DYNAMICS FEATURES OF A VIBRATING MACHINE WITH ELASTIC ELEMENT HAVING EXPONENTIAL CHARACTERISTIC OF RESILIENT FORCE

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Abstract

The article analyzes the possibility of using nonlinear elastic elements as a suspension of the working element of resonant vibrating machines with two unbalance vibration exciters is analyzed. The elastic characteristic of the suspension is described by an exponential law, which ensures that the natural frequency remains unchanged regardless of the system mass. Static characteristics of the vibration exciter motors are taken into account. A system of differential equations describing movement of the system depending on the processed material mass is obtained. Amplitude-frequency characteristics depending on the power supply voltage, as well as on the debalance rotational speed are obtained for different values of material mass. The stability of the obtained periodic solutions is analyzed. The constancy of resonant amplitude and frequency of the working element vibrations at various values of material mass is shown. The results obtained confirm the advisability of using an equal-frequency suspension of the working element for resonant vibrating machines.

Key words

vibrating machine, non-linear elastic element, resonance.

This article is dedicated to the memory of Professor Ilya Izrailevich Blekhman, who died from COVID-19 at the very beginning of this year (2021). We were all connected with Professor Ilya Blekhman not only by common scientific interests, but also by long-term personal relationships. Almost all of our research, and not only on this issue, we discussed with him in personal meetings. One of us, Professor Grigory Panovko, knew Ilya Blekhman from childhood. The families of Professor Yakov Panovko (father of Grigory) and Ilya Blekhman have been friends for many decades. The memory of Ilya Blekhman - an outstanding mechanical scientist, who had the deepest understanding of mechanics and its manifestations in nature and technology, a very bright and noble person with amazing intuition, will always be in our hearts. And we will pass this memory on to our colleagues and students.

1 Introduction

One of the main ways to excite vibrations of technological vibromachines is force excitation using unbalanced vibrators [Blekhman, 1994; Vaisberg, 1986; Lavendel, 1981]. Usually vibromachines of this class operate in a above-resonant mode. However, this mode requires the use of overpowered electric motors, which is necessary to overcome the resonance.

The use of the most effective resonance modes is sophisticated by their instability, which appears even with minor changes in the system parameters, as well as due to nonlinearities arising from the working element interaction with the processed material and/or with a vibration drive of limited power [Gouskov and Panovko, 2012; Cveticanin et al., 2017; Nayfeh and Mook, 1995; Sinha et al., 2020; Yaroshevich, 2020]. All this increases the complexity of maintaining near resonance modes and leads to the need to use systems for controlling the electric motors rotational speed [Fradkov et al., 2013; Panovko et al., 2015].

This article explores the possibility of using a nonlinear elastic suspension in order to ensure the constancy of the machine resonant frequency at variable mass of the processed material.

The design scheme of a single-mass vibrating machine is shown in Fig. 1. The working element is an absolutely rigid tray *1* mounted on non-linear springs *2* with the identical resilient force characteristics. To excite oscillations, two identical unbalanced vibration exciters *3* are used, connected to a single source of electric voltage U. The driving torques of the motors $M(U)$ ensure their synchronous in-phase rotation in opposite directions, which creates a unidirectional disturbing force. It is assumed that the mass of material *4* during vibration treatment can slowly change over time. The working element oscillatory movement only in the vertical direction is considered. The moment of friction forces on the motor shaft is taken into account, as well as the dissipative force in the springs.

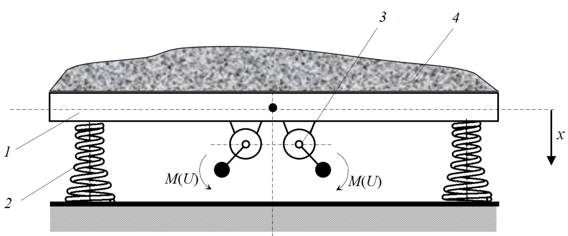

Figure 1. Vibrating machine design scheme.

The main idea in the selection of the type of elastic suspension nonlinear characteristics is based on the principle of an equal-frequency shock absorber, which ensures the constancy of the natural vibration frequency of the system regardless of the change in mass [Panovko and Gubanova, 1987]. Assuming that the elastic properties of both springs are equal, their total reaction will be described by the function $F(X)$, where X is the shortening (deflection) of the equivalent spring under applied load $F = mg,\ \ m = m_0 g\ (1 + \mu),\ \ \mu = m_w/m_0$, where m_0 is the mass of the working element with vibration exciters; $m_w \in (0,\ m_{w\,\max})$ is the processed material mass. Vibrations of the system will be considered in the direction of the x axis relative to the static equilibrium position of the system under the applied load. In accordance with [Panovko and Gubanova, 1987], the elastic characteristic of the equivalent spring relative to the coordinate $x = X - X_{st}$ (X_{st} - static deflection) is determined by the expression

$$F\left(x\right) = m_0 g \left(1+\mu\right) \left[\exp\left(\frac{p^2 x}{g}\right) - 1\right],$$

p – resonance frequency.

2 Equations of motion

In the case of equality of vibration exciters inertial and torque characteristics and synchronous in-phase rotation of the debalances, the motion of the system will be described by the following system of differential equations

$$\begin{cases} \left(1+\mu\right) m_0 \ddot{x} + b\dot{x} + m_0\left(1+\mu\right) g \exp\left(p^2 x/g\right) - \\ \quad - \left(1+\mu\right) m_0 g = -m_e e \left[\ddot{\varphi}\cos\varphi - \left(\dot{\varphi}\right)^2 \sin\varphi\right], \\ J\ddot{\varphi} + m_e e\left[\ddot{x}\cos\varphi + g\cos\varphi\right] = M(U) - M_C, \end{cases} \tag{1}$$

where $\varphi = \varphi(t)$ – debalances rotation angle, J – debalance inertia moment relative to its axis of rotation, e – debalances eccentricity, b – damping coefficient of equivalent spring, $M(U) = c_0 U\left(1 - \dot{\varphi}/(c_1 U)\right)$ – driving torque, U – power supply voltage, $c_0,\ c_1$ - electric constants of the motor; $M_C = \gamma_C \dot{\varphi}^2$ – moment of friction forces in the debalances bearings, γ_C– coefficient of friction.

To present the results in general form, let us bring system (1) to a dimensionless form:

$$\begin{cases} \xi'' + \frac{2\zeta}{(1+\mu)}\xi' - \exp\left(-\xi\right) + 1 = \\ \quad = -\frac{\varepsilon}{(1+\mu)}\left[\varphi''\cos\left(\varphi\right) - \left(\varphi'\right)^2 \sin\left(\varphi\right)\right], \\ \varphi'' + \eta\left[\xi''\cos\left(\varphi\right) + \cos\left(\varphi\right)\right] = \\ \quad = \gamma_0\left(u - \frac{\varphi'}{\omega_0}\right) - \gamma(\varphi')^2, \end{cases} \tag{2}$$

where

$$x = \frac{g}{p^2}\xi;\ t = \tau/p;\ U = U_0 u;\ \zeta = \frac{b}{2m_0 p};\ \gamma = \frac{\gamma_C}{J};$$
$$\varepsilon = \frac{m_e}{m_0}\frac{ep^2}{g};\ \eta = \frac{m_e e g}{Jp^2};\ \gamma_0 = \frac{c_0 U_0}{Jp^2};\ \omega_0 = \frac{U_0}{c_1 p}.$$

3 Simulation results analysis

The system of equations (2) was solved by Newton's method in combination with the parameter continuation method [Nayfeh and Balachandran, 2004] for the following values of the system parameters: $\zeta = 0.025$; $\varepsilon = 0.29$; $\eta = 0.035$; $\omega_0 = 4.23$; $\gamma_0 = 0.55$; $\gamma = 0.04$. The μ parameter had been varied from 0 to 0.5. Fig. 2 shows the graphs of dimensionless amplitudes (half of peak-peak amplitudes) of the working element vibrations A_ξ depending on the dimensionless voltage u at various values of the processed material relative mass μ.

Here and in the following figures, numbers indicate: 1 - $\mu = 0$; 2 - $\mu = 0.16$; 3 - $\mu = 0.33$; 4 - $\mu = 0.5$.

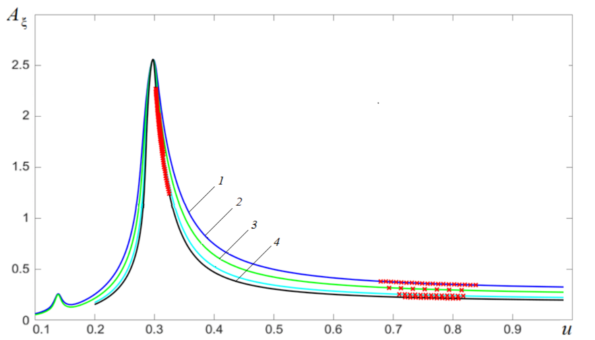

Figure 2. Vibration amplitude A_ξ depending on the power supply voltage u: 1 - $\mu = 0$; 2 - $\mu = 0.16$; 3 - $\mu = 0.33$; 4 - $\mu = 0.5$.

One can see that the amplitudes maxima for all considered values of the material mass appear at the same value of voltage $u \approx 0.3$. Unstable states of the system are shown by bold lines and crosses. Note that unstable modes in the main resonance area appear only at $\mu \geq 0.33$, which is associated with an increase in the total moment of all resistance forces.

Fig. 3 shows graphs of dimensionless amplitudes depending on the average rotation speed of the debalances ω_{avg} at various values of the material mass. It is important to note here that for all graphs the skeleton curves (dashed line) coincide with each other.

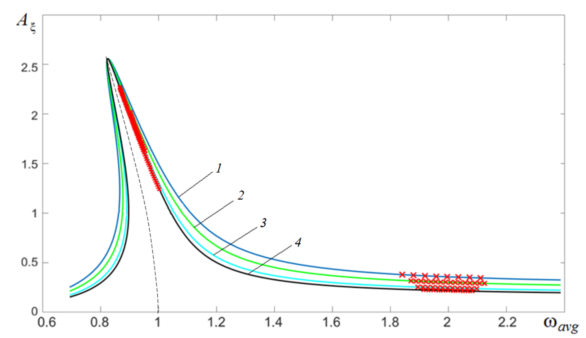

Figure 3. Vibration amplitude A_ξ depending on average rotation speed ω_{avg}: 1 - $\mu = 0$; 2 - $\mu = 0.16$; 3 - $\mu = 0.33$; 4 - $\mu = 0.5$.

Fig. 4 shows the evolution of multiplicators for $\mu = 0$ (Fig. 4(a)) and $\mu = 0.5$ (Fig. 4(b)) for all values of the material mass from a given range, a period doubling bifuraction is observed, and Neimark-Sacker bifurcation also occurs at $\mu = 0.35$.

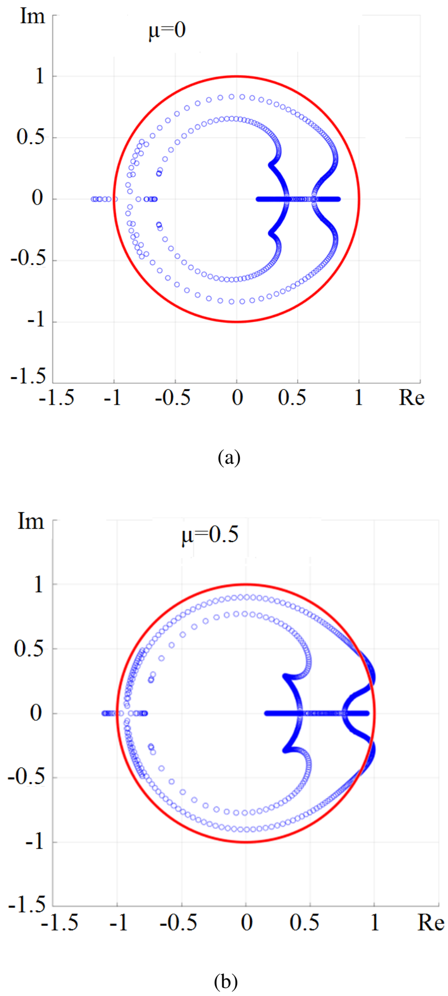

Figure 4. Multiplicators evolution at $\mu = 0$ (a) and $\mu = 0.5$ (b).

4 Conclusion

The dynamics of a vibrating machine with an unbalance vibration exciter and a nonlinear elastic suspension of the working element has been investigated. The resilient force characteristic of the elastic element is described by exponential law. As a result of the numerical solution of the obtained equations system, taking into account the static characteristic of the driving motor, the constancy of the resonance frequency and vibrations amplitude was established, regardless of the system mass in the selected range of its variation. The results obtained show that the use of such a spring makes it possible to obtain a constant resonant vibration frequency of the machine in a wide range of material mass changes.

Acknowledgements

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