Realization of Variable Structure System for Astatic Aircraft Control

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Abstract: A solution of astatic attitude control problem for an aircraft is considered. The traditional solution method on the base of integral for the error can lead to the system dynamic accuracy breakdown. In the paper a contradiction between dynamic accuracy and the request of an astatic control is solved on the base of variable structure control principle.

Keywords: aircraft, astatic control, variable structure system, dynamic accuracy

1. INTRODUCTION

In tasks of aircraft control there is the problem to ensure conflicting requests to dynamic accuracy. So for attitude high maneuverability aircraft control there exists the request of astatism to constant input actions and the request of high dynamic accuracy for transient functions. Usually, to attain the astatism, an integral for a control error is used. But the integral is used to slow down transient functions. Sometimes, this deceleration could be inadmissible for an aircraft to fulfill a task of their mission. In particular, such a problem appeared in designing the control system for the Buran aircraft (Puchkov et al., 1989).

Especially, this problem arises for aircraft angular control under wind disturbances and disturbances that are determined by interconnections between channels (Glumov et al., 1997). One of possible problem solution for the Buran aircraft was the use of the variable structure control principle (SVS) (Puchkov et al., 1989; Emel'yanov, 1967a).

2. THE PROBLEM STATEMENT

Let us take an aircraft motion equations relative to the angle of roll in the following form (Bushgens, 1998):

$$\begin{aligned} &\frac{d}{dt}\gamma(t) = \omega_x(t), \\ &\frac{d}{dt}\omega_x(t) + c_1\omega_x(t) = c_3\delta_e(t) + c_2\beta_w(t), \\ &\Delta\gamma(t) = \gamma(t) - \gamma_{ref}(t), \end{aligned} \qquad (1)$$

where $\gamma(t)$ is the attack angle; $\gamma_{ref}(t)$ is a desired function of the attack angle; $\delta_e(t)$ is an aileron angle; $\beta_w(t)$ is the action on an object that is equivalent to wind and interconnection disturbances; c_1, c_2, c_3 are the dynamic coefficients that are determined during an aircraft flight .

During a flight the dynamic coefficients c_1, c_2, c_3 in the aircraft mathematical model (MM) (1) are changed with time depending on the dynamic head pressure and other factors (Bushgens, 1998). But to derive some analytical results, we accept the hypothesis of quasi-stationary regime, namely, we will take into account some relatively small interval of time where the velocity of dynamic coefficients c_1, c_2, c_3 is changed so small that these velocities can be neglected. Such a hypothesis is typical and justified for preliminary engineering synthesis of an object control laws. For the clarity of presenting the main results of this paper, we consider one point of the aircraft trajectory where the coefficients c_1, c_2, c_3 are the following:

$$c_1 = 0,915;\ c_2 = 45;\ c_3 = 300\,. \qquad (2)$$

For this point it is required:

1) the condition of astatism for the coordinate $\gamma(t)$ relative to the input control $\gamma_{ref}(t)$;

2) the transient function duration on $\gamma(t)$ be not more 3 seconds (relatively, the accuracy interval $\pm 5\%$);

3) the overshooting for the transient function on $\gamma(t)$ be not more than 20%;

4) the condition of astatism for the coordinate $\gamma(t)$ relative to the input disturbance $\beta_w(t)$;

5) outlier of the coordinate $\Delta\gamma(t)$ under conditions $\gamma_{ref}(t) = \text{const}$ and step-function $\beta_w(t)$ from $\beta_w(t) = 1^\circ$ to $\beta_w(t) = -1^\circ$ be not more than 0.7°;

6) the transient function duration on $\Delta\gamma(t)$ under conditions $\gamma_{ref}(t) = \text{const}$ and step-function $\beta_w(t)$ from $\beta_w(t) = 1^\circ$ to $\beta_w(t) = -1^\circ$ be not more than 10 seconds (relatively, the accuracy interval $\pm 0.1^\circ$).

3. THE MAIN IDEA OF THE CONTROL PRINCIPLE ON THE BASE OF VARIABLE STRUCTURE SYSTEM

The author of the control principle based on the variable structure system origin defined it simply in Emel'yanov (1967a): "as systems with variable structure (SVS), we will understand the systems where connections between functional elements are changed depending on the system state". Of course, the essence of this principle is contained in deriving these algorithms which lead to a problem solution.

It should be noted that the main line in SVS development was the use of sliding modes (Emel'yanov, 1967b). But for aircraft with uncertain and no taking into account dynamics in mathematical models, it is not always permisiible to use sliding modes. In this paper, use is made of another no less interesting idea that was formulated in Emel'yanov (1967a), Syrov and Puchkov (2006). The idea consists in switching several structures of a system during the transient functions to avoid negative properties of every structure and to associate positive properties.

4. AN AUTOPILOT STRUCTURE CHOICE FROM REQUIREMENTS TO CONTROL INPUT AND ANALYSIS OF CONTROL SYSTEM DYNAMICS

For the conditions

$$\beta_w(t) \equiv 0, \quad \gamma_{ref}(t) \neq 0 \tag{3}$$

an autopilot structure for the angle of roll channel could be chosen in the form presented in Fig. 1, where OBJ is an object with MM (1); ACT is an actuator; i_e, ρ_e are the autopilot coefficients.

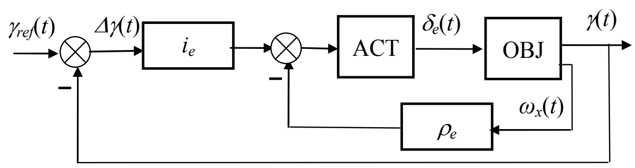

Fig.1. An integral autopilot structure for the angle of roll channel

To simplify the analytical investigation:

1) nonlinearities and delay in the autopilot structure are not taken into account;

2) actuator, roll and rate-of-roll meters are assumed to be ideal.

But it is necessary to note that these factors are essentially affected the system control dynamic properties. It is difficult , to take into account this influence analytically so the results derived on the base of a simplified MM of the system then are subjected to mathematical and imitative simulation and to development test.

A differential equation that determines the motion of coordinate $\gamma = \gamma(t)$ as the reaction to $\gamma_{ref}(t)$ in the case under consideration can be written in the form

$$[T^2 D^2 + 2\xi T D + 1]\gamma(t) = \gamma_{ref}(t)\,, \tag{4}$$

where $T = \sqrt{\dfrac{1}{i_e c_3}}$, $\xi = \dfrac{(c_1 + \rho_e c_3)}{2\sqrt{i_e c_3}}$, $D = \dfrac{d}{dt}$ is the differentiation operator.

In Fig. 2 the oscillograms for the coordinates of control system motion at the angle of roll channel are presented. Here the autopilot coefficients are the following:

$$i_e = 0{,}2\,; \quad \rho_e = 0{,}055\,. \tag{5}$$

From Eq. (4) with coefficients (5) and from oscillograms in Fig. 2 it is seen that the first three specifications for dynamic accuracy are satisfied.

In this paper, we will not pay our attention to the choice of concrete autopilot coefficients numbers (5). This choice is dictated by a set of conditions. We will take them as preassigned ones.

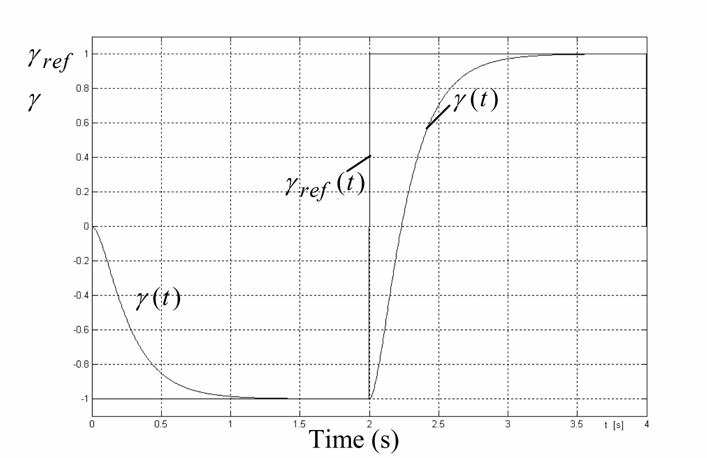

Fig.2. Oscillograms for coordinates of control system motion in the angle of roll channel

5. AN AUTOPILOT STRUCTURE CHOICE FROM REQUIREMENTS TO THE DISTURBANCE $\beta_w(t)$ AND ANALYSIS OF CONTROL SYSTEM DYNAMICS

Let us consider the conditions

$$\beta_w(t) \neq 0, \quad \gamma_{ref}(t) \equiv 0\,, \tag{6}$$

and the autopilot structure presented in Fig.1. For this case, the coordinate $\gamma(t)$ motion depending on the disturbance is determined by the differential equation in the following form

$$[T^2 D^2 + 2\xi T D + 1]\gamma(t) = k_{st}\beta_w(t)\,, \tag{7}$$

where T, ξ are determined in (4), $k_{st} = \dfrac{c_2}{i_э c_3}$.

For the object dynamic coefficients (2) and the autopilot coefficients (5) we get the number $k_{st} = 0.75$. It means that requirement 4 is not satisfied. To satisfy this requirement, we add an integral link respectively to the misalignment $\Delta\gamma(t)$. As a result, we obtain the autopilot structure presented in Fig. 3 where k_{int} is the integral coefficient.

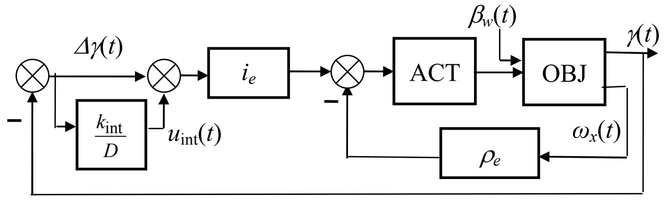

Fig. 3. Integral autopilot structure for the roll channel

As before we take the value of the integral coefficient as the preassigned one

$$k_{\text{int}} = 0,2\,. \tag{8}$$

For this case, the coordinate $\gamma(t)$ motion depending on the disturbance $\beta_w(t)$ is determined by the differential equation in the following form

$$(D^3 + a_2 D^2 + a_1 D + a_0)\gamma(t) = (b_0 D)\beta_w(t) \tag{9}$$

where $a_0 = k_{\text{int}} i_e c_3$; $a_1 = i_e c_3$; $a_2 = (c_1 + \rho_e c_3)$; $b_0 = c_2$.

From differential equation (9) it is clear that for the autopilot structure presented in Fig. 3 requirement 4 is satisfied.

To test requirements 5 and 6, we will get oscillograms for the coordinates of control system motion. Such oscillograms are presented in Fig. 4, where through $u_{\text{int}}(t)$ we denote a voltage from the output of integral link. From these oscillograms we see that requirement 4 is naturally satisfied but requirements 5 and 6 are not satisfied.

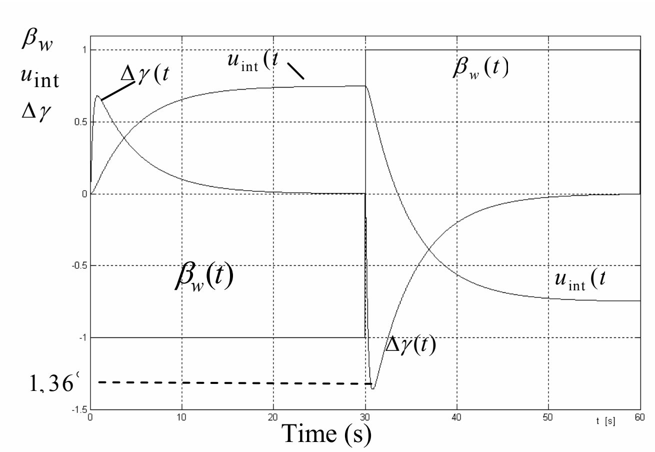

Fig. 4. Oscillograms for coordinates of control system motion for the integral autopilot structure

6. SOLUTION BASED ON THE PRINCIPLE OF VARIABLE STRUCTURE CONTROL SYSTEMS

The problem is that for the task solution all possibilities are exhausted: the structure is presented in Fig. 3, coefficients are prescribed by equalities (5) and (8). So it is remained only to change the autopilot structure during the dynamic process. It means that it is remained to use the principle of variable structure control systems. For example, if we nullify the value of the coefficient k_{int} then we get the autopilot structure without an integral, see Fig.1. This structure has the good speed of acting (see Fig. 2) but it does not provide the required astatism to the disturbance $\beta_w(t)$. The integral autopilot structure (see Fig. 3) provides the astatism condition but does not satisfy requirements relative to the speed of acting and the overshooting for the transient function on $\Delta\gamma(t)$ under condition of the step-function $\beta_w(t)$. It is necessary to combine positive properties for each structure and reduce negative ones. For this goal it is necessary to derive an algorithm for switching these structures during the dynamic process. Naturally, this algorithm has to be realizable.

Statement: For a case under consideration it is sufficient to nullify the integral device output in the autopilot at a moment when the disturbance $\beta_w(t)$ polarity changes.

The essence of the statement is clear. Really, if the disturbance $\beta_w(t)$ polarity changes (see Fig. 4) then the value $u_{\text{int}}(t)$ has to change its polarity too to compensate the $\beta_w(t)$ influence on the coordinate $\gamma(t)$, for example, from $(+u_{\text{int}}(t))$ to $(-u_{\text{int}}(t))$. But in the integral scheme (see Figs. 3 and 4) changing $(+u_{\text{int}}(t))$ to zero value increases not only the duration of transient function but also the overshooting. If at the time moment of the disturbance $\beta_w(t)$ polarity change the value $(+u_{\text{int}}(t))$ to nullify instantly then it is possible to expect the improvement of the transient function. It is confirmed by the oscillograms presented in Fig. 5. Here we can see that the overshooting of the coordinate $\Delta\gamma(t)$ is decreased to the permissible value 0.68°; the duration of transient function is decreased to the permissible 10 seconds.

So it is shown that all requirements are fulfilled with the algorithm. But for the realization aboard an aircraft it is necessary to solve some principle tasks. For example,

- in a real flight the disturbance $\beta_w(t)$ is not measured and so the information about the disturbance $\beta_w(t)$ polarity changing is absent;
- from the point of the reliability it is more advisable to nullify the integral not instantly but during a relatively small not zero time interval;
- it is necessary to eliminate false switching of the autopilot structures

All these questions will be considered in the next section.

7. REALIZATION OF THE ASTATIC CONTROL

7.1. The indication of the time moment for the disturbance $\beta_w(t)$ polarity changing

From oscillograms (see Fig. 4) it is possible to see that after changing the disturbance $\beta_w(t)$ polarity, the misalignment

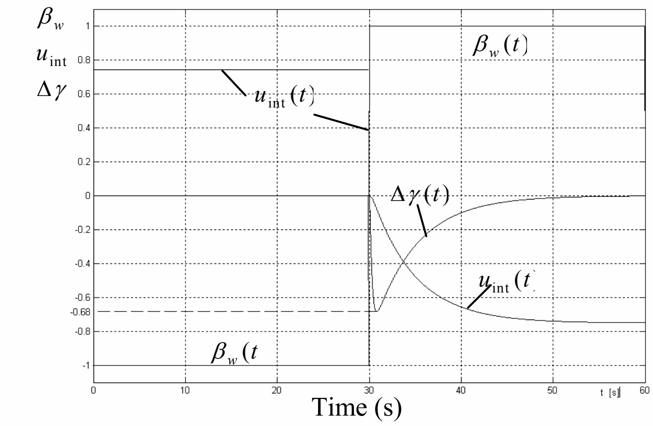

Fig. 5. Oscillograms for coordinates of control system motion in the integral autopilot structure with the ideal nullification

$\Delta\gamma(t)$ appears and this misalignment has the sign opposite to the sign of $u_{int}(t)$. Then in the integral scheme the misalignment $\Delta\gamma(t)$ goes to zero and $u_{int}(t)$ goes to the value that is opposite to initial sign. Therefore, the appeared difference of signs for the values $\Delta\gamma(t)$ and $u_{int}(t)$ indicates the time moment for changing the disturbance $\beta_w(t)$ polarity.

7.2. The solution of the task of nullifying the integral not instantly but during a relatively small not zero time interval

To solve the task it is possible to join a negative feedback to the integral link of the autopilot with the large coefficient. Such a feedback under its engagement nullifies quickly the integral output. After nullifying the feedback needs to be disconnected.

7.3. Eliminating false the autopilot structures switching

The misalignment $\Delta\gamma(t)$ does not keep the null position. This coordinate performs small or not small oscillations near the null position. There are a lot of reasons for these oscillations. Really, the most essential reason is the disturbance $\beta_w(t)$. But other relatively small reasons could lead to false a switching the autopilot structures. Here it is appropriately to use the principle of the "threshold restrictions" that is wildly used for solving the reliability problem. The sense is simple: to make the nullification scheme to work not at the moment $\Delta\gamma(t) \approx 0$ but after $|\Delta\gamma(t)| \geq \varepsilon$, $\varepsilon = \text{const} > 0$.

7.4. Analog solution for the astatic control algorithm

A possible analog solution for the astatic control algorithm is presented in Fig. 6 on the base of the Matlab + Simulink system. Here the time constant for the integral nullification is 0.01 s. As threshold restrictions are the values $|\Delta\gamma(t)| = 0.1^{\circ}$.

In Fig. 7 the oscillograms of the motion for the integral scheme with real nullification are presented. If we compare the oscillograms presented in Figs. 5 and 7.a, then it could be

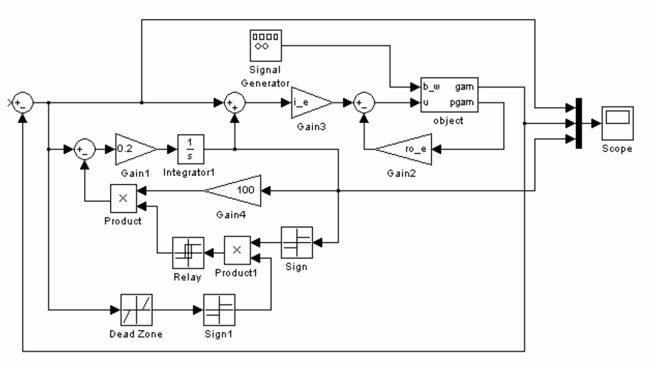

Fig. 6. Analog solution of the astatic control algorithm

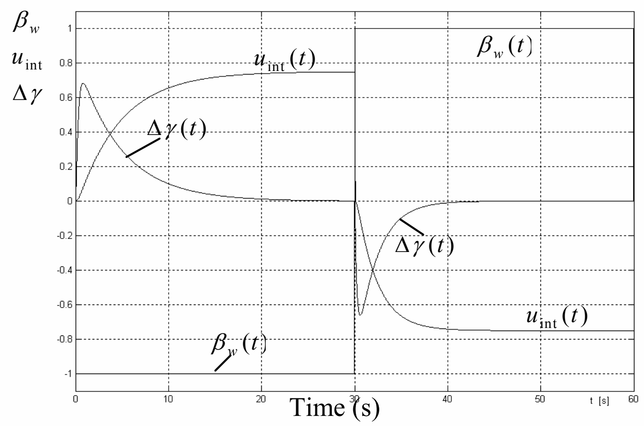

Fig. 7.a. Coordinates of control system motion the integral autopilot structure with the real nullification

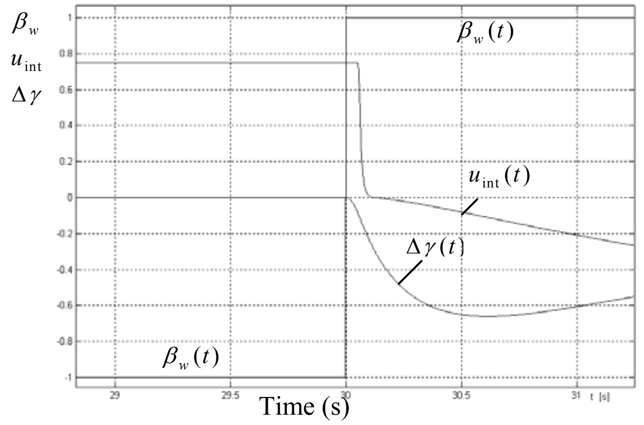

Fig. 7.b. Coordinates of a control system motion for the integral autopilot structure with the ideal nullification and with the time scale increased

remarked that real analog realization practically does not change the dynamic accuracy. In Fig. 7.b a piece of the same oscillograms is presented but with the time scale increased. Here we see that the beginning the integral output nullification is delayed to the point $|\Delta\gamma(t)| = 0.1^{\circ}$ and the process of nullification is not instantaneous but it is exponential.

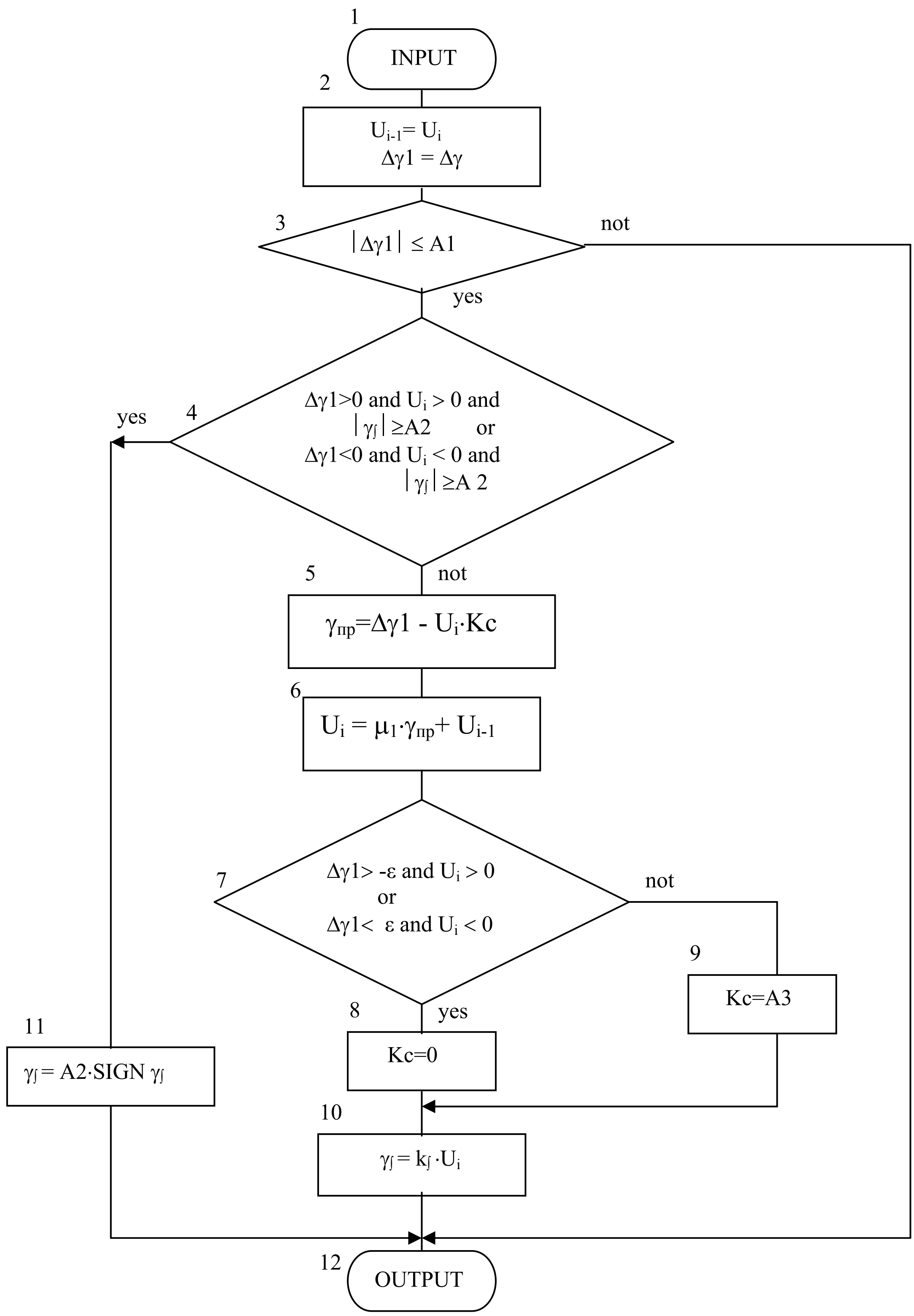

Fig. 8. Block-scheme for a possible digital realization of the astatic control algorithm

7.5. Digital solution for the astatic control algorithm

A possible digital solution for the astatic control algorithm aboard aircraft is presented in Fig. 8. Here the value ε determines the threshold restrictions $|\Delta\gamma(t)| = \varepsilon$ that eliminate the autopilot structures from false switching. The value A_2 determines a limitation of the integral output that is needed in the control system for the concrete aircraft. The time constant for the integral output nullification $T = 1/B$.

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