. \quad (21)$$

Integrating by parts taking into account boundary conditions (8), we will obtain inequalities

$$\int_{y_0}^{y_*}w''^2(y,t)dy\ge\lambda_1\int_{y_0}^{y_*}w'^2(y,t)dy,$$
$$\int_{y_0}^{y_*}w''^2(y,t)dy\ge\eta_1\int_{y_0}^{y_*}w^2(y,t)dy. \quad (22)$$

Similar considering the boundary value problems for the equations $\psi''''=-\lambda\psi''$, $y\in(y_0,y_*)$ with boundary conditions (8) for the function $\dot{w}(y,t)$ we obtain the estimate

$$\int_{y_0}^{y_*}\dot{w}''^2(y,t)dy\ge\eta_1\int_{y_0}^{y_*}\dot{w}^2(y,t)dy\quad. \quad (23)$$

Using the inequality (23) to (20), we obtain

$$\frac{\partial J}{\partial t}\le-\int_{y_0}^{y_*}\left((\beta_2\eta_1+\beta_1)\dot{w}^2(y,t)-\frac{1}{2}\dot{N}(t)w'^2(y,t)\right)dy-$$
$$-\mu\iint_S\left(v_{1x}^2+v_{1y}^2+v_{2x}^2+v_{2y}^2\right)dS. \quad (24)$$

Let the conditions (9) are satisfied, then from (24) it follows, that $\frac{\partial J}{\partial t}\le 0$. Integrating from 0 to t, we obtain the inequality

$$J(t)\le J(0). \quad (25)$$

We make the evaluations for functional with the boundary conditions (8). Using the inequalities (22), we obtain the upper bound of $J(0)$:

$$J(0)=\frac{1}{2}\iint_S\rho\left(v_{10}^2+v_{20}^2\right)dS+\frac{1}{2}\int_{y_0}^{y_*}\left(EF\left(u_0'+\frac{1}{2}w_0'^2\right)^2+\right.$$
$$\left.+M(\dot{u}_0^2+\dot{w}_0^2)+\left(D+\frac{|N(0)|}{\lambda_1}+\frac{\beta_0}{\eta_1}\right)w_0''^2\right)dy. \quad (26)$$

Here are introduced the designations $v_{10}=v_1(x,y,0)$, $v_{20}=v_2(x,y,0)$, $\dot{u}_0=\dot{u}(x,0)$, $u_0'=u'(x,0)$, $\dot{w}_0=\dot{w}(x,0)$, $w_0'=w'(x,0)$, $w_0''=w''(x,0)$.

Using the first inequality (22), we obtain the estimate $J(t)$ from below:

$$J(t)\ge\frac{1}{2}\iint_S\rho V^2dS+\frac{1}{2}\int_{y_0}^{y_*}\Big(M(\dot{u}^2(y,t)+$$
$$+\dot{w}^2(y,t))+(\lambda_1D-N(t))w'^2(y,t)\Big)dy. \quad (27)$$

Using the Cauchy-Bunyakovsky inequality at the boundary conditions (8), we obtain the estimate

$$w^2(y,t)\le(y_*-y_0)\int_{y_0}^{y_*}w'^2(y,t)dy\,. \quad (28)$$

Let the condition (12) is satisfied, then the inequality (27) takes the form

$$J(t)\ge\frac{1}{2}\iint_S\rho\left(v_1^2+v_2^2\right)dS+\frac{1}{2}\int_{y_0}^{y_*}M(\dot{u}^2(y,t)+$$
$$+\dot{w}^2(y,t))dy+\frac{\lambda_1D-N(t)}{2(y_*-y_0)}w^2(y,t). \quad (29)$$

Thus, taking into account (25), (26), (29), we obtain the inequality

$$\iint_S\rho\left(v_1^2+v_2^2\right)dS+\int_{y_0}^{y_*}M(\dot{u}^2(y,t)+\dot{w}^2(y,t))dy+$$
$$+\frac{\lambda_1D-N(t)}{y_*-y_0}w^2(y,t)\le\iint_S\rho\left(v_{10}^2+v_{20}^2\right)dS+ \quad (30)$$

$$+\int_{y_0}^{y_*}\left(M(\dot{u}_0^2+\dot{w}_0^2)+EF\left(u_0'+\frac{1}{2}w_0'^2\right)^2+\right.$$

$$\left.+\left(D+\frac{|N(0)|}{\lambda_1}+\frac{\beta_0}{\mu_1}\right)w_0''^2\right)dy.$$

From inequalities $\|f_i(y)\|<\varepsilon,\ i=\overline{1,4},\ \|f_i(x,y)\|<\varepsilon,$ $i=5,6$ follows that $|v_{10}|<\varepsilon, |v_{20}|<\varepsilon,\ |\dot{u}_0|<\varepsilon,\ |u_0'|<\varepsilon,$ $|\dot{w}_0|<\varepsilon, |w_0'|<\varepsilon, |w_0''|<\varepsilon$. And as $u(y,t)\in$ $\in C^{2,2}\left\{[y_0,y_*]\times R^+\right\}$, $w(y,t)\in C^{4,2}\left\{[y_0,y_*]\times R^+\right\}$, $v_1(x,y,t),\ \ v_2(x,y,t)\in C^{2,2}\left\{S_1\bigcup S_2\times R^+\right\}$, then from (30) implies, that for any arbitrarily small positive number $\delta>0$ exist number $\varepsilon=\varepsilon(\delta)>0$, such that the inequalities $|w(y,t)|<\delta, |\dot{w}(y,t)|<\delta, |\dot{u}(y,t)|<\delta,$ $y\in[y_0,y_*]$ and $|v_1(x,y,t)|<\delta,$ $|v_2(x,y,t)|<\delta,$ $(x,y)\in S_1\bigcup S_2$ will be performed for any time $t>0$.

The theorem is proved.

On the basis of inequality (30) it is possible to receive an assessment of amplitude of the greatest possible fluctuations of an elastic plate at any timepoint:

$$|w(y,t)|\le\sqrt{\frac{y_*-y_0}{\lambda_1 D-N(t)}J(0)}.$$

4 Conclusion

In work on the basis of the constructed mathematical model and the Lyapunov's functional constructed for this model the research of dynamic stability of the elastic plate contacting to the fluctuating viscous incompressible liquid is conducted. Further, using results of this work, it is planned to conduct a research of dynamic stability of elastic elements of various designs which are flowed round by a stream of the viscous incompressible liquid.

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