Algorithm of multiple observer-based synchronization for time-varying two-rotor vibration system

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1. Introduction

During recent years the speed-gradient algorithms developed in control engineering area [1,2] have been applied intensively to control of oscillatory motion and particularly to control of vibration units. Among problems solved by speed-gradient approach are control of vibration units in start-up modes (swing-up and passage through resonance) [3-5], synchronization [6-8], etc.

One of the main arising problems is keeping stable synchronous working mode in order to achieve maximum working amplitude of the platform vibrations. Additional opportunities for development of vibration equipment, especially for vibrational transportation of materials can be provided by using multiple synchronous modes. It keeps constant the ratio of average velocities and/or phases of vibroactuators. Unlike simple synchronization modes which can arise spontaneously, stable multiple synchronous mode can only be achieved by means of advanced control systems.

In this paper an algorithm of multiple synchronization of two-rotor vibration unit with time-varying payload is proposed. The performance of the proposed system is analyzed by computer simulation for model of the 2-rotor vibration set-up consisting of two rotors driven by DC motors and a rigid platform mounted on an unmovable base. A time-varying payload attached to a platform allows one to study dynamics of material processing. Additional problem arises owing to incompleteness of measurements: only rotor phases are available for measurement. To solve this problem an observer (state estimator) is introduced into the system structure.

2. Model of the two-rotor vibration set-up dynamics

The scheme of the two-rotor vibration set-up is presented in Fig.1. Here φ_1, φ_2 are rotation angles of the rotors measured from the lowest vertical position, y is the vertical displacement of the supporting body from the equilibrium position, m, M are the masses of the rotors and the supporting body, respectively, J_1, J_2 are the inertia moments of the rotors, ρ is the eccentricity of rotors, c_0, c_1 are the spring stiffness, g is the gravitational acceleration, m_r is the mass of the payload, y_1 is the vertical displacement of the payload, $m_0 = M + 2m$. Let us consider only vertical motion of the system.

Kinetic and potential energies T and Π are as follows:

$$T = \frac{1}{2} m_0 \dot{y}^2 + \frac{1}{2} J_1 \dot{\varphi}_1^2 + \frac{1}{2} J_2 \dot{\varphi}_2^2 + m\rho\, \dot{y}[\sin\varphi_1\, \dot{\varphi}_1 + \sin\varphi_2\, \dot{\varphi}_2] + \frac{1}{2} m_r(t)\, \dot{y}_1^2 .$$
$$\Pi = m_0 g y - m g \rho(\cos\varphi_1 + \cos\varphi_2) + \frac{1}{2} c_1 (y - y_1)^2 + c_0 y^2 + m_r(t) g y_1 \qquad (1)$$

Denoting friction coefficient in the bearings of unbalanced rotors by k_c and dissipation of the lower springs by b, we obtain dynamics equations of the two-rotor vibration set-up with payload:

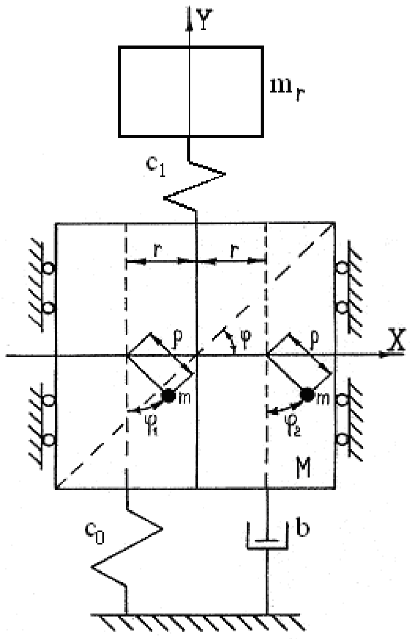


Fig.1. Scheme of the two-rotor vibration set-up.

$$\begin{cases} m_0\ddot{y} + m\rho\sin\varphi_1\ddot{\varphi}_1 + m\rho\sin\varphi_2\ddot{\varphi}_2 + m\rho\cos\varphi_1\dot{\varphi}_1^{\ 2} + m\rho\cos\varphi_2\dot{\varphi}_2^{\ 2} + \\ \qquad + 2c_0 y + c_1(y - y_1) + m_0\cdot g + b\cdot\dot{y} = 0 \\ m\rho\sin\varphi_1\ddot{y} + J_1\ddot{\varphi}_1 + mg\rho\sin\varphi_1 + k_c\dot{\varphi}_1 = M_1; \\ m\rho\sin\varphi_2\ddot{y} + J_2\ddot{\varphi}_2 + mg\rho\sin\varphi_2 + k_c\dot{\varphi}_2 = M_2; \\ m_r(t)\ddot{y}_1 + c_1(y_1 - y) + m_r(t)g + \dot{m}_r(t)\dot{y}_1 = 0, \end{cases} \tag{2}$$

where M_1, M_2 are the motor torques (controlling variables).

3. The base control algorithm for multiple synchronization

The first step of design of control algorithm by the speed-gradient method [1, 2] is the choice of the goal functional according to the desired control goal. In our case the control goal is achievement of the desired ratio of angular velocities of the rotors. For n-ple synchronization it means achievement of minimum (zero) value of the term $(\dot{\varphi}_1 \pm n\dot{\varphi}_2)^2$.

Another goal is to achieve the desired level of average angular velocities of the rotors. It corresponds to achievement of the desired average kinetic energy or total energy of the system. Therefore the goal functional can be chosen as follows:

$$Q(z) = 0.5\{(1-\alpha)(H - H^*)^2 + \alpha(\dot{\varphi}_1 \pm n\dot{\varphi}_2)^2\}, \tag{3}$$

where $z = (\varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2, y, \dot{y})^T$ is the state vector of the system; $0 < \alpha < 1$ is weighting coefficient; H^* is the desired level of total mechanical energy.

Obviously the goal is achieved if Q (z) = 0, otherwise Q (z)> 0. At this stage of design we neglected friction ($k_c = 0, b = 0$). Applying the speed-gradient methodology we evaluate the speed of changing (3) along trajectories of controlled system, assuming that payload mass is frozen. At this stage we neglect friction. Then evaluate the gradient of the speed with respect to controlling variables (torques). The designed base control algorithm is as follows

$$M_1 = \gamma_1\{(1-\alpha_1)(H - H^*)\dot{\varphi}_1 + \frac{\alpha_1}{J_1}(\dot{\varphi}_1 \pm n\dot{\varphi}_2)\};$$

$$M_2 = \gamma_2\{(1-\alpha_2)(H - H^*)\dot{\varphi}_2 \pm \frac{\alpha_2 n}{J_2}(\dot{\varphi}_1 \pm n\dot{\varphi}_2)\}, \tag{4}$$

where $\gamma_1>0$ and $\gamma_2>0$ are control gains. The next step is to introduce observer to overcome incompleteness of measurements.

4. Nonlinear observer: design and analysis

In order to design a simple observer for a nonlinear system we first simplify the system model. Omitting the rotor number and neglecting dynamics of optimized current loop, we consider the following model of rotor (vibroactuator)

$$J\ddot{\varphi}(t)+\kappa_c\dot{\varphi}(t)+mg\rho\sin\varphi(t)=M_m(t), \qquad (5)$$

where M_m is controlling motor torque. Then the following observer structure can be used. Let $x=[x_1,x_2]^{\mathrm{T}}=[\varphi,\dot{\varphi}]^{\mathrm{T}}$ be the state vector of the rotor and $\hat{x}=[\hat{x}_1,\hat{x}_2]^{\mathrm{T}}$ - be the state vector of the observer. Neglecting friction in the bearings (k_c=0) and rewriting the rotor equation in the state space form

$$\dot{x}_1(t)=x_2(t),$$
$$\dot{x}_2(t)=-a_{21}\sin x_1+a_{23}M_m,$$

where $a_{23}=1/J$, $a_{21}=mg\rho/J$ the observer equations for the case when only $x_1(t)$ is measured can be written as follows :

$$\begin{aligned}\dot{\hat{x}}_1(t)&=\hat{x}_2(t)+\kappa_1[x_1(t)-\hat{x}_1(t)],\\ \dot{\hat{x}}_2(t)&=-a_{21}\sin\hat{x}_1(t)+a_{23}M_m+\kappa_2[x_1(t)-\hat{x}_1(t)],\end{aligned} \qquad (6)$$

where $\kappa_1>0,\kappa_2>0$ are observer gains. In order to establish conditions providing proper work of observer write down the error equations for $\dot{e}=\dot{x}(t)-\dot{\hat{x}}(t)$:

$$\dot{e}=\begin{bmatrix}e_2-k_1e_1\\ -a_{21}[\sin x_1-\sin\hat{x}_1]-k_2e_1\end{bmatrix}=\begin{bmatrix}-k_1 & 1\\ -k_2 & 0\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix}+\begin{bmatrix}0\\ -a_{21}(\sin x_1-\sin\hat{x}_1)\end{bmatrix}=A_Ne(t)+f_N(e,t). \quad (7)$$

Since the characteristic polynomial of the matrix A_{N} is

$$\det(pI_2-A_N)=\begin{bmatrix}p+k_1 & -1\\ k_2 & p\end{bmatrix}=p^2+k_1p+k_2, \qquad (8)$$

the matrix A_{N} is Hurwitz (stable) if both coefficients k_1 k_2 are positive. In this case the limit value of the estimation error $|e(t)|$ is bounded for bounded disturbance $f_N(e(t),t)$ and it is proportional to the upper bound of the disturbance $f_N(e(t),t)$ which is obviously equal to $2|a_{21}|$. Moreover, the disturbance nonlinear function $f_N(e(t),t)$ is Lipschitz with Lipschitz constant equal to one since

$$|\sin x_1-\sin\hat{x}_1|\le 2\left|\sin\left(\frac{x_1-\hat{x}_1}{2}\right)\right|\left|\cos\left(\frac{x_1+\hat{x}_1}{2}\right)\right|\le 2\left|\sin\frac{x_1-\hat{x}_1}{2}\right|\le|x_1-\hat{x}_1|=|e_1|.$$

Therefore for sufficiently small $|a_{21}|$ the error equation is asymptotically stable. Stability condition is valid provided eccentricity ρ is sufficiently small. It also can be shown that the reduced observer estimates velocities with asymptotically vanishing error.

Then control algorithms (4) in the observer based system are as follows:

$$M_1=-\gamma_1\left[(1-\alpha_1)\left(\hat{H}(t)-H^*\right)\dot{\varphi}_1+\alpha_1\left(\dot{\hat{\varphi}}_1\pm n\dot{\hat{\varphi}}_2\right)\right]; \qquad (9)$$

$$M_2=-\gamma_2\left[(1-\alpha_2)\left(\hat{H}(t)-H^*\right)\dot{\hat{\varphi}}_2\pm\alpha_2n\left(\dot{\hat{\varphi}}_1\pm n\dot{\hat{\varphi}}_2\right)\right];$$

where $\hat{H}$ depends on estimates of velocities $\dot{\hat{\varphi}}_1$, $\dot{\hat{\varphi}}_2$.

5. **Comparison of observer based double synchronization system with full measurement case**

Let us compare efficiency of proposed observer based algorithm (9) for double synchronization *(n = 2)* with the algorithm (3) designed for the case of all state vector measured.

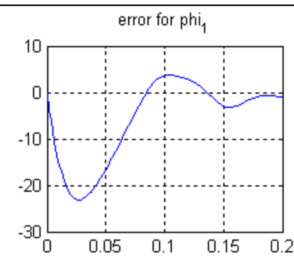
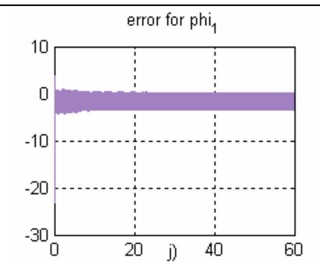
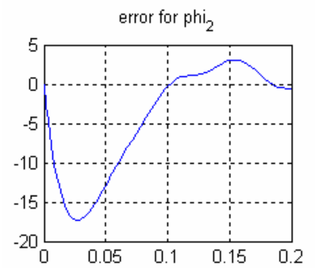
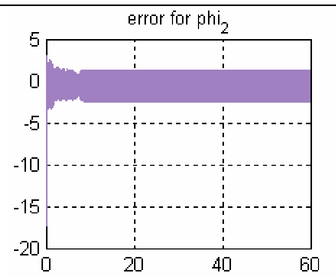
	$0 \le t \le 0{,}2$ s	$0 \le t \le 60$ s
$\dot{e}_1 = \dot{\varphi}_1 - \dot{\hat{\varphi}}_1$		
$\dot{e}_2 = \dot{\varphi}_2 - \dot{\hat{\varphi}}_2$		

Fig.2. Dynamics of velocity estimation errors $\dot{e}_i = \dot{\varphi}_i - \dot{\hat{\varphi}}_i$.

In Fig.2 the plots of velocity estimation errors $\dot{e}_i = \dot{\varphi}_i - \dot{\hat{\varphi}}_i$ at the initial time interval and in the steady state mode are shown. It is seen that the chosen observer gains $\kappa_1 = 50, \kappa_2 = 2000$ ensure convergence of the estimation error practically to zero after 1s - 2 s which is significantly less than synchronization time t_{sync} .

The table 1 contains values of varying parameters: rate of payload mass change V , initial loading time t_1, final loading time t_2 bounds of varying mass m_r algorithm parameters γ_i , α_i as well as experimental results taken from simulation plots: synchronization time t_{sync} which is the time until the multiplied phase shift enters 5% zone near its steady state mode, usually a multiple of π), t_p – which is the transient time for rotor velocities, $\Delta\varphi_{st}$ which is the steady state value of the multiplied phase shift. Symbol (I) marks results for algorithm (4) with full state measurement while symbol (II) corresponds to the observer based algorithm (9). The final payload mass in our experiments varies up to the value of *75%* from the mass of the supporting platform which is equal to *9 kg*. The maximum value of the rate of payload mass change is *V*=0.8kg/s*.

Table 1. – Characteristics of the system with time varying payload

Payload mass	Algorithm parameters	Rate of payload mass change	t_1, *s*	$\Delta\varphi_{st}$, *rad*		t_{sync}, *s*		t_p, *s*	
				I	II	I	II	I	II
$1 \le m_r \le 2$	$\gamma_1 = 0{,}02$ $\gamma_2 = 0{,}02$ $\alpha_1 = 0{,}05$ $\alpha_2 = 0{,}003$	V = 0,1 V = 0,2 V = 1/3 V = 2/3	$t_1 = 0$; $t_2 = 10$ $t_1 = 0$; $t_2 = 5$ $t_1 = 0$; $t_2 = 3$ $t_1 = 0$; $t_2 = 1{,}5$	3,5252 3,5357 3,5388 3,5411	3,8548 3,8554 3,8556 3,8557	11 6 4 3	11 8 6 4	1,5 1,5 1,5 1,5	1 1 1 1
$1 \le m_r \le 2.66$	$\gamma_1 = 0{,}002$ $\gamma_2 = 0{,}002$ $\alpha_1 = 0{,}03$ $\alpha_2 = 0{,}003$	V = 0,3	$t_1 = 0$; $t_2 = 5$	1058,4	47,314	15	5	6,5	25
		V = 2/3	$t_1 = 0$; $t_2 = 2{,}5$	844,95	22,11	16	7	30	2,5
	$\gamma_1 = 0{,}003$ $\gamma_2 = 0{,}003$ $\alpha_1 = 0{,}09$ $\alpha_2 = 0{,}003$	V = 1/3	$t_1 = 0$; $t_2 = 5$	10,035	16,442	18	10	12	7,5
$1 \le m_r \le 3$	$\gamma_1 = 0{,}003$ $\gamma_2 = 0{,}003$ $\alpha_1 = 0{,}09$ $\alpha_2 = 0{,}003$	V = 0,1	$t_1 = 0$; $t_2 = 21$	9,296	21,338	34	47	24	45
$1 \le m_r \le 3{,}5$	$\gamma_1 = 0{,}02$ $\gamma_2 = 0{,}03$ $\alpha_1 = 0{,}25$ $\alpha_2 = 0{,}05$	V = 0,05	$t_1 = 0$; $t_2 = 50$	16,571	47,671	28	---	22	90
$3{,}5 \le m_r \le 7$	$\gamma_1 = 0{,}02$ $\gamma_2 = 0{,}03$ $\alpha_1 = 0{,}25$ $\alpha_2 = 0{,}05$	V = 0,05	$t_1 = 0$; $t_2 = 70$	5,2838	7,7588	50	40	1	2

In Fig.3,4 simulation results for algorithm (3) with full state measurements and for algorithm (9) with observer are shown according to the following notation: a) Platform position *y(t), m* ; b) Platform velocity $\dot{y}$, *m/s*; c) Current value of rotor phases φ_1, φ_2, *rad*; d) Current value of multiple phase shift $\Delta\varphi = (\varphi_1 - 2\varphi_2)$, *rad*; e) Total mechanical energy *Energy, J*; f) Controlling torques *M1, M2, N·m*; g) Rotor velocities $\dot{\varphi}_1, \dot{\varphi}_2$, s^{-1}; h) Multiple velocity shift $\Delta\dot{\varphi} = (\dot{\varphi}_1 - 2\dot{\varphi}_2)$, s^{-1}.

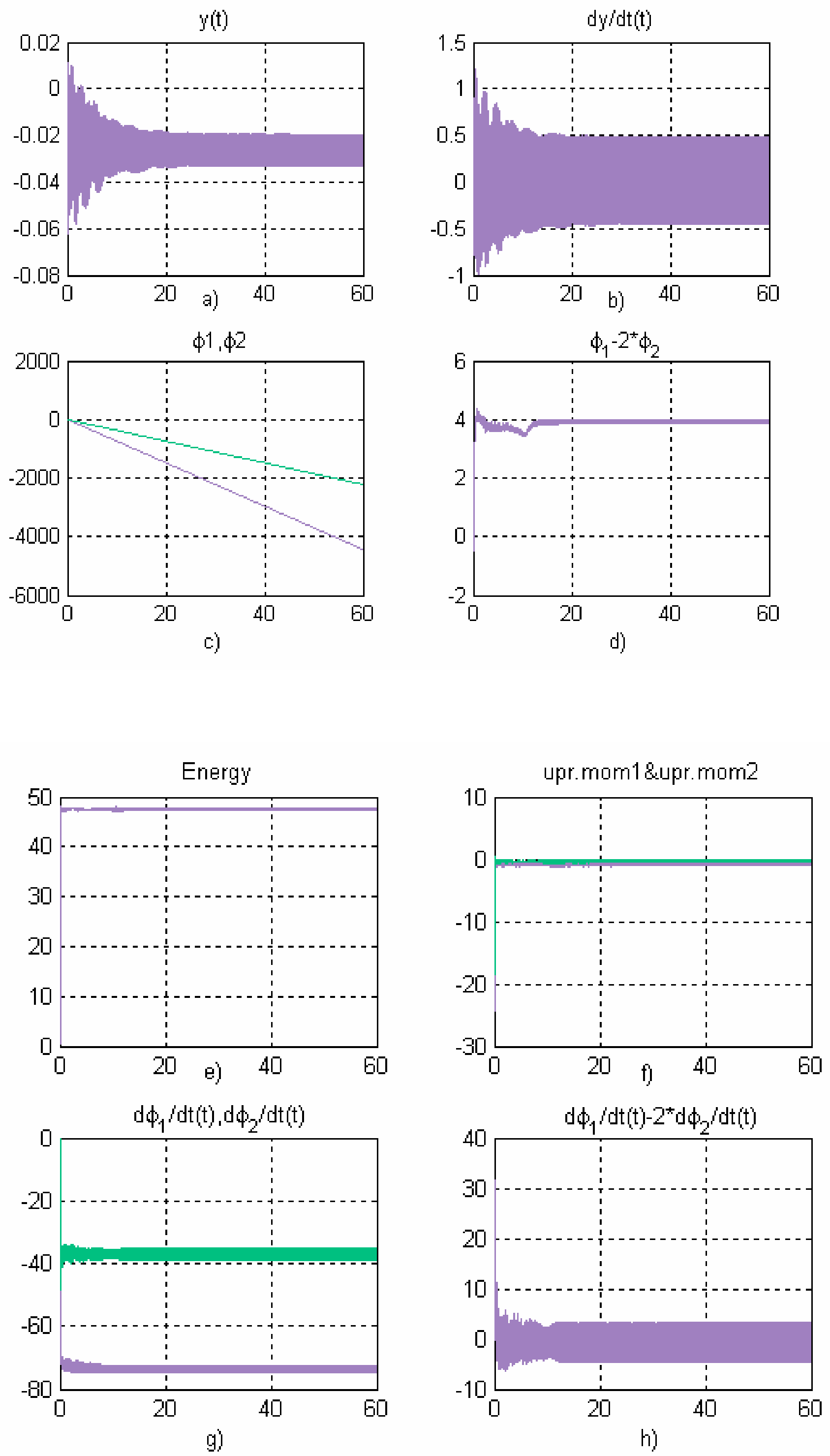


Fig.3. Simulation results for algorithm (4) with full state measurement.

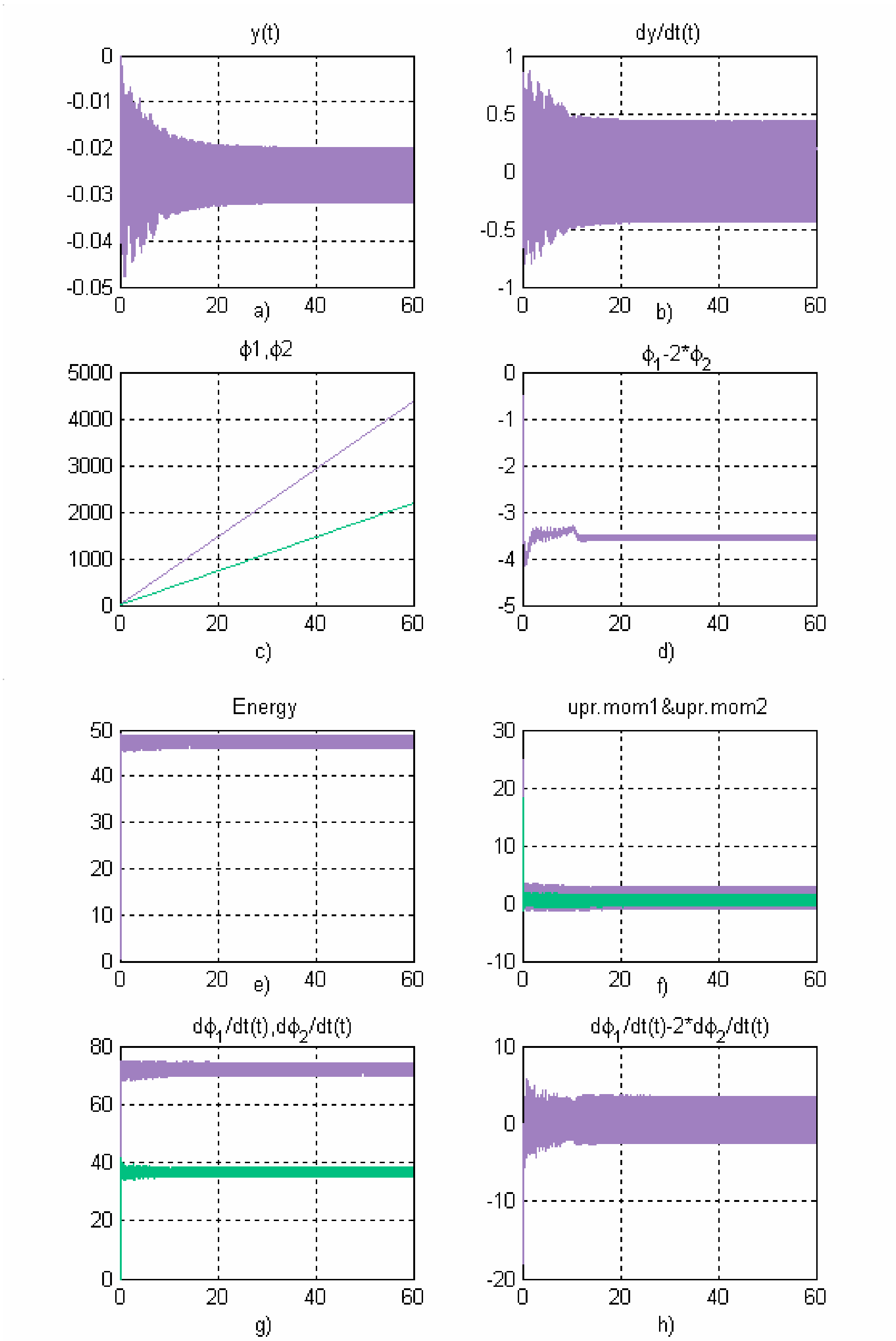


Fig.4. Simulation results for algorithm (9) with observer.

6. Conclusions

The main result of our study is demonstration of a stable multiple synchronous mode in vibration units with changing payload mass for systems with incomplete measurements. Comparison of systems with and without observer shows that the system dynamics in both cases are equivalent with respect to synchronization time and value of control signals when the payload mass changes slowly. Using observer does not introduces serious errors into the system: observation error approaches zero after 1-2s. However, it is seen from the Table 1 that if the rate of the mass change is large synchronization may be absent in the system with full measurements while stable synchronous mode is achieved in the system with observer.

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